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Global stability of nonlinear systems with positive linear parts and positive dynamical feedbacks

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The global stability of nonlinear continuous-time standard and fractional order with linear dynamical positive feedback systems and of positive linear parts is investigated. New sufficient conditions for the global stability of this class of positive nonlinear systems are established. Procedures for computation of gains characterizing the class of nonlinear elements are given and illustrated on simple examples.

Key words: global stability, fractional order, positive, nonlinear, dynamical feedback, system

1. Introduction

In positive systems inputs, state variables and outputs take only nonnegative values for any nonnegative inputs and nonnegative initial conditions [1, 2, 10, 12, 16, 20]. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollutions models and electrical circuits. A variety of models having positive behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc. An overview of state of the art in positive systems theory is given in the monographs [1, 2, 10, 12].

Mathematical fundamentals of the fractional calculus are given in the monographs [17, 18]. The positive fractional linear systems have been investigated in [3–12, 20]. Positive linear systems with different fractional orders have been addressed in [8, 9, 20]. Linear positive electrical circuits have been investigated

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in [12]. The global stability of nonlinear systems with positive feedbacks and positive stable linear parts has been investigated in [5–7, 13] and the stability of discrete-time systems with delays in [19].

In this paper the global stability of nonlinear standard and fractional positive systems with dynamical positive feedbacks will be addressed.

The paper is organized as follows. In Section 2 the basic definitions and theorems concerning the positive standard and fractional orders linear systems are recalled. The global stability of nonlinear systems with standard positive dynamical linear systems and dynamical positive feedbacks is analyzed in Section 3. New sufficient conditions for the global stability of the class of nonlinear systems are established and procedures for computation of the gains characterizing the class of characteristics of nonlinear elements are given. In Section 4 the results of Section 3 are extended to fractional nonlinear positive systems. The procedures are illustrated by numerical examples. Concluding remarks are given in Section 5.

The following notation will be used: $\Re$ – the set of real numbers, $\Re_+^{n\times m}$ – the set of $n \times m$ real matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n – the set of $n \times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n – the $n \times n$ identity matrix.

2. Positive integer and different fractional orders linear systems

Consider the continuous-time linear system

$$\dot{x} = Ax + Bu, \tag{1a}$$

$$y = Cx, \tag{1b}$$

where $x = x(t) \in \Re^n$, $u = u(t) \in \Re^m$, $y = y(t) \in \Re^p$ are the state, input and output vectors and $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{p\times n}$.

Definition 1. [10, 12] *The continuous-time linear system (1) is called (internally) positive if $x(t) \in \Re_+^n$, $y(t) \in \Re_+^p$, $t \geqslant 0$ for any initial conditions $x(0) \in \Re_+^n$ and all inputs $u(t) \in \Re_+^m$, $t \geqslant 0$.*

Theorem 1. [10, 12] *The continuous-time linear system (1) is positive if and only if*

$$A \in M_n\,, \qquad B \in \Re_+^{n\times m}, \qquad C \in \Re_+^{p\times n}. \tag{2}$$

Definition 2. [10, 12] *The positive continuous-time system (1) for $u(t) = 0$ is called asymptotically stable if*

$$\lim_{t\to\infty} x(t) = 0 \ \textit{ for any } \ x(0) \in \Re_+^n\,. \tag{3}$$

Theorem 2. [10, 12] *The positive continuous-time linear system (1) for $u(t) = 0$ is asymptotically stable if and only if one of the following equivalent conditions is satisfied:*

1. *All coefficient of the characteristic polynomial*

$$p_n(s) = \det[I_n s - A] = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \tag{4}$$

are positive, i.e. $a_i > 0$ for $i = 0, 1, \ldots, n-1$.

2. *There exists strictly positive vector $\lambda^T = \begin{bmatrix} \lambda_1 & \cdots & \lambda_n \end{bmatrix}^T$, $\lambda_k > 0$, $k = 1, \ldots, n$ such that*

$$A\lambda < 0 \quad or \quad \lambda^T A < 0. \tag{5}$$

If the matrix A is nonsingular then we can choose $\lambda = A^{-1}c$, where $c \in \Re^n$ is strictly positive.

In this paper the following Caputo definition of the fractional derivative of α order will be used [10, 12]

$${}_0D_t^\alpha f(t) = \frac{\mathrm{d}^\alpha f(t)}{\mathrm{d}t^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot{f}(\tau)}{(t-\tau)^\alpha} \mathrm{d}\tau, \quad 0 < \alpha < 1, \tag{6}$$

where $\dot{f}(\tau) = \dfrac{\mathrm{d}f(\tau)}{\mathrm{d}\tau}$ and $\Gamma(z) = \displaystyle\int_0^\infty t^{x-1}e^{-t}\mathrm{d}t$, $\mathrm{Re}(x) > 0$ is the Euler gamma function.

Consider the fractional continuous-time linear system

$$\frac{\mathrm{d}^\alpha x(t)}{\mathrm{d}t^\alpha} = Ax(t) + Bu(t), \tag{7a}$$

$$y(t) = Cx(t), \tag{7b}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state, input and output vectors and $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{p\times n}$.

Definition 3. [10, 12] *The fractional system (7) is called (internally) positive if $x(t) \in \Re_+^n$ and $y(t) \in \Re_+^p$, $t \geqslant 0$ for any initial conditions $x(0) \in \Re_+^n$ and all inputs $u(t) \in \Re_+^m$, $t \geqslant 0$.*

Theorem 3. [10, 12] *The fractional system (7) is positive if and only if*

$$A \in M_n, \qquad B \in \Re_+^{n\times m}, \qquad C \in \Re_+^{p\times n}. \tag{8}$$

Definition 4. [10, 12] *The positive continuous-time system (1) for $u(t) = 0$ is called asymptotically stable if*

$$\lim_{t\to\infty} x(t) = 0 \ \textit{for any} \ x(0) \in \mathfrak{R}_+^n. \tag{9}$$

Theorem 4. [10, 12] *The positive linear system (1) is asymptotically stable if and only if one of the following equivalent conditions is satisfied:*

3. *All coefficient of the characteristic polynomial*

$$p_n(s) = \det[I_n s - A] = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \tag{10}$$

are positive, i.e. $a_i > 0$ for $i = 0, 1, \ldots, n-1$.

4. *There exists strictly positive vector $\lambda^T = \begin{bmatrix} \lambda_1 & \cdots & \lambda_n \end{bmatrix}^T$, $\lambda_k > 0$, $k = 1, \ldots, n$ such that*

$$A\lambda < 0 \quad \textit{or} \quad \lambda^T A < 0. \tag{11}$$

Theorem 5. *The positive system (1) (and (7)) is asymptotically stable if the sum of entries of each column (row) of the matrix A is negative.*

Proof. Using (11) we obtain

$$A\lambda = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ldots & \vdots \\ a_{n1} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} a_{11} \\ \vdots \\ a_{1n} \end{bmatrix} \lambda_1 + \ldots + \begin{bmatrix} a_{n1} \\ \vdots \\ a_{nn} \end{bmatrix} \lambda_n < \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{12}$$

and the sum of entries of each column of the matrix A is negative since $\lambda_k > 0$, $k = 1, \ldots, n$. The proof for rows is similar. □

3. Global stability of nonlinear systems with positive dynamical feedbacks

Consider the nonlinear feedback system shown in Fig. 1 which consists of the positive linear part, the nonlinear element with characteristic $u = f(e)$ and positive dynamical feedback. The linear part is described by the equations

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned} \tag{13a}$$

with interval matrices

$$\underline{A} \leqslant A \leqslant \overline{A}, \quad \underline{B} \leqslant B \leqslant \overline{B}, \quad \underline{C} \leqslant C \leqslant \overline{C}, \tag{13b}$$

where $x = x(t) \in \mathfrak{R}_+^{n_1}$, $u = u(t) \in \mathfrak{R}_+$, $y = y(t) \in \mathfrak{R}_+$ is the state, input and output vectors of the system (13) and $A \in M_{n_1}$, $B \in \mathfrak{R}_+^{n_1\times 1}$, $C \in \mathfrak{R}_+^{1\times n_1}$. It is

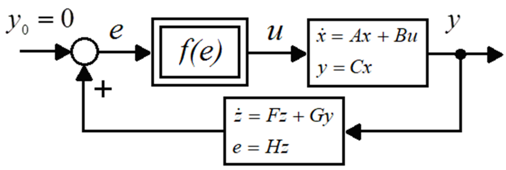

Figure 1: The nonlinear feedback system

assumed that the positive linear part described by (13) is asymptotically stable (the matrix $A \in M_{n_1}$ is Hurwitz).

The characteristic of the nonlinear element is shown in Fig. 2 and it satisfies the condition

$$0 \leqslant \frac{f(e)}{e} \leqslant k < \infty. \tag{14}$$

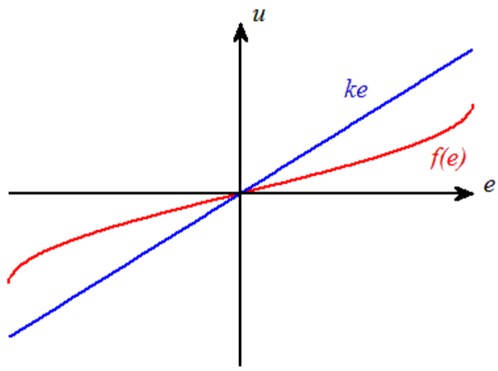

Figure 2: Characteristic of the nonlinear element

The positive feedback system is described by the equations

$$\begin{aligned} \dot{z} &= Fz + Gy, \\ e &= Hz, \end{aligned} \tag{15a}$$

with interval matrices

$$\underline{F} \leqslant F \leqslant \overline{F}, \quad \underline{G} \leqslant G \leqslant \overline{G}, \quad \underline{H} \leqslant H \leqslant \overline{H}, \tag{15b}$$

where $z = z(t) \in \Re_+^{n_2}$, $e = e(t) \in \Re_+$ are the state vector and output vectors.

It is assumed that the matrix $F \in M_{n_2}$ is also asymptotically stable.

From (13) and (15) we have

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \hat{A} \begin{bmatrix} x \\ z \end{bmatrix} + \hat{B}u, \tag{16a}$$

where

$$\hat{A} = \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix} \in M_n, \quad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \in \Re_+^{n_1 \times 1}, \quad n = n_1 + n_2. \tag{16b}$$

Definition 5. *The nonlinear positive system is called globally stable if it is asymptotically stable for all nonnegative initial conditions* $x(0) \in \Re_+^{n_1}$.

The following theorem gives sufficient conditions for the global stability of the positive feedback nonlinear system.

Theorem 6. *The nonlinear system consisting of the positive linear part (13) and the nonlinear element satisfying the condition (14) and positive asymptotically stable dynamical feedback system (15) is globally stable if the matrix*

$$\begin{bmatrix} \underline{A} & k\overline{B}\overline{H} \\ \overline{G}\overline{C} & \underline{F} \end{bmatrix} \in M_n \tag{17}$$

is asymptotically stable.

Proof. The proof will be accomplished by the use of the Lyapunov method [14, 15]. As the Lyapunov function $V(x,z)$ we choose

$$V(x,z) = \lambda^T \begin{bmatrix} x \\ z \end{bmatrix} \geqslant 0 \quad \text{for } x \in \Re_+^{n_1}, \quad z \in \Re_+^{n_2}, \tag{18}$$

where $\lambda \in \Re_+^n$ is strictly positive vector, i.e. $\lambda_k > 0$, $k = 1, \ldots, n$.

Using (18), (13) and (15) we obtain

$$\dot{V}(x,z) = \lambda^T \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \lambda^T \left\{ \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \right\}$$

$$= \lambda^T \left\{ \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} kHz \right\} = \lambda^T \begin{bmatrix} A & kBH \\ GC & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \leqslant 0 \tag{19}$$

since $Bu = Bf(e) \leqslant kBHz$ by the condition (14).

From (19) it follows that $\dot{V}(x,z) \leqslant 0$ if the condition (17) is satisfied and the nonlinear positive system is globally stable.

Theorem 6 can be applied to solve the following two problems.

Problem 1. *Given matrices A, B, C and F, G, H of the positive systems (13), (15) and the nonlinear characteristic* $u = f(e)$ *of the nonlinear element. Knowing the value of k satisfying the condition (14) check the global stability of the nonlinear system.*

Problem 2. *Given matrices A, B, C and F, G, H of the positive systems (13), (15) and the nonlinear characteristic $u = f(e)$ of the nonlinear element. Find the maximal value of k for which the characteristic $u = f(e)$ of the nonlinear element satisfies the condition (14).*

The Problem 1 can be solved by the use of the following:

Procedure 1.

Step 1. Knowing the characteristic $u = f(e)$ find the minimal value of k satisfying the condition (14).

Step 2. Using Theorem 6 find the sum of entries of each column(row) of the matrix (17). If all these sums are negative then the nonlinear system is globally stable.

The Problem 2 can be solved by the use of the following:

Procedure 2.

Step 1. Using Theorem 6 find the sum of entries of each column(row) of the matrix (17).

Step 2. Find the maximal value of k_c (k_r) for which the sums of entries of all columns (rows) are negative.

Step 3. Find $k_{\max} = \min(k_c, k_r)$.

The nonlinear system is globally stable for all nonlinear characteristics $u = f(e)$ satisfying the condition

$$0 < f(e) < k_{\max} e. \tag{20}$$

Remark 1. *The value of $k_{\max}$ depends only on the first n_1 rows and of the last n_2 columns of the matrix (17).*

Example 1. Consider the nonlinear system shown in Fig. 1 with linear positive parts described by (13) and (16) with

$$\begin{aligned}
&\underline{A} = \begin{bmatrix} -4.5 & 1 \\ 2 & -4.2 \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} -5 & 1.5 \\ 2.3 & -4.5 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix}, \\
&\underline{C} = \begin{bmatrix} 0.4 & 0.5 \end{bmatrix}, \qquad \overline{C} = \begin{bmatrix} 0.5 & 0.6 \end{bmatrix}
\end{aligned} \tag{21}$$

and

$$\begin{aligned}
&\underline{F} = \begin{bmatrix} -4 & 2 \\ 1.6 & -5 \end{bmatrix}, \quad \overline{F} = \begin{bmatrix} -4.2 & 2.2 \\ 1.8 & -5.3 \end{bmatrix}, \quad \underline{G} = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix}, \quad \overline{G} = \begin{bmatrix} 1 \\ 0.8 \end{bmatrix}, \\
&\underline{H} = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}, \quad \overline{H} = \begin{bmatrix} 0.5 & 0.4 \end{bmatrix},
\end{aligned} \tag{22}$$

respectively and the nonlinear element with characteristic satisfying the condition (14).

Case 1. Using $k = 1$ check the global stability of the nonlinear system.

In this case using (17), (21) and (22) we obtain

$$\begin{bmatrix} \underline{A} & k\overline{B}\overline{H} \\ \overline{G}\overline{C} & \underline{F} \end{bmatrix} = \begin{bmatrix} -4.5 & 1 & 0.3 & 0.24 \\ 2 & -4.2 & 0.4 & 0.32 \\ 0.5 & 0.6 & -4 & 2 \\ 0.4 & 0.48 & 1.6 & -5 \end{bmatrix}. \tag{23}$$

The sums of the entries of columns of the matrix (23) are: column 1: $= -1.6$, column 2: $= -2.12$, column 3: $= -1.7$, column 4: $= -2.44$. Therefore, by Theorem 6 the nonlinear system is globally stable.

Case 2. Find the maximal value of $k_{\max}$ satisfying the condition (14) for which the nonlinear system is globally stable.

Using Procedure 2 we obtain the following:

Step 1. The sums of entries of each column (row) of the matrix

$$\begin{bmatrix} \underline{A} & k\overline{B}\overline{H} \\ \overline{G}\overline{C} & \underline{F} \end{bmatrix} = \begin{bmatrix} -4.5 & 1 & 0.3k & 0.24k \\ 2 & -4.2 & 0.4k & 0.32k \\ 0.5 & 0.6 & -4 & 2 \\ 0.4 & 0.48 & 1.6 & -5 \end{bmatrix} \tag{24}$$

are: column 1: $= -1.6$, column 2: $= -2.12$, column 3: $= 0.7k - 2.4$, column 4: $= 0.56k - 3$, row 1: $= -2.96$, row 2: $= -1.48$, row 3: $= -3.5 + 0.54k$, row 4: $= -2.2 + 0.72k$.

Step 2. From Theorem 6 we have: for column 3: $k < 3.428$ and for column 4: $k < 5.357$ and for row 1: $k < 6.482$, row 2: $k < 3.0555$.

Step 3. The desired value of k is $k_{\max} = \min(k_c,\ k_r) = 3.0555$.

Therefore, the nonlinear system is globally stable for the nonlinear systems with characteristics satisfying the condition (14) with $k < 3.0555$. .

Remark 2. *From matrix (17) and the computation procedure it follows that the k depends only on the matrices F, G, H and is independent of the matrices A, C, G.*

4. Global stability of fractional positive nonlinear feedback dynamical systems

Consider the fractional nonlinear feedback system with the similar structure as shown in Fig. 1 which consists of the fractional positive linear part, the nonlinear element with characteristic shown in Fig. 2 and dynamical positive feedback element.

The fractional positive linear part is described by the equations (7) and the fractional positive feedback element by the equations

$$\frac{\mathrm{d}^{\beta} z}{\mathrm{d}t^{\beta}} = Fz + Gy, \tag{25a}$$

$$e = Hz, \tag{25b}$$

where $z = z(t) \in \Re_+^{n_2}$, $y = y(t) \in \Re_+$, $e = e(t) \in \Re_+$ are the state, input and output vectors and the fractional derivative is defined by (6).

It is assumed that the fractional positive linear systems (7) and (25) are asymptotically stable and the nonlinear characteristic $u = f(e)$ satisfies the condition (14).

From (7) and (25) we obtain

$$\begin{bmatrix} \dfrac{\mathrm{d}^{\alpha} x}{\mathrm{d}t^{\alpha}} \\ \dfrac{\mathrm{d}^{\beta} z}{\mathrm{d}t^{\beta}} \end{bmatrix} = \hat{A} \begin{bmatrix} x \\ z \end{bmatrix} + \hat{B}u, \tag{26}$$

where the matrices $\hat{A}$ and $\hat{B}$ are defined by (16b).

Definition 6. *The fractional nonlinear positive system is called globally stable if it is asymptotically stable for all nonnegative initial conditions* $x(0) \in \Re_+^{n_1}$.

The following theorem gives sufficient conditions for the global stability of the fractional positive nonlinear system.

Theorem 7. *The fractional nonlinear system consisting of the positive linear part (7), the nonlinear element satisfying the condition (14) and the positive fractional dynamical feedback (25) is globally stable if the matrix*

$$\begin{bmatrix} \underline{A} & k\overline{B}\overline{H} \\ \overline{G}\overline{C} & \underline{F} \end{bmatrix} \in M_n \tag{27}$$

is asymptotically stable.

Proof. The proof will be accomplished by the use of the Lyapunov method [14, 15]. As the Lyapunov function $V(x, z)$ we choose the scalar function defined by (18).

Using (18) and (26) we obtain

$$\begin{bmatrix} \frac{d^\alpha V(x,z)}{dt^\alpha} \\ \frac{d^\beta V(x,z)}{dt^\beta} \end{bmatrix} = \lambda^T \begin{bmatrix} \frac{d^\alpha x}{dt^\alpha} \\ \frac{d^\beta z}{dt^\beta} \end{bmatrix} = \lambda^T \left\{ \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \right\}$$
$$= \lambda^T \left\{ \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} kHz \right\} = \lambda^T \begin{bmatrix} A & kBH \\ GC & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \leqslant 0 \quad (28)$$

since $Bu = Bf(e) \leqslant kBHz$.

From (28) it follows that the fractional derivatives of the Lyapunov function are negative if the condition (27) is satisfied and the fractional nonlinear system is globally stable. □

For the fractional nonlinear feedback systems we can also formulate and solve similar two problems as for the standard nonlinear systems in Section 3.

Example 2. Consider the nonlinear fractional positive electrical circuit shown in Fig. 3 and Fig. 4 with positive and known resistances R_1, R_2, R_3, inductances L_1, L_2 and capacitance C.

The characteristic of the nonlinear element satisfying the condition (14) is given in Fig. 2.

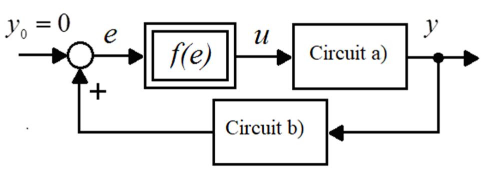

Figure 3: Nonlinear fractional positive electrical circuit

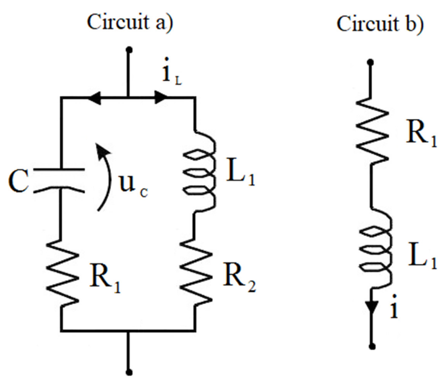

Figure 4: Positive electrical circuits

The fractional linear positive part of the system is shown in Fig. 4a and it is described by the equations

$$\begin{bmatrix} \dfrac{d^\alpha u_C}{dt^\alpha} \\ \dfrac{d^\beta i_L}{dt^\beta} \end{bmatrix} = A \begin{bmatrix} u_C \\ i_L \end{bmatrix} + Bu, \qquad y = C \begin{bmatrix} u_C \\ i_L \end{bmatrix}, \tag{29a}$$

where

$$A = \begin{bmatrix} -\dfrac{1}{R_1 C} & 0 \\ 0 & -\dfrac{R_2}{L_1} \end{bmatrix}, \qquad B = \begin{bmatrix} \dfrac{1}{R_1 C} \\ \dfrac{1}{L_1} \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{29b}$$

The fractional dynamical feedback linear electrical circuit is shown in Fig. 4b and it is described by the equations

$$\frac{d^\beta i}{dt^\beta} = Fi + Gu_C, \qquad e = Hi, \tag{30a}$$

where

$$F = -\frac{R_3}{L_2}, \qquad G = \frac{1}{L_2}, \qquad H = 1. \tag{30b}$$

In this case the Metzler matrix (17) has the form

$$\begin{bmatrix} \dfrac{A}{GC} & \dfrac{k\overline{BH}}{F} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{R_1 C} & 0 & \dfrac{k}{R_1 C} \\ 0 & -\dfrac{R_2}{L_1} & \dfrac{1}{L_1} \\ \dfrac{1}{L_2} & 0 & -\dfrac{R_3}{L_2} \end{bmatrix}. \tag{31}$$

By Theorem 5 the matrix (31) is asymptotically stable if the following conditions are satisfied:

$$\text{1) row conditions:} \quad k < 1, \quad R_2 > 1 \ \text{ and } \ R_3 > 1 \tag{32a}$$

and

$$\text{2) column conditions:} \quad R_1 C < L_2, \quad k < R_1 C \left(\frac{R_3}{L_2} - \frac{1}{L_1} \right). \tag{32b}$$

The matrix is asymptotically stable if k satisfies the conditions (32).

Therefore, the fractional nonlinear electrical circuit is globally stable if k satisfies the condition (32).

5. Concluding remarks

The global stability of integer and fractional orders nonlinear systems with positive linear parts and dynamical feedbacks has been investigated. New sufficient conditions for the global stability of this class of positive nonlinear systems have been established (Theorems 6, 7). The procedures for calculation of gain k characterizing the class of nonlinear element have been presented (Procedures 1 and 2). The applications of the new global stability conditions have been illustrated by examples. The considerations can be extended to nonlinear systems with interval matrices of the linear parts and to the discrete-time fractional nonlinear systems with interval matrices of positive linear parts. An open problem is an extension of the considerations to nonlinear different orders fractional systems with interval matrices of their positive nonlinear parts.

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