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Research paper

2D affine transform parameters by Gaussian elimination with pivoting

Tariq N. Ataiwe[1], Israa Hatem[2], Hisham M. Jawad Al Sharaa[3]

Abstract: In many geomatics, computer vision, and computer-aided applications, coordinate transformations are needed to transform from one coordinate system to another, especially in geodesy and photogrammetry. In photogrammetry one of the important coordinates transformation methods used to transform photo coordinates is the 2D affine transformation which takes into consideration the change in the differences in scale factor in the x and y directions. In this paper, a new method for computing the 2D affine transform parameters will be introduced, the problem of the 2D affine transform method has been solved by Gaussian elimination with pivoting. We have derived equations by which to find transformation parameters. Geometric transformation is a technique used to define the properties of common features between different images using the same coordinates basis, This method can be effectively used in image processing and computer vision to facilitate the computation process throughout eliminating the need for solving the inverse of the matrix.

Keywords: 2D, affine, Gaussian, photogrammetry, pivoting, transform

[1]Prof., PhD., Eng., University of Technology – Iraq, Civil Engineering Department, Baghdad, Iraq, e-mail: tariq.n.ataiwe@uotechnology.edu.iq, ORCID: 0000-0002-5838-3687
[2]Assist. Prof., Eng., University of Technology – Iraq, Civil Engineering Department, Baghdad, Iraq, e-mail: israa.h.mohammed@uotechnology.edu.iq, ORCID: 0000-0002-8904-0097
[3]Assist. Prof., PhD., Eng., University of Technology – Iraq, Civil Engineering Department, Baghdad, Iraq, e-mail: Hisham.M.AlSharaa@uotechnology.edu.iq, ORCID: 0000-0002-1026-8866

1. Introduction

The frequent problem in Photogrammetry is converting an arbitrary (unknown) rectangular coordinate system to another calibrated (known) rectangular coordinate system [1, 2]. In computer vision and photogrammetry, unknown coordinates are usually selected for a series of points, with respect to an arbitrary rectangle coordinate system [3, 4]. 2D coordinate transformation is the process of transferring data between different coordinate systems [5, 6].

Coordinate transformations can be considered the mathematical model through which the amount of transformation of any element can be numerically recognized by (reflecting, rotating, translating, and changing the scale).

For the affine transformation, there are two parameters of scale factors, one in the x-axis and the second one in the y-axis [7–10]. In photogrammetry and computer vision frequently employ this transformation in the process of interior orientation [10, 11]. When one two-dimensional coordinate system (arbitrary coordinate) is projected onto another nonparallel system (calibrated coordinates) [12, 13]. This type of transformation may convert coordinates for geodetic applications and is frequently used in photogrammetry and cadastral surveying [14–17] and is considered one of the methods in the geometric correction in GIS and image processing [18, 19].

The points themselves must have their known coordinates in two systems in order to complete the procedure, the arbitrary and calibrated coordinate system, so that a number of repeated fuctions in the x, and y coordinates are achieved. This system of functions can be solved directly with linear and nonlinear least squares [20, 21], or the transformation coefficients can be estimated by the multiple regression coefficients computation method [22, 23].

It is challenging to estimate all six affine parameters using the traditional Radon transform methods without iterations [24]. Hence, novel methods and approaches in this field have been explored and suggested, such as utilizing a line integral transform that is invariant to affine deformation to compute a 2D matrix of affine invariants are efficient [25], and an approach based on the Radon transform is proposed to directly estimate the six affine parameters in a novel method. Such new methods are useful for 2D affine transformation parameters, which enhances the accuracy and efficiency.

In this research, a new approach for computing the transformation parameters is presented which can give a fast and direct solution that can be used in computer vision and image processing.

2. Transformation parameters

In general, there are four sets of parameters that might be involved in the coordinates transformation:

1. Scale change "scaling", occurs due to a change in size which might be one-dimensional, two-dimensional, or three-dimensional.
2. Reflection this occurs in the case of coordinates transformation, from a left-hand system to a another right-handed system and vice versa.

3. Rotation, this occurs if the coordinates axes of the two systems are not parallel to each other.
4. Translation, this occurs if the coordinate systems have a different origin [2].

2.1. Two-dimensional conformal coordinate transformation

The conformal transformation applied to the figures or shapes does not change their true form after applying transformation. To apply 2D conformal transformation, it is indispensable to know the coordinates of two points in at least two systems. The accuracy of the transformation is improved, if the points are chosen as much as possible far from each other [26, 27].

With reference to the Fig. 1.

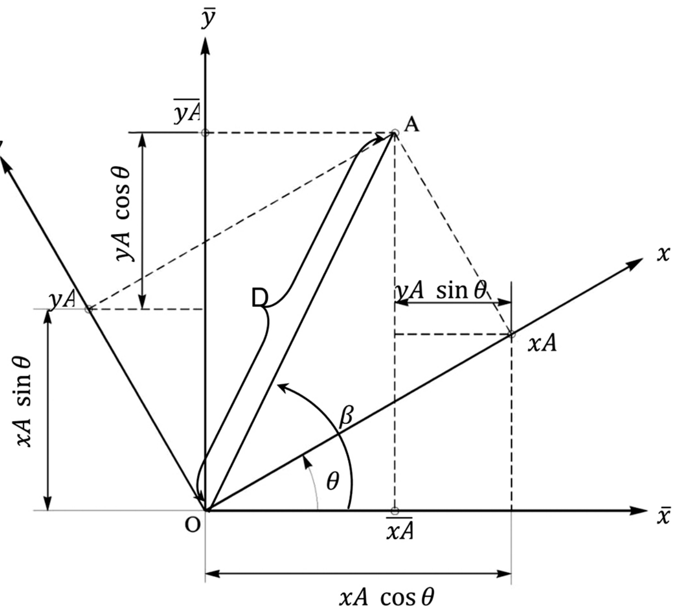

Fig. 1. Coordinate systems rotated by an angle θ

We have:

$$x = D\cos(\beta - \theta) \tag{2.1}$$

$$y = D\sin(\beta - \theta) \tag{2.2}$$

$$x = D\cos\beta\cos\theta + D\sin\beta\sin\theta \tag{2.3}$$

$$y = D\sin\beta\cos\theta - D\cos\beta\sin\theta \tag{2.4}$$

$$\overline{x} = D\cos\beta \tag{2.5}$$

$$\overline{y} = D\sin\beta \tag{2.6}$$

Substitute Eq. 2.5 and Eq. 2.6 into Eq. 2.3 Eq. and 2.4 we get:

$$x = \overline{x}\cos\theta + \overline{y}\sin\theta \tag{2.7}$$

$$y = \overline{y}\cos\theta - \overline{x}\sin\theta \tag{2.8}$$

Or in matrix form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix} \tag{2.9}$$

Or:

$$X = M\overline{X} \tag{2.10}$$

Regarding that M = Rotation matrix = Orthogonal matrix:

$$M^T = M^{-1} \tag{2.11}$$

$$\therefore\ \overline{X} = M^T X \tag{2.12}$$

After adding the scale factor:

$$\begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix} = s \begin{bmatrix} \cos\theta & -sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{2.13}$$

And after adding the translation:

$$\begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix} = s \begin{bmatrix} \cos\theta & -sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} Tx \\ Ty \end{bmatrix} \tag{2.14}$$

If $a = s\cos\theta$ and $b = s\sin\theta$ we get:

$$\begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} Tx \\ Ty \end{bmatrix} \tag{2.15}$$

$$\overline{x} = ax - by + Tx, \quad \overline{y} = bx + ay + Ty \tag{2.16}$$

To find the rotation angle and the scale factor:

$$\tan\theta = \frac{b}{a} \tag{2.17}$$

$$s = \sqrt{a^2 + b^2} \tag{2.18}$$

2.2. Two-dimensional 2D affine transformation

The transformation of 2D affine coordinates is only a small change of the 2D conformal or similarity transformation. Affine transformation is the existing relationship (or conversion to be made) between the 2D different coordinate systems at the origin point and in the direction of the axes and the scale. The scale is constant for each axis but not fixed for both axes (it has two scales). The two-dimensional affine transformation has five transformation parameters:

– 2 scales (S_X, S_Y)
– 2 translations (T_X, T_Y)
– 1 rotation θ.

Recall Eq. 2.14 and add 2 scales (S_X, S_Y).

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix} + \begin{bmatrix} Tx \\ Ty \end{bmatrix} \tag{2.19}$$

If $a_1 = S_x \cos\theta$, $b_1 = S_y \sin\theta$, $a_o = Tx$, $a_2 = S_x \sin\theta$, $b_2 = S_y \cos\theta$ and $a_o = Tx$, $b_o = Ty$ we get:

$$\begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix} = \begin{bmatrix} a_1 & -a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_o \\ b_o \end{bmatrix} \tag{2.20}$$

Or

$$\begin{aligned} \overline{x} &= a_o + a_1 x - a_2 \\ \overline{y} &= b_o + b_1 x + b_2 y \end{aligned} \tag{2.21}$$

3. Gaussian elimination with pivoting

In linear algebra, the Gaussian elimination with pivoting is a technique used to solve the inverse of the matrix. The mathematical derivation method that we implemented in this paper is presented in this section. Let's consider the following general linear system of equations.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \tag{3.1}$$

From these equations, three matrices can be extracted:

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{3.2}$$

In general, the solution of any system of linear equations having a form similar to the above form (number of equations = number of unknown) can be done by the triangular factorizations with Gauss elimination. The problem is to find the values of X and to solve it the following steps must be considered.

Given the $n(n+1)$ matrix, W contains the matrix A of order n in its first column and the vector b in its last column.

$$W = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1n} & w_{1,n+1} \\ w_{21} & w_{22} & \dots & w_{2n} & w_{2,n+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nn} & w_{n,n+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} & b_n \end{bmatrix} \tag{3.3}$$

Initialize the pivoting n-vector P

$$P_i = i, \quad i = 1, \dots, n$$

$$P = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix} \tag{3.4}$$

The triangular factorizations with Gauss elimination can be done in two steps, step one (factorization) and step two (forward elimination and backward elimination).

In step one, the application of factorization applies only to the coefficients of the matrix (A) by eliminating of x_1 using Eq. 3.3 as a pivoting value.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \dfrac{a_{21}}{a_{11}} & a_{22} - \dfrac{a_{21}}{a_{11}} a_{12} & \dots & a_{2n} - \dfrac{a_{21}}{a_{11}} a_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{a_{n1}}{a_{11}} & a_{n2} - \dfrac{a_{n1}}{a_{11}} a_{12} & \dots & a_{nn} - \dfrac{a_{n1}}{a_{11}} a_{1n} \end{bmatrix}$$

Or

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \grave{a}_{21} & \grave{a}_{22} & \dots & \grave{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \grave{a}_{n1} & \grave{a}_{n2} & \dots & \grave{a}_{nn} \end{bmatrix} \tag{3.5}$$

Where $\grave{a}_{22} = a_{22} - \dfrac{a_{21}}{a_{11}} a_{12}$ and so on.

This procedure was repeated for the remaining equation and from x_1 to x_n. At the end of the factorization steps the final matrices look like the following:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \grave{a}_{21} & \grave{a}_{22} & \dots & \grave{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1}^{n-2} & a_{n2}^{n-1} & \dots & a_{nn}^{n-1} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{3.6}$$

In step two (forward elimination and backward elimination) after taking into consideration the arrangement of pivoting, the forward elimination will be applied to the following matrices:

$$A = \begin{bmatrix} 1 & 0 \dots & 0 \\ \grave{a}_{21} & 1 \quad \dots & 0 \\ \vdots & \vdots \quad \vdots & \vdots \\ a_{n1}^{n-2} & a_{n2}^{n-1} \quad \dots & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{3.7}$$

Where

$$\begin{aligned} y_1 &= b_1 \\ y_2 &= b_2 - \grave{a}_{21} b_1 \\ &\vdots \\ y_n &= b_n - a_{n1}^{n-2} b_1 + a_{n2}^{n-1} b_2 + \dots \end{aligned}$$

The final step is backward elimination which applies to the following matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & \grave{a}_{22} & \dots & \grave{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & a_{nn}^{n-1} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \tag{3.8}$$

Since it only has one value, the final unknown will be calculated first.

$$x_n = \frac{y_n}{a_{nn}^{n-1}}$$

The second unknown will be solved as x_{n-1} using the x_n that solved previously, and the following general formula can be used to solve the remaining unknowns.

$$x_i = \frac{y_n - \sum\limits_{j=i+1}^{n} a_{ij}^{(i-1)} x_j}{a_{ii}^{(i-1)}}, \qquad \text{for } i = n-1, n-2, \dots, 1$$

3.1. Solve the affine transformation with Gaussian elimination

In this section, the derivation of the six 2D affine linear transformation parameters is described In general, the 2D transformation formula between any two coordinate systems has two equations, one in the X direction (X axis) and the other in the (Y axis) Y direction. In general, and can be used between points in the two systems to find the relationship between the systems (transformation parameters). The general form formula of the 2D affine transformation for the X direction is as follows:

$$X = a_o + a_1 x + a_2 y$$

And in matrix form:

$$\underset{A}{\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{bmatrix}}, \underset{L}{\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ \vdots \\ X_n \end{bmatrix}} \tag{3.9}$$

An equation in matrix form is of the form $AX = B$, where A represent the matrix of parameters, X is the vector of unknowns in the column, and B is the vector of function value in the column on the right side of the equations in a system.

$$AX - B = V$$
$$N = A^T A$$
$$D = A^T B$$
$$NX = D$$
$$X = N^{-1} D$$

By solving the normal equation the following matrices can be derived:

For the matrix N, the final solution of the X-direction has 3 transformation parameters so that the dimension of the N matrix will be 3×3 regardless of the number of points between the two systems

$$N = \begin{bmatrix} n & \sum_1^n x & -\sum_1^n y \\ \sum_1^n x & \sum_1^n x^2 & \sum_1^n xy \\ \sum_1^n y & \sum_1^n xy & \sum_1^n y^2 \end{bmatrix} \tag{3.10}$$

and so the D matrix has the dimension 3×1

$$D = \begin{bmatrix} \sum_1^n X \\ \sum_1^n xX \\ \sum_1^n yX \end{bmatrix} \tag{3.11}$$

substitution matrices N and D in Eq. 3.3 yield

$$W = \begin{bmatrix} n & \sum_1^n x & \sum_1^n y & \sum_1^n X \\ \sum_1^n x & \sum_1^n x^2 & \sum_1^n xy & \sum_1^n xX \\ \sum_1^n y & \sum_1^n xy & \sum_1^n y^2 & \sum_1^n yX \end{bmatrix} \tag{3.12}$$

When solving the Eq. 3.12 by step two (forward elimination and backward elimination) after taking into consideration the arrangement of pivoting which gives:

$$= \begin{bmatrix} n & \sum_1^n x & \sum_1^n y & \sum_1^n X \\ \dfrac{\sum_1^n x}{n} & \sum_1^n x^2 - \dfrac{\sum_1^n x^2}{n} & \sum_1^n xy - \dfrac{\sum_1^n x \sum_1^n y}{n} & \sum_1^n xX - \dfrac{\sum_1^n x}{\sum_1^n X} \\ \dfrac{\sum_1^n y}{n} & \sum_1^n xy - \dfrac{\sum_1^n x \sum_1^n y}{n} & \sum_1^n y^2 - \dfrac{\sum_1^n y^2}{n} & \sum_1^n yX - \dfrac{\sum_1^n y \sum_1^n X}{n} \end{bmatrix} \tag{3.13}$$

And solving will give:

$$\Rightarrow N_{33} = \frac{n \cdot \sum y^2 - \left(\sum y\right)^2}{n} - \frac{\left(n \cdot \sum xy - \sum x \cdot \sum y\right)^2}{n^2 \cdot \sum x^2 - n \cdot \left(\sum x\right)^2} \tag{3.14}$$

And

$$\begin{aligned} D_{31} = {} & \frac{n \cdot \sum yX - \sum y \cdot \sum X}{n} - \\ & - \left[\frac{\left(n \cdot \sum xy - \sum x \cdot \sum y\right)\left(n \cdot \sum xX - \sum x \cdot \sum X\right)}{n^2 \cdot \sum x^2 - n \cdot \left(\sum x\right)^2}\right] \end{aligned} \tag{3.15}$$

From the following equations, the parameters of the X-axis can be derived.

$$\Rightarrow a_2 = \frac{D_{31}}{N_{33}} \tag{3.16}$$

$$a_1 = \frac{n \cdot \sum xX - \sum x \cdot \sum X - \left(n \cdot \sum xy - \sum x \cdot \sum y\right) \cdot a_2}{n \cdot \sum x^2 - \left[\sum x\right]^2} \tag{3.17}$$

$$a_0 = \frac{\sum X - \sum x \cdot a_1 - \sum y \cdot a_2}{n} \tag{3.18}$$

The first unknown a_2 represent the coefficient of x, the second unknown a_2 represent the coefficient of y, the second unknown a_0 represent the translation in the X-direction.

Similarly for Y-direction:

$$Y = b_o + b_1 x + b_2 y =$$

$$= \begin{bmatrix} n & \sum_1^n x & \sum_1^n y & \sum_1^n Y \\ \frac{\sum_1^n x}{n} & \sum_1^n x^2 - \frac{\sum_1^n x^2}{n} & \sum_1^n xy - \frac{\sum_1^n x \sum_1^n y}{n} & \sum_1^n xY - \frac{\sum_1^n x}{\sum_1^n Y} \\ \frac{\sum_1^n y}{n} & \sum_1^n xy - \frac{\sum_1^n x \sum_1^n y}{n} & \sum_1^n y^2 - \frac{\sum_1^n y^2}{n} & \sum_1^n yY - \frac{\sum_1^n y \sum_1^n Y}{n} \end{bmatrix}$$

The final equations in the Y-direction are:

$$\Rightarrow N_{33} = \frac{n \cdot \sum y^2 - \left(\sum y\right)^2}{n} - \frac{\left(n \cdot \sum xy - \sum x \cdot \sum y\right)^2}{n^2 \cdot \sum x^2 - n \cdot \left(\sum x\right)^2} \tag{3.19}$$

$$D_{31} = \frac{n \cdot \sum yY - \sum y \cdot \sum Y}{n} - \left[\frac{\left(n \cdot \sum xy - \sum x \cdot \sum y\right)(n \cdot \sum xY - \sum x \cdot \sum Y)}{n^2 \cdot \sum x^2 - n \cdot \left(\sum x\right)^2}\right] \tag{3.20}$$

From the following equations, the parameters of the Y-axis can be derived.

$$b_2 = \frac{D_{31}}{N_{33}} \tag{3.21}$$

$$b_1 = \frac{n \cdot \sum xY - \sum x \cdot \sum Y - \left(n \cdot \sum xy - \sum x \cdot \sum y\right) \cdot b_2}{n \cdot \sum x^2 - \left[\sum x\right]^2} \tag{3.22}$$

$$b_0 = \frac{\sum Y - \sum x \cdot b_1 - \sum y \cdot b_2}{n} \tag{3.23}$$

3.2. Numerical example

The following Table 1. represents the 2D Cartesian coordinates in two systems, it is desired to find the transformation coordinates.

Table 1. 2D cartesian coordinates in two systems

Point	**X (m)**	**Y (m)**	**x (m)**	**y (m)**
A	125	281	10	50
B	123	364	90	120
C	178.8	378.5	150	77

Solution:
According to the new method presented in this paper, the following matrix can be calculated:

$$W = \begin{bmatrix} n & \sum_1^n x & \sum_1^n y & \sum_1^n X \\ \sum_1^n x & \sum_1^n x^2 & \sum_1^n xy & \sum_1^n xX \\ \sum_1^n y & \sum_1^n xy & \sum_1^n y^2 & \sum_1^n yX \end{bmatrix}$$

Solving the matrix by the previous equation the following results can be found:

$$W = \begin{bmatrix} 3 & 250 & 247 & 426.8 \\ 250 & 30700 & 22850 & 39140 \\ 247 & 22850 & 22829 & 34777.6 \end{bmatrix}$$

Applying these values in the following equations

$$\Rightarrow N_{33} = \frac{n \cdot \sum y^2 - \left(\sum y\right)^2}{n} - \frac{\left(n \cdot \sum xy - \sum x \cdot \sum y\right)^2}{n^2 \cdot \sum x^2 - n \cdot \left(\sum x\right)^2}$$

$$\Rightarrow N_{33} = \left(\frac{3 \cdot 22829 - (247)^2}{3}\right) - \left(\frac{\left((3 \cdot 22850 - 250 \cdot 247)^2\right)}{\left(3^2 \cdot 30700 - 3 \cdot (250)^2\right)}\right) = 1971.945946$$

$$D_{31} = \frac{n \cdot \sum yX - \sum y \cdot \sum X}{n} - \left[\frac{\left(n \cdot \sum xy - \sum x \cdot \sum y\right)\left(n \cdot \sum xX - \sum x \cdot \sum X\right)}{n^2 \cdot \sum x^2 - n \cdot \left(\sum x\right)^2}\right]$$

$$D_{31} = \left(\frac{(3 \cdot 34777.6 - 247 \cdot 426.8)}{3}\right) - \left[\frac{(3 \cdot 22850 - 250 \cdot 247)(3 \cdot 39140 - 250 \cdot 426.8)}{\left(3^2 \cdot 30700 - 3 \cdot (250)^2\right)}\right] =$$
$$= -1183.167568$$

$$a_2 = \frac{D_{31}}{N_{33}} = \frac{-1183.167568}{1971.945946} = -0.6$$

$$a_1 = \frac{n \cdot \sum xX - \sum x \cdot \sum X - \left(n \cdot \sum xy - \sum x \cdot \sum y\right) \cdot a_2}{n \cdot \sum x^2 - \left[\sum x\right]^2}$$

$$a_1 = \frac{(3 \cdot 39140 - 250 \cdot 426.8 - (3 \cdot 22850 - 250 \cdot 247) \cdot -0.6)}{\left(3 \cdot 30700 - 250^2\right)} = 0.5$$

$$a_0 = \frac{\sum X - \sum x \cdot a_1 - \sum y \cdot a_2}{n}$$

$$a_0 = \frac{(426.8 - 250 \cdot 0.5 - 247 \cdot -0.6)}{3} = 150$$

The final transformation parameters in the X-direction are:

$$a_0 = 150$$
$$a_1 = 0.5$$
$$a_2 = -0.6$$

When applying the derived equations in the Y-direction the parameters of b_0, b_1, b_2 are:

$$b_2 = 0.5$$
$$b_1 = 0.6$$
$$b_0 = 250$$

The example can also be computed by the least squares observation method and the result found to be in Table 2.

Table 2. Transformation parameters by least squares observation method

Transformation Parameter	**Values**
a_0	150
a_1	0.5
a_2	-0.6
b_0	250
b_1	0.6
b_2	0.5

The results after applying the regression with two independent variables the computed 2D transformation parameters are given in Table 3.

Table 3. Transformation parameters by regression with two independent variables

Transformation Parameter	Vlaues
a_0	150
a_1	0.5
a_2	-0.6
b_0	250
b_1	0.6
b_2	0.5

4. Discussion

This paper discusses a new approach to finding the transformation parameters between two coordinated systems, this is achieved by deriving new equations to compute the six affine 2D transformation parameters using Gaussian elimination with pivoting.

The transformation parameters in the X-direction derived by the method presented in the paper were 150, 0.5, and –0.6 for the a_0, a_1 and a_2 respectively, and for the Y-direction were 250, 0.6, and 0.5 for the transformation parameters b_0, b_1 and b_2 respectively, to justify the computed parameters, the 2D affine six transformation parameters were computed by traditional method using the Least Squares Observation method LSQ and Regression with Two Independent Variables RTIV the results were illustrated in Table 4.

Table 4. Comparison between the three methods

Transformation Parameter	LSQ	RTIV	New method
a_0	150	150	150
a_1	0.5	0.5	0.5
a_2	–0.6	–0.6	-0.6
b_0	250	250	250
b_1	0.6	0.6	0.6
b_2	0.5	0.5	0.5

The results presented in Table 4 indicated that the new method has the same accuracy as other traditional methods, and the advantage of the new method over other methods is the speed and ease which can be used to reduce the processing time in image processing and photogrammetry software.

5. Conclusions

The frequent problem in photogrammetry is converting one rectangular coordinate system to another. The operation requires that the points themselves have their known coordinates in both systems, the arbitrary and calibrated coordinate system.

This paper introduces a new method for computing the 2D affine transform parameter, the problem of the 2D affine transform method has been solved by Gaussian elimination with pivoting. In this paper, we have derived equations by which to find transformation parameters. The main usefulness of this method is that numerical calculations can be performed without solving the inverse matrix. The results indicated that the method has high accuracy computation to the 2D transformation parameters furthermore the method is regarded as fast and easy to implement which can be used to reduce the time and processing routines in image processing and photogrammetry software. The results presented from the method in this paper are completely competitive with other methods.

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