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Estimation of filter order for prescribed, reduced group delay FIR filter design

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Abstract. FIR filters are often applied, as they possess many advantages, including linear-phase response and well elaborated design methods. However, group delay introduced by FIR filters is usually large. The reduction of group delay can be obtained by restriction of the linear phase requirement only to the passband. One of the problems that appear while designing FIR filters with a prescribed value of group delay is the choice of the filter order. In the paper a formula for filter order calculation for the given filter parameters and dedicated for equiripple or quasi-equiripple approximation of the magnitude response has been derived based on experiments. Numerous examples that explain how to use the derived formula have been included.

Key words: digital filters, FIR filters design, filter order estimation.

1. Introduction

Digital finite impulse response (FIR) filters are often applied because of their numerous advantages, including inherent stability, well elaborated design methods and easiness of obtaining a linear-phase response. Moreover, non-recursive structures can be easily implemented in programmable systems. A significant drawback of linear-phase FIR filters is large group delay, in particular when the order of the filter is large. In order to reduce group delay, the requirement for an exact linear phase can be restricted only to the passband. In many applications, such as communication systems [1–3], medical imaging [4], or active noise control systems [5], reduced group delays are required. For the design of such filters, methods dedicated for the broader category of nonlinear-phase FIR filters design can be applied. A comprehensive review of different approaches can be found in [6, 7]. One of the problems that appear while applying the given methods is calculation of the filter order. Formulas for filter order calculation presented in the literature [8–11] have been elaborated for linear-phase FIR filters. In the given paper, the formula for filter order calculation based on a prescribed group delay in the passband as well as other filter parameters (the width of the transition band and magnitude response approximation error in the passband and stopband) has been derived. The derivation has been shown for lowpass filters, and the procedure for other types of filters has been formulated. The elaborated formula enables less time to be spent on FIR filter design. It can be also applied for the design of IIR filters based on the FIR prototype [12, 13].

It should be noted that filters with reduced group delay can be designed as frequency-response masking filters [14, 15] and can also be realized by block convolution techniques for long sequence filtering [16].

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The structure of this paper is as follows: in the second section a review of formulas for calculation of the linear-phase FIR filter order, known from the reference, is presented. This section also includes derivation of a new formula for calculation of the filter order, which can be applied for FIR filters with linear phase only in the passband and the prescribed value of group delay. The precision of the proposed formula has been estimated. Four examples explaining how to use the proposed formula for filter design are presented in section three. The last example shows group delay reduction in the decimation filter of an analog-to-digital sigma-delta converter. The conclusions are formulated in section four.

2. Calculation of FIR filters order

The filters considered in the paper are described by the following system function $H(z)$:

$$H(z) = \sum_{i=0}^{M} h_i z^{-i} \tag{1}$$

and fulfil the requirement for equiripple or quasi-equiripple approximation of the magnitude response. Known methods for filter design require specification of frequency response parameters and filter order M, for calculation of filter coefficients h_i. The value of filter order M is dependent on frequency response parameters, so formulas which enable the calculation or estimation of M are desirable.

2.1. Linear-phase FIR filters. For linear-phase FIR filters and an equiripple magnitude response, the formula (derived in an experimental manner) was proposed by Hermann, Rabiner and Chan [8, 9]:

$$M = \frac{D}{\Delta F} - B\Delta F, \tag{2}$$

where

$$D = [0.005309\,(\log\delta_1)^2 + 0.07114\log\delta_1 - 0.4761]\ \log\delta_2 - 0.00266\,(\log\delta_1)^2 - 0.5941\log\delta_1 - 0.4278$$

$$B = 0.51244\,(\log\delta_1 - \log\delta_2) + 11.012,$$

ΔF denotes the transition band width: $\frac{|\omega_s - \omega_p|}{2\pi}$, δ_1 and δ_2 denote the maximum magnitude error in the passband and stopband, respectively, ω_p and ω_s denote the edges of the passband and stopband (in rad/sample – normalized radian frequency).

Formula (2) is quite complicated and therefore inconvenient for fast estimation of the filter order. This formula has been implemented in the MATLAB procedure *firpmord*. Kaiser has proposed a simpler method [10]:

$$M = \frac{-20\log\sqrt{\delta_1\delta_2} - 13}{2.32\,|\omega_s - \omega_p|}. \tag{3}$$

Formulas (2) and (3) have been recommended in many books, e.g. [17–20]. The value obtained from (2) is smaller by one when compared with source formulas given in [8] and [9], because M in this paper denotes the filter order, whereas in [8] and [9] it denotes the number of coefficients (filter length). The formulas can be applied for wide ranges of passband edges ω_p for lowpass and highpass filters. They can also be implemented for bandpass and bandstop filters, assuming for calculation the width of the narrower transition band.

The formulas yield better precision for FIR filters with an odd number of coefficients, as they were derived for such filters [8]. Moreover, the precision of filter order estimation decreases for filters with a very large number of coefficients. This drawback has been eliminated in a method elaborated almost 30 years later [11], which is unfortunately even more complicated than (2) and has not been commonly implemented.

2.2. Nonlinear-phase filters. None of the so far presented formulas is suitable for estimation of filter order in the case of a filter with prescribed linear-phase only in the passband, and a reduced value of group delay τ smaller than $M/2$. Figure 1 presents attenuation $A = 20\log\delta_2$ in the stopband of a lowpass filter as a function of M for $\tau = M/2$ (solid line), calculated according to formula (3), as well as attenuation obtained experimentally for a filter with reduced group delay $\tau = 15$ samples (points denoted by asterisks). The other parameters of the filters were as follows: $\delta_1 = \delta_2$, $\omega_p = 0.5\pi$ and $\omega_s = 0.6\pi$ rad/sample. It can be seen that for increasing attenuation and constant τ, the difference between the filter order calculated from (3) and the real value of M, which gives the prescribed value of attenuation, increases. The formula which enables estimation of the FIR filter order with a group delay smaller than $M/2$ is derived in the next subsection.

Derivation of a new formula for order estimation ($\delta_1 = \delta_2$). The background for formula derivation was an experiment that included the design of a few hundreds of FIR lowpass filters with different parameters. Calculations were performed by the MATLAB procedure available in [21], based on the method described in [7]. (For nonlinear-phase FIR filter design, the MATLAB procedure *cfirpm* based on the algorithm presented in [22] can be applied too, but the solution is worse than that of [21], in particular for filters with a significantly reduced group delay).

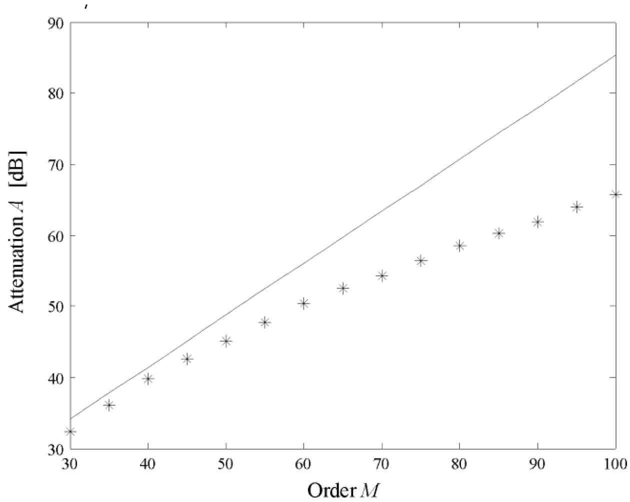

Fig. 1. The attenuation A versus order M determined by experiment for $\tau = 15$ (stars) and calculated by formula (3) for $\tau = M/2$ (solid line)

The following filter parameters have been considered:

- passband width $\omega_p = 0.2\pi,\ 0.5\pi,\ 0.7\pi$ rad/sample,
- transition band width $\omega_t = \omega_s - \omega_p = 0.05\pi,\ 0.1\pi,\ 0.15\pi$ rad/sample,
- equal magnitude approximation errors in the passband and stopband $\delta_1 = \delta_2$,
- filter order $M = 20, 25,\ 30,\ \ldots,\ 100$,
- constant group delay in the passband $\tau = M/2,\ M/3,\ \ M/4,\ \ M/6$.

Some of the results obtained are presented in Fig. 2 as plots $A = f(M, \omega_t, \tau)$, where A has been averaged for three filters, as it was found that they do not depend on the passband width ω_p. As seen from the plots, attenuation A is a linear function of M, whose slope depends on ω_t and τ.

In order to find the form and parameters of the function $A = f(M, \omega_t, \tau)$, which approximates the obtained results, a method successfully applied for solution of a similar problem in the case of IIR filters has been applied [23]. The method consists of two stages. In the first stage, the parameters α and β of a linear function, approximating real values A are calculated:

$$A = \alpha(\omega_t, \tau)M + \beta(\omega_t, \tau). \tag{4}$$

In the second stage, a quadratic function showing the dependence of coefficients α and β on ω_t and τ is to be found. Finally, the following formula has been obtained:

$$A = \left[\left(a_2\frac{\tau^2}{M^2} + a_1\frac{\tau}{M} + a_0\right)\omega_t + b\right] M + c_1\omega_t + c_0, \tag{5}$$

where $a_0 = 1.0562$, $a_1 = 4.9148$, $a_2 = -5.2582$, $b = 0.044$, $c_0 = 7.3341$, $c_1 = 9.8399$.

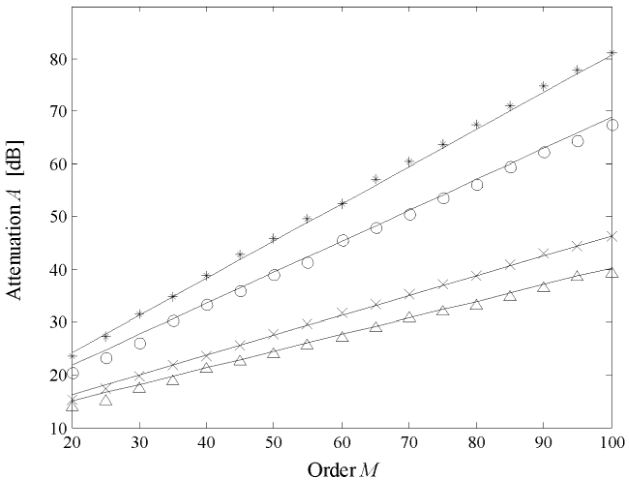

Fig. 2. The average values of attenuation A versus order M for: $\omega_t = 0.15\pi$, $\tau = M/3$ (*); $\omega_t = 0.15\pi$, $\tau = M/6$ (o); $\omega_t = 0.05\pi$, $\tau = M/3$ (×); $\omega_t = 0.05\pi$, $\tau = M/6$ (△) and their approximation (solid lines) by (5)

Equation (5) requires substitution of group delay τexpressed in samples and transition band width ω_t in rad/sample. The lines in Fig. 2 have been plotted according to (5), and it can be observed that they approximate the real values very well. After transformation of (5), the following equation for filter order M can be obtained:

$$(a_0\omega_t + b)\,M^2 + (a_1\tau\omega_t + c_1\omega_t + c_0 - A)\,M + a_2\tau^2\omega_t = 0. \tag{6}$$

Calculation of the filter order requires solution of the quadratic Eq. (6). A positive root, rounded to the nearest integer value, should be considered as the desired solution. Equation (5) has been obtained for lowpass filters, but is suitable for highpass filters as well. It can also be applied for bandpass or band-stop filters, if ω_t is replaced with the width of the narrower transition band.

Evaluation of precision of the proposed formula. Evaluation of the precision of formula (5) has been performed for two test sets. The first set consisted of the same results of the experiments that were used for derivation of (5). The second set included the results of new filter design simulations, where design specification parameters were different by at least one parameter from the set listed in subsection "Derivation of a new formula for order estimation ($\delta_1 = \delta_2$)".

The following parameters were subject to change: passband width $\omega_p = 0.1\pi$ or 0.4π rad/sample, filter order M (up to 150), group delay $\tau = M/5$, transition band width $\omega_t = 0.08\pi$ rad/sample. The condition $\delta_1 = \delta_2$ remained constant. For all the filters considered, precision error ε, defined as the difference between attenuation A calculated from (5) and the real attenuation obtained in the experiment, was calculated. The results are summarized in Table 1. It can be observed that the errors are similar for both test sets. In the worst case, for 90.4% of filters $|\varepsilon|$ is not greater than 2 dB.

In Figs. 3 and 4 the minimum values of the order of a lowpass filter that fulfils the imposed parameters as a function of passband width ω_p have been plotted as circles (found by a trial-and-error method). For comparison, the values estimated by equation (6) have been plotted as a solid line. As can be observed, the FIR filter order is practically constant in the wide range of ω_p and the value obtained from Eq. (6) differs on average by ± 1 from the real one in this range. There is a greater difference for the filters characterized by either a very narrow or a very wide transition band. The same trend can be observed in the case of group delay $\tau = M/2$ and filter order derived from (2) or (3) [11].

Table 1
Approximation error ε for lowpass filters

Testing set	Number of filters	Percentage of filters that fulfill the given condition: [%]		$\varepsilon_{\max}$ [dB]	$\varepsilon_{\min}$ [dB]
		$\lvert\varepsilon\rvert < 1$ [dB]	$\lvert\varepsilon\rvert > 2$ [dB]		
Set 1	612	67.6	5.7	2.67	−3.61
Set 2	480	60.8	9.6	3.34	−4.18

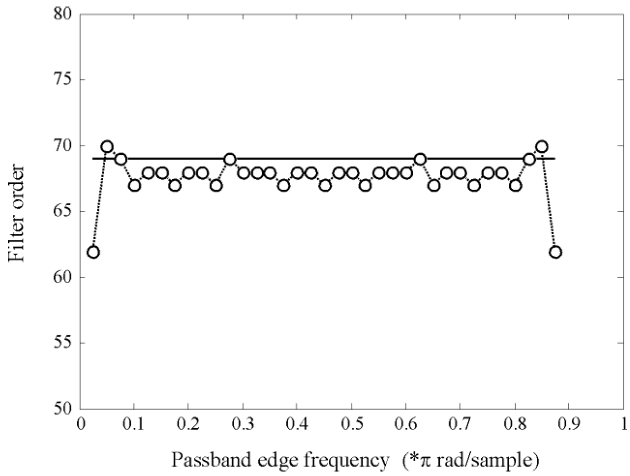

Fig. 3. Required filter order (circles) and estimated order (solid line) as a function of passband edge frequency ω_p in the case of $A = 60$ dB, $\omega_t = 0.1\pi$ rad/sample, $r = 25$ samples

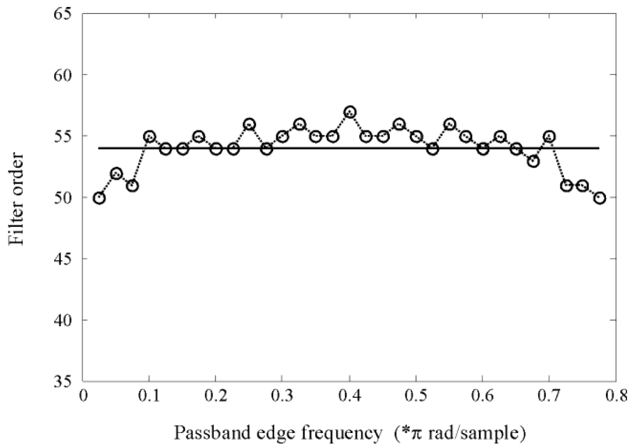

Fig. 4. Required filter order (circles) and estimated order (solid line) as a function of passband edge frequency ω_p in the case of $A = 80$ dB, $\omega_t = 0.2\pi$ rad/sample, $r = 12$ samples

The results of the experiments lead to a recommendation for the application of formula (6) with the parameters that fulfil the following restrictions: $\tau > M/7$, $0.02\pi \leq \omega_t \leq 0.3\pi$, $\omega_p > 0.025\pi$, $\omega_p + \omega_t \leq 0.95\pi$ and $M \leq 150$. Formula (6) can be applied with parameters outside the given range, but the approximation of the filter order will be less accurate.

Case $\delta_1 \neq \delta_2$.

Quite often it is desired to obtain smaller ripples in the stopband than in the passband, i.e. $\delta_1 > \delta_2$. In the optimization procedures applied for FIR filter design, this condition is forced by choosing an appropriate weighting function for the desired magnitude response approximation error. If the weighting function for the passband and stopband has constant values W_p and W_s, respectively, then in order to obtain the ripple ratio $K = \delta_1/\delta_2$, it is necessary to preserve $W_p/W_s = 1/K$. For example, assuming $W_p = 1$ and $W_s = 10$, we obtain $K = 10$, but δ_1 will be $\sqrt{10}$ times greater, and $\delta_2\sqrt{10}$ times smaller than in the case with $W_p = W_s = 1$. Generally, by increasing W_s, an increase of attenuation in the stopband (decrease of δ_2) by approximately $10 \log W_s$ can be obtained. This value results from (3), but as shown in Fig. 5, it is also valid for FIR filters with the linear phase requirement restricted only for the passband. However, application of W_p and W_s different from one for the considered method for filter design is unfavorable. Increasing W_s in the stopband enables greater attenuation in this band, but it also increases the approximation error of the desired group delay in the passband (see example 3). Increasing W_p in the passband will cause a reverse effect. Therefore, the assumption $W_p = W_s = 1$ seems to be a reasonable trade-off.

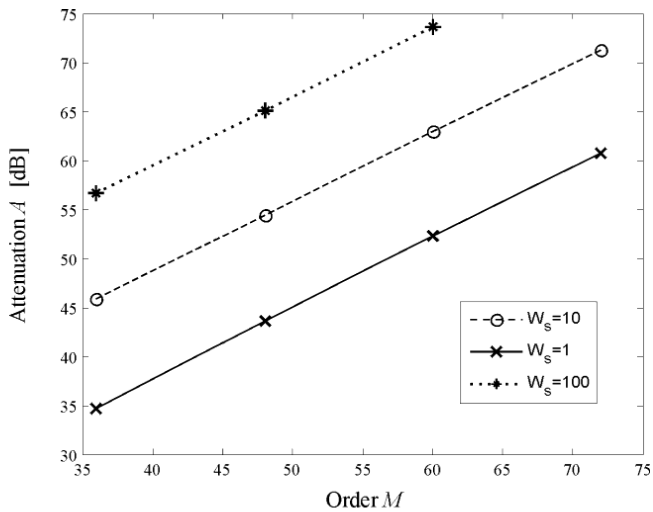

Fig. 5. Attenuation of lowpass filter with $\omega_p = 0.2\pi$, $\omega_t = 0.1\pi$ rad/sample, $\tau = M/3$ versus filter order M and weighting function W_s in the stopband

3. Examples

Application of the proposed approach (6) for filter design will be shown in the examples given below. Calculations have been implemented in MATLAB – the codes are available at http://iele.polsl.pl/∼jkon/FIR/FIR_order.zip.

Example 1.

Design a lowpass FIR filter with the given parameters: passband edge $\omega_p = 0.2\pi$ rad/sample, stopband edge $\omega_s = 0.325\pi$ rad/sample, group delay in the passband $\tau = 18$ samples, attenuation in the stopband $A = 60$ dB.

Substituting $\omega_t = 0.125\pi$ and $\tau = 18$ into (6) gives the following condition:

$$0.4588\, M^2 - 14.0611\, M - 669.02 = 0$$

which has two solutions: $M = 56.47$ and $M = -25.83$. The obtained value of filter order M equals 56; this value should be substituted into the procedure given in [21] jointly with the remaining parameters. The magnitude response and group delay of the designed filter are shown in Fig. 6. As can be seen, the design requirements for the considered filter have been fulfilled. The attenuation in the stopband equals 60.1 dB, the maximum error of group delay equals 0.186 sample, but in the frequency range from 0 up to 0.195π it is smaller than 0.045 sample, and increases only at the end of the passband.

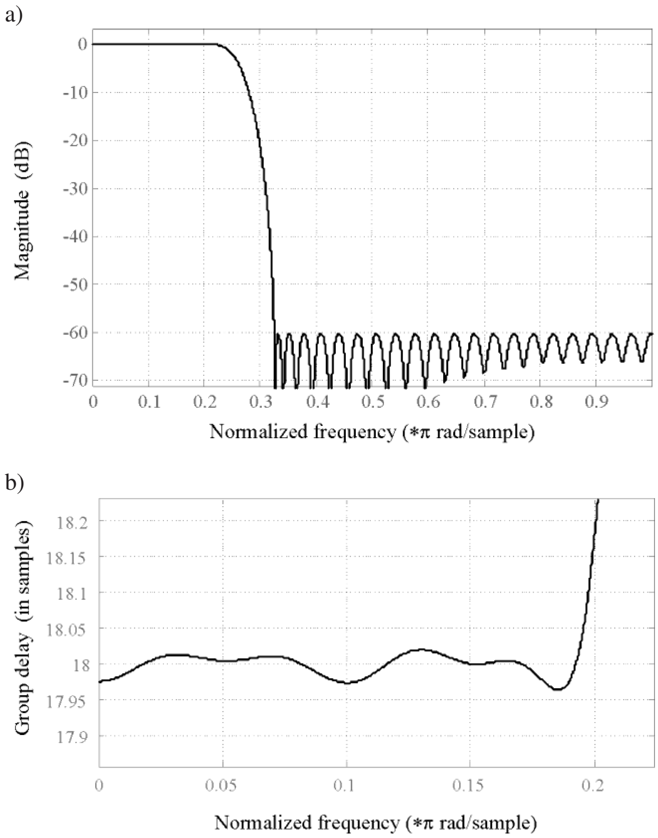

Fig. 6. Frequency responses of lowpass filter designed in Example 1 a) magnitude and b) group delay (in passband)

Example 2.

Design a bandstop FIR filter with the given parameters: lower passband edge $\omega_{p1} = 0.15\pi$, upper passband edge

$\omega_{p2} = 0.67\pi$, lower stopband edge $\omega_{s1} = 0.2\pi$, upper stopband edge $\omega_{s2} = 0.6\pi$ (all frequencies in rad/sample), minimum attenuation in the stopband $A = 40$ dB, constant group delay in the passband $\tau = 20$ samples.

In this case, $\omega_t = 0.05\pi$ – the width of the narrower transition band – should be substituted into (6). The positive root value equals 91.84, therefore the calculated filter order $M = 92$. The frequency responses of the considered filter are shown in Fig. 7. The minimum attenuation in the stopband equals 40.23 dB. The group delay ripple in almost the whole passband is smaller than 0.5 sample and increases only at the edge of the passband (maximum up to 1.9 sample).

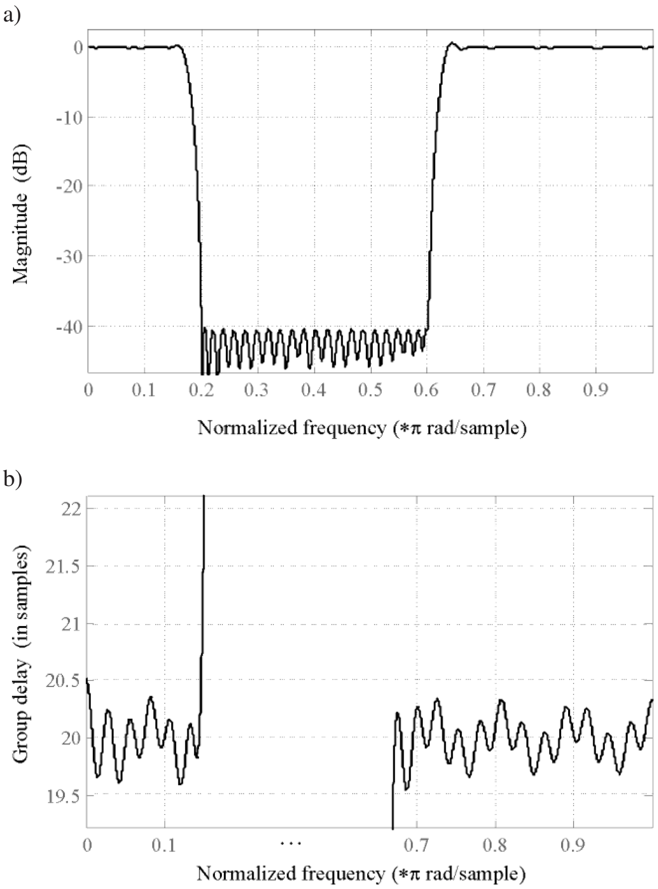

Fig. 7. Frequency responses of bandstop filter designed in Example 2 a) magnitude and b) group delay (in passband)

Example 3.
Calculate the order, and then design the following highpass filter: passband and stopband edges $\omega_p = 0.75\pi$, $\omega_s = 0.65\pi$ rad/sample, group delay in the passband $\tau = 15$ samples, stopband attenuation $A = 45$ dB.

The proposed procedure (6), applied as in the previous examples, gives the filter order $M = 50$. The frequency responses of the filter are shown in Fig. 8. The minimum attenuation in the stopband equals 45.9 dB. The maximum group delay error equals 0.71 sample. In the same plot the frequency responses of the filter designed for $W_s = 10$ have been presented. As can be observed, the attenuation in the stopband has increased by 9 dB, but the approximation error of group delay has increased about 4 times.

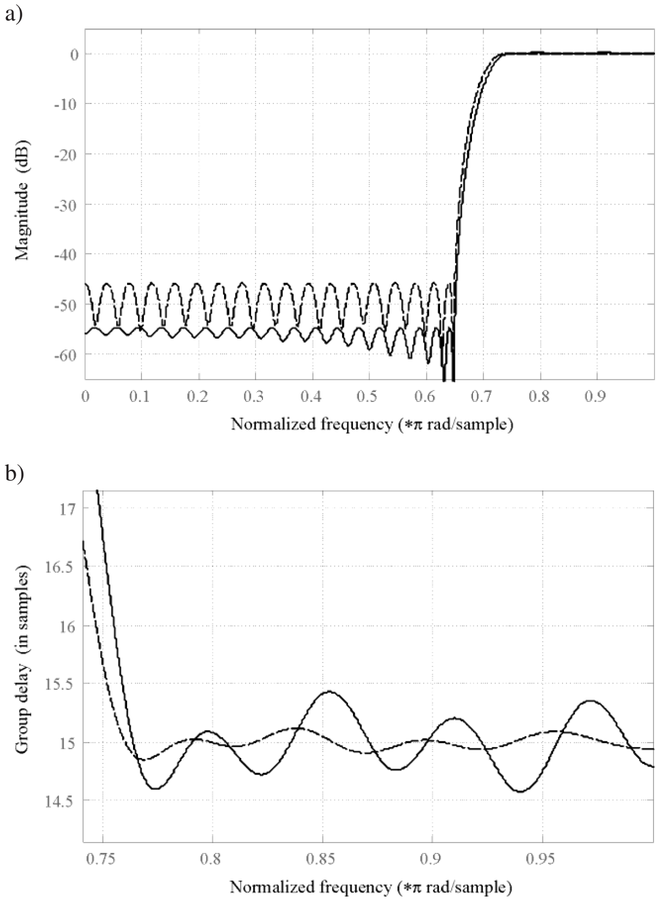

Fig. 8. Frequency responses of the highpass filter designed in Example 3 for $W_s = 1$ (dashed line) and $W_s = 10$ (solid line): a) magnitude, b) group delay (in the passband)

Frequency responses of the filters from the examples 1, 2 and 3 have been obtained for non-quantized filter coefficients. In the case of realization of a filter in fixed-point arithmetic, the number of bits for coefficients representation must be properly chosen, so that the filter preserves the prescribed parameters. For FIR filters, quantization of coefficients mainly causes a loss of attenuation in the stopband. In Fig. 9 a fragment of the stopband frequency response of the example 1 filter has been shown, for exact values of coefficients (dashed line) and quantized to 16 bits accuracy (solid line). The format 1 sign bit and 15 fractional bits have been applied. The chosen fragment shows the biggest difference in the whole frequency range between responses in the case of 16 bits representation. As can be observed, in some frequency ranges the attenuation is smaller than the required 60 dB. However, in the case of 17 bits representation, the decrease of attenuation is almost unnoticeable. The filters from examples 2 and 3 will fulfil the design requirements, when at least 13-bit registers will be applied for coefficients storage. The presented results have been obtained by the MATLAB *fdatool* procedure.

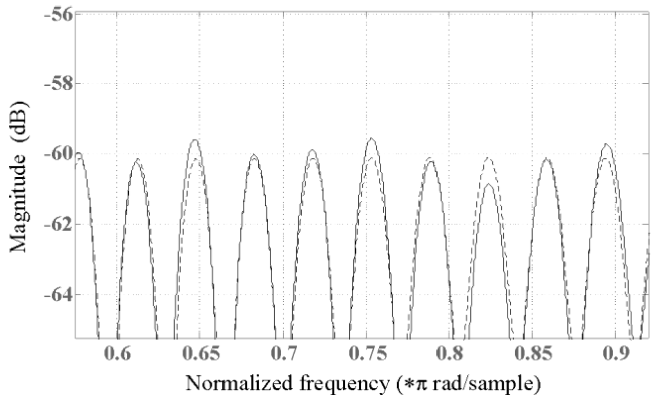

Fig. 9. Magnitude response in the stopband of lowpass filter designed in Example 1 with no quantization (dashed line) and quantization of coefficients to 16 bits (solid line)

Example 4.
Generally, in digital feedback control applications a large delay in the feedback loop is not acceptable. This causes a reduction in processing speed. For example, this situation appears in active noise control systems [5]. Below, the method for reduction of delay in the case of an analog-to-digital sigma-delta converter, applied for AD7722 parameters [24] will be presented. Sigma-delta converters operate with oversampling, followed by lowpass filtering and decimation. In AD7722, a two-stage FIR filter has been applied; the first stage is a 384-tap filter (i.e., the filter order is 383), and the second is a 151-tap half-band filter. The digital signal is decimated by 32 and again by 2 after passing these stages, respectively. Both stages introduce a group delay of 215.5 μs (with a nominal clock frequency of 12.5 MHz), of which only 18.4 μs is caused by the first stage because of the 32-times higher operating frequency. Application of the filter, with reduced group delay $\tau = 45$ samples for the second stage, allows a significant reduction of delay of the whole converter. The half-band FIR filter in the second stage possesses the following parameters [24]: passband edge 90.625 kHz, stopband edge 104.6875 kHz, stopband attenuation $A = 90$ dB, passband ripple 0.00025 dB (as 0.0005 dB is specified for the whole filter, equal shares have been assumed for both stages). The filter operates at a sampling frequency of 12.5 MHz/32 = 390.625 kHz. The normalized values of the passband and stopband edges are $\omega_p = 0.464\pi$, $\omega_s = 0.536\pi$ rad/sample and the transition band $\omega_t = 0.072\pi$ rad/sample. Substituting the specified parameters for A, τ, ω_t in (6) yields a filter order of $M = 161$. The designed filter possesses an attenuation slightly less than required (88.3 dB), whereas in order to obtain the specified attenuation, $M = 164$ is necessary. The group delay of the second stage filter is:

$$\frac{45}{390.625\text{ kHz}} = 115.2\ \mu\text{s}.$$

Considering the group delay introduced by the first stage, as well as additional delay caused by decimation after the second stage, one obtains the total group delay of the two-stage filter equal to 138.7 μs.This means reduction by more than 35%, when compared to the 215.5 μs specified in the AD7722 data sheet. Frequency responses of the filter have been shown in Fig. 10 and Fig. 11, separately for the passband and the stopband. The dashed line shows the responses for non-quantized coefficients, and a solid line shows quantized at 22 bits. A slight difference in the responses can be observed only in the stopband.

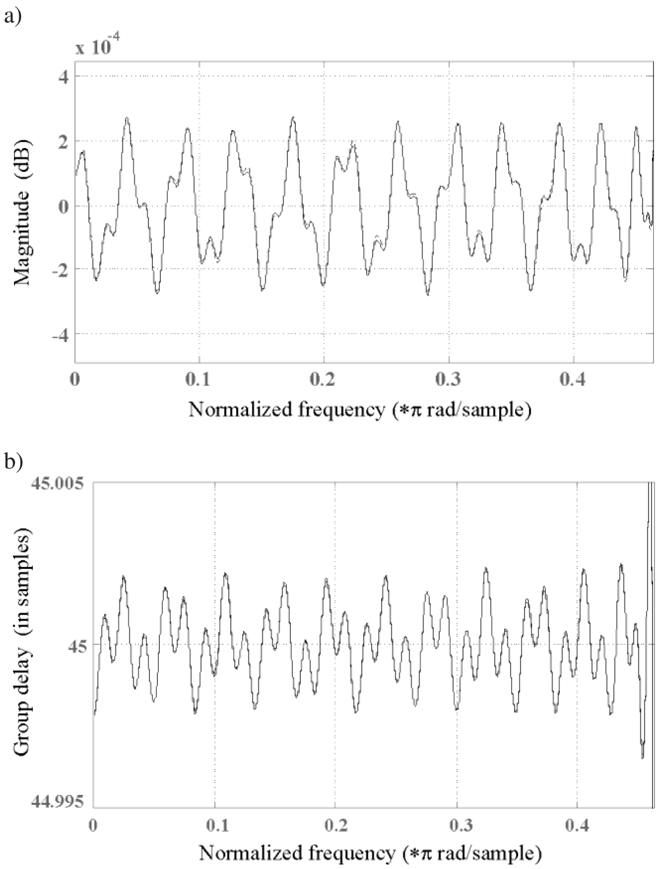

Fig. 10. Frequency response in the passband of lowpas filter designed in Example 4 with no quantization (dashed line) and quantization of coefficients to 22 bits (solid line): a) magnitude, b) group delay

Realization of a filter in finite-precision arithmetic requires a proper choice of digital wordlength. The considered converter operates at 16 bits, therefore the variance of the quantization noise of the input signal equals:

$$\sigma_x^2 = \frac{2^{-32}}{3} = 7.76 \cdot 10^{-11}.$$

After filtering, the variance at the output equals:

$$\sigma_{yx}^2 = \sigma_x^2 \sum_{i=0}^{164} |h_i|^2 = 3.96 \cdot 10^{-11}.$$

Whereas noise caused by multiplication rounding at L bits at the output of a direct form FIR filter has variance (see chapter 6.9 in [25]):

$$\sigma_y^2 = N \frac{2^{-2L}}{3},$$

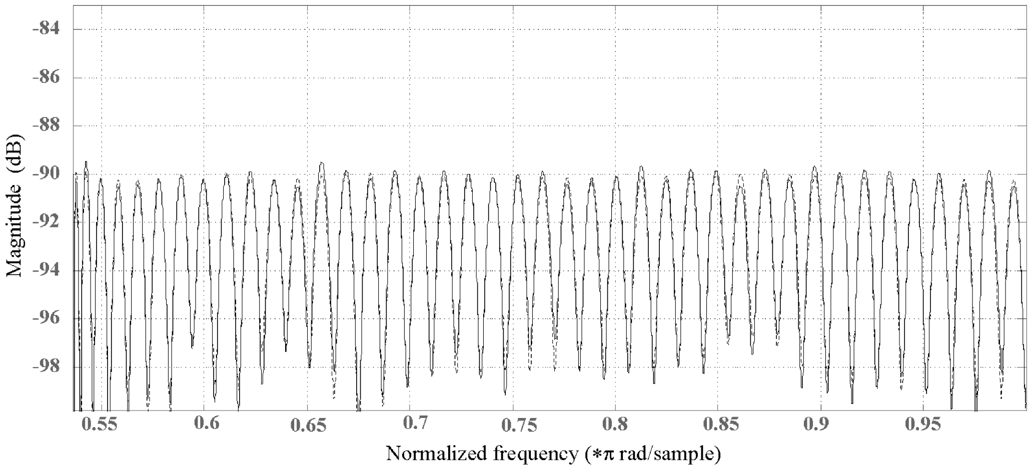

Fig. 11. Magnitude response in the stopband of lowpass filter designed in Example 4 with no quantization (dashed line) and quantization of coefficients to 22 bits (solid line)

where N denotes the number of multiplications. The designed reduced group delay filter has a non-symmetric impulse response, but almost half of its coefficients equal 0 and its realization requires $N = 83$ multiplications. Assuming $L = 24$ bits, one obtains $\sigma_y^2 = 9.83 \cdot 10^{-14}$, which is more than 100 times smaller than σ_{yx}^2. Therefore for $L = 24$ bits wordlength, the signal to noise ratio at the filter input is practically the same as at the output. The reduction of group delay has been obtained at a cost of an increased number of multiplications. However, fortunately this does not increase the overall delay, as input samples appear with a frequency 32 times smaller than at the first stage, and multiplications and summations are over before the next sample comes.

4. Conclusions

In the paper the experimental procedure that enables calculation of the filter order for FIR filters with the prescribed constant value of group delay in the passband has been presented. The formula has been verified in the course of numerous filter design examples, including over 1000 simulations with various parameters. The order estimation error can increase in the case of filters characterized by a very narrow or very broad passband. The proposed formula has been derived for lowpass filters, but the manner of its application in the case of other types of filters has been shown, too. A limitation of the proposed formula is the requirement for equal values of magnitude of the frequency response errors in the passband and stopband. Moreover, it should be noted that the significant reduction of group delay with respect to $M/2$ may cause (depending on the applied design method) an increase of the group delay approximation error as well as the gain overshoot in the transition band.

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