10.2478/v10170-010-0010-y

Archives of Control Sciences
Volume 20(LVI), 2010
No. 2, *pages 151–163*

Robust control and modal analysis of flexible rotor magnetic bearings system

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Key words: active magnetic bearings, uncertainty design, flexible rotor model, bending modes, μ-synthesis control, rotor vibrations control

1. Introduction

Active magnetic bearings (AMBs) are used to support a rotor without mechanical contact and to control the vibrations [1]. Rotor vibrations compensation is very important in machines operating with high rotational speeds [4]. The AMBs system uses magnetic forces to levitate the rotor between two opposing electromagnetic poles. The rotor is inherently unstable. Thus, the magnetic bearing system must be stabilized with an active control system.

In the paper, the robust optimal vibrations control system of the rotor supported magnetically is presented. The flexible rotor supported by AMBs was analyzed using finite element method (FEM) such that all flexible modes up to bandwidth of 1106 [Hz] are considered for non-collocation sensors-actuators. The all matrix of flexible rotor model were computed in MATLAB and frequencies of the flexible modes are verified in the ANSYS program. Model of components of AMBs system which included dynamics of magnetic bearings, rotor model, power amplifiers and sensors dynamics is presented.

The dynamics of the open-loop system is influenced by external disturbances (steady sinusoidal loads), nonlinearities, uncertainties and signal limits [5, 6]. Thus, a design methodology which covers the practical issues of optimal robust control based on μ-synthesis like the modeling of uncertainties, selection of optimal weighting functions is presented [9]. Then μ-controllers synthesized for the augmented plant model which meet analysis objectives ($\mu \leqslant 1.0$) stabilize the actual plant and meet specified performance objectives. The μ-synthesis control method for AMB rotor was investigated successfully. The μ-synthesis control permits the design of multivariable optimal robust controllers for complex linear systems with any type of uncertainties in their structure. It is convenient

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Received 30.09.2009.

method for MIMO systems like AMB and offers many knobs that a control designer can turn. The optimized performance index is well related to real AMB system [5]. The goal of this method is to design a controller which is robust to variations in plant dynamics. This procedure is natural augmentation of H_∞ control theory with the analysis of the structured singular value. However, in the case of μ-synthesis control, the uncertainties are much simpler to be considered than in H_∞ case. What is more, the nonlinearity of the controlled plant can be also considered. Generally, robust control theory says that if the control is unconstrained, the better performance of robust control can be achieved if the greater control effort is applied. Thus, if there is a limit on achievable H_∞ attenuation strongly depends on the relative position of the disturbances and the actors. However, the best control performances can be achieved unfortunately with infinite controller gain.

This paper is focused on the experimental evaluation of the robust performance. The laboratory stand with the high speed rotor (21 000 [rpm]) supported magnetically was built. The rigid rotor is supported by two radial active heteropolar magnetic bearings. Four closed-loops are used to control displacement of the rotor in the air gap of the radial bearings. The dynamical behavior of the closed-loop system in wide range of rotation speed was evaluated. The stable operation, good stiffness of the rotor and robust performances of the closed-loop magnetic bearings systems is reached. Finally, the success of the robust control is demonstrated through the results of computer simulations and experiments. The experimental results show the effectiveness of the control system as well as good vibrations reduction and robustness of the designed controllers. The dynamical behavior of the closed-loop system in wide range of rotation speed (from 0 to 21 000 [rpm]) was investigated.

2. Model of AMB rotor system

To measure the rotor displacement in two radial directions, the precision eddy-current sensors were used. They have high resolution and wide bandwidth up to 10 [kHz], which is over the rotor drive maximal speed. Thus, the model of the sensor was assumed as a simple proportional gain. The measured signals are filtered by anti-aliasing filters [2].

The AMBs were operated with current control, i.e. at each sampling step the digital signal processor calculated the size of the magnitude of the control current that was to supply the electromagnetic coils. These currents were generated by digital power amplifiers based on pulse-width-modulation. Each of the amplifiers had an internal control loop with a simple proportional gain controller. Thus, the coil current was measured and subtracted from the set current which was proportional to a voltage signal of the digital signal processor. The demanded set current was created by means of switching among a positive or negative voltage ($U = 180$ [V]). The switching frequency was equal to 18 [kHz]. Thus, the true current oscillated around the switching frequency. For a good dynamics performance of the operating AMBs, the current should be as smooth as possible. Thus, the amplifier output filter $L-C$ was used. The AMBs coils were modeled as the series interconnection of a copper resistance R and an inductance L (where: $L = L_{coil} + L_{add}$). The rate of the current change had to be fast enough to follow the current command. However, the amplitude and frequency ranges were limited by the $R-L$ curve of the power amplifier. The coil's model was a first low-pass filter with cut-off frequency $\omega_c = R/L$. The maximal value of the current was limited to 10 [A]. In the low frequency range up to the crossover frequency ω_c, the output current was limited by the limit $i_{\max}$. Beyond the frequency ω_c, the current was limited by the coil's low-pass characteristic denoted by the $R-L$ curve. The model of power amplifiers was designed as a low-pass filter with cut-off frequency at 700 [Hz] and gain at 1[A/V]. To simplify the AMBs open-loop model the dynamics of the digital signal processor was neglected and only the gains of A/C and D/C converters are considered.

2.1. Modal analysis of a flexible rotor in AMB

The complex flexible rotor was modeled by using FEM in order to compute frequency modes. Thus, the rotor was partitioned into 20 discrete elements of simple geometry based on the model of Timoshenko's beam. Connecting nodes were introduced between neighboring elements. Each of nodes had 6 degrees of freedom (DOF), so the rotor had 120 DOF. Thus, the equation of motion for the free rotor (without external forces) can be stated as follows:

$$M\ddot{q} + (D + \Omega G)\dot{q} + Kq = 0 \tag{1}$$

where: M – symmetrical, positive definite mass matrix, D – symmetrical damping matrix, G – skew-symmetric gyroscopic matrix, K – symmetrical, positive semi-definite stiffness matrix, Ω – rotational speed, q – displacement vector.

The model of the rotor motion can be transformed to state space model of the following form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{2}$$

where:

$$A = \begin{bmatrix} 0^{q\times q} & I^{q\times q} \\ -M^{-1}K & -M^{-1}(D+\Omega G) \end{bmatrix}, B = \begin{bmatrix} 0^{q\times 2n} \\ M^{-1}F \end{bmatrix}, C = \begin{bmatrix} S & 0^{2l\times q} \end{bmatrix}, D = \begin{bmatrix} 0^{2l\times 2n} \end{bmatrix}.$$

The model of external forces acting on the rotor and model of displacement sensors is as follows:

$$F = \begin{bmatrix} F_x & 0 \\ 0 & F_y \end{bmatrix}, \; S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}. \tag{3}$$

The total rotor mass is 18.5 [kg] and total length is 0.902 [m]. The matrices of flexible rotor model were computed in MATLAB. Bode plot of the free-free rotor is presented in

Mode	MATLAB [Hz]
1st rigid	0
2nd rigid	0
1st flexible	520.01
2nd flexible	944.95
3rd flexible	1639.68
4th flexible	2648.73
5th flexible	3762.84
6th flexible	5116.51

Table 1. Rigid and flexible natural frequencies of free-free rotor

Fig. 1. The first two flexible modes for free-free rotor appear at 520 [Hz] and 944 [Hz] respectively. For the free-free rotor, the rigid modes are equal to 0. For the modal analysis of bending frequencies the non-rotating rotor without couplings by the gyroscopic effects was assumed.

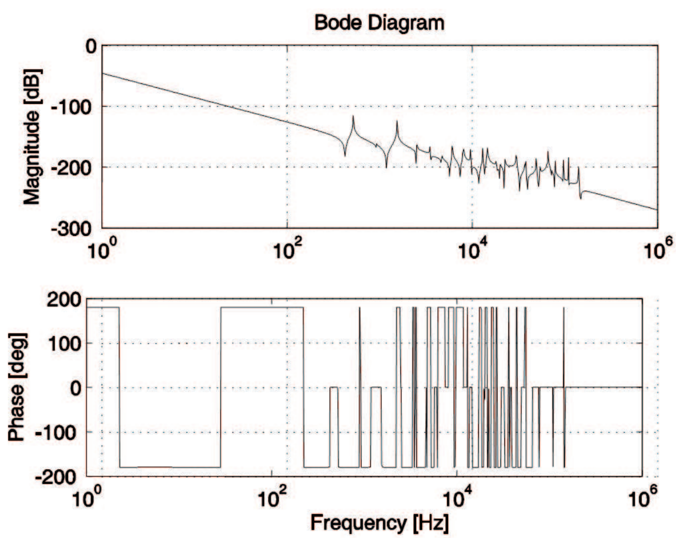

Figure 1. Bode plot of the free-free rotor, for $\Omega = 0$ [rpm].

Based on the equation (2) the pole-zero distribution of the flexible rotor is shown in Fig. 2. Due to symmetry of the system, each pole occurs twice, once for each plane and all poles are very weakly damped. The values of the flexible modes for free-free rotor computed in MATLAB are presented in Tab. 1.

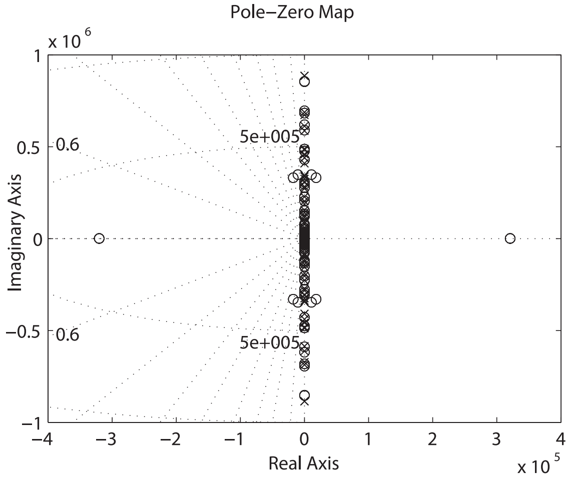

Figure 2. Pole-zero distribution of the free-free rotor, for $\Omega = 0$ [rpm].

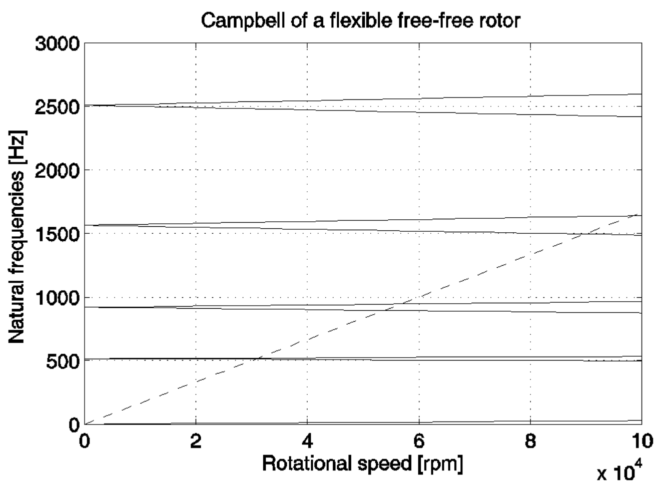

Figure 3. Campbell diagram of the free-free rotor.

In the case of rotating rotor (i.e. $\Omega > 0$), the motion in the two planes are coupled by the gyroscopic term ΩG. This causes the flexible rotor poles moving with increasing rotational speed Ω along the imaginary axis towards increasing and decreasing frequencies. The modes with increasing frequencies which expose a rotation in the same direction as the rotor are called forward modes (*nutations*). The modes with decreasing frequencies rotate in the opposite direction and are called backward modes (*precessions*). Fig. 3 shows the splitting of the flexible eigenfrequencies with rotational speed from 0 to 100

000 [rpm]. Dashed line in Fig. 3 corresponds to rotational frequency while the solid lines correspond to backward and forward flexible modes frequencies respectively.

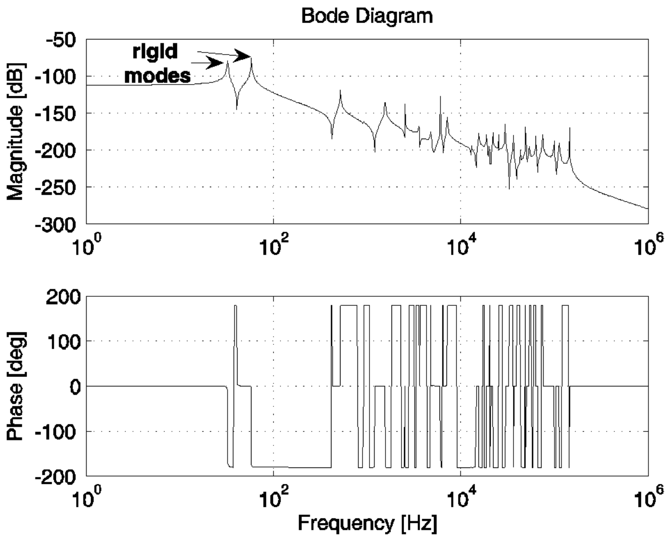

Figure 4. Bode plot of the flexible rotor in AMBs, for $\Omega = 0$ [rpm].

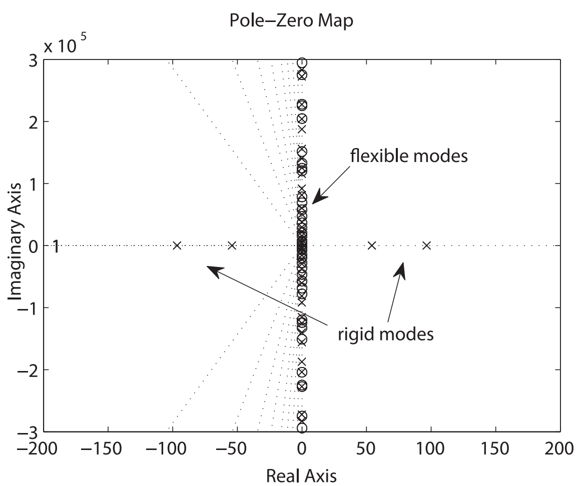

Figure 5. Pole distribution of the flexible rotor in AMB.

2.2. Model of AMB flexible rotor

Flexible rotor supported in AMBs is modeled by combining the model of the flexible rotor with the model of AMBs. Thus, the model of the flexible rotor in AMBs can by described as follows:

$$M\ddot{q}+(D+\Omega G)\dot{q}+Kq=K_sq+K_ii \tag{4}$$

where: K_s, K_i – matrices of the AMBs displacement and current stiffness coefficients respectively. Transformation to modal coordinates yields:

$$M_r\ddot{q}_r+(D_r+\Omega G_r)\dot{q}+(K_r-K_{sr})q=K_{ir}i. \tag{5}$$

Obtained stiffness matrix K_r-K_{sr} is not exactly diagonal, but the diagonal elements are significantly larger than other elements. The simulation were performed for the following parameters of the magnetic bearings system: air gap $x_0=0.25$ [mm], nominal bias current $i_0=2$ [A], $k_i=50$ [A/m], $k_s=400000$ [N/m].

The flexible rotor supported by magnetic bearings was also analyzed using FEM such that all flexible modes up to the bandwidth of $1\dot{1}0^6$ [Hz] were considered. Fig. 4 shows the rigid and flexible modes of the rotor with respect to the frequency. If the system is collocated (sensors and actuators act at the same point along the shaft), then the poles and zeros are interlaced and phase is between 0 and -180^0. However in AMBs system discussed here, the distance between sensors and magnetic actuators was 0.035 [m]. Thus, AMBs system is non-collocated. Comparing with Fig. 1, we can notice that the poles of the AMBs model are independent on the sensor location while the zeros are strongly sensitive to sensor location. The poles and zeros are no longer interlaced which produces additional stability problems. This can be explained as follows: the poles do not stay in the left half plane if the frequency rises (root locus analysis) but travel into the right half plane.

Fig. 5 shows the poles distribution of the flexible rotor in uncontrolled AMBs. Design process of stabilizing controllers of the AMBs which brings all poles to the left half plane is rather difficult. If the controller gain increases, the poles firstly move to the left half plane, but if the gain is further increased, they split and follow the positive and negative imaginary axis. Thus, the controller with added damping behavior should be applied. This illustrates that the process of controller design that stabilizes the rigid body modes of the flexible rotor without destabilizing the weakly damped flexible modes is difficult and complicated task.

The rigid and flexible modes of the AMBs rotor are presented in Tab. 2. Fig. 6 shows the form of the first 4 flexible modes of the analyzed AMB flexible rotor conducted in ANSYS program. Fig. 7 shows the splitting of the flexible eigenfrequencies with rotational speed from 0 to 100 000 [rpm]. The solid lines in Fig. 7 corresponds to the rigid and flexible modes frequencies respectively.

Mode	MATLAB [Hz]
1st rigid	32.34
2nd rigid	58.63
1st flexible	522.95
2nd flexible	945.59
3rd flexible	1640.47
4th flexible	2648.90
5th flexible	3763.04
6th flexible	5116.66

Table 2. Rigid and flexible natural frequencies of AMBs rotor

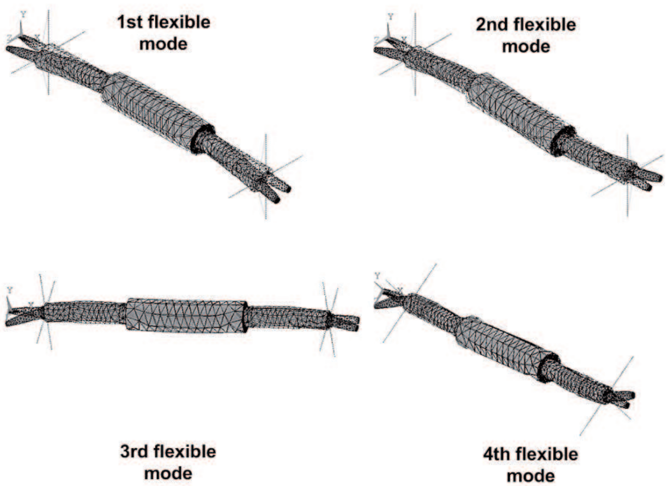

Figure 6. Flexible modes of the AMB flexible rotor.

3. μ-controller design

In the μ-synthesis control, the cost factor is a minimal value of the norm $||T_{zw}||_\infty$, where T_{zw} is a closed-loop transfer function. The μ-synthesis algorithm bases on the $D-K$ iteration procedure. After performed the singular value analysis, the μ-controller should pass the condition in frequency domain [9]:

$$\sup_{\omega \in \mathbb{R}} \bar{\sigma}\left(T\left(j\omega\right)\right) \leqslant 1. \tag{6}$$

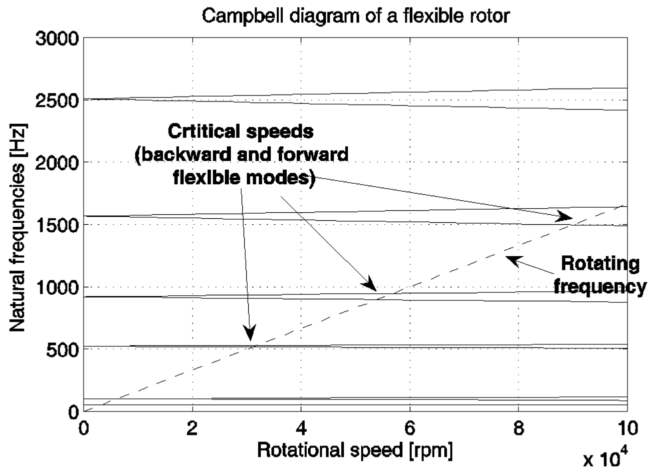

Figure 7. Campbell diagram of the flexible rotor in AMBs for Ω from 0 to 100 000 [rpm].

The robust controller is designed for augmented control plant. In this case, the augmented plant includes nominal models of: rotor, magnetic bearings, power amplifiers, sensors, delayed model of digital signal processor and also models of uncertainties and weighting functions. The first two flexible modes for free-free rotor condition appear at 520 [Hz] and 944 [Hz] respectively, which is over the drive maximal angular speed. Therefore, for control design the rigid rotor model was partitioned. The weighting functions were putted on the input and output signals like error signal, control signal and rotor displacement signal. Thus, the AMB signals are scaled and limited and also the performances of the real control loop are considered [2,3].

The model of magnetic bearing is uncertain. The uncertainty and nonlinearity follow mainly form the characteristics of power amplifiers and electromagnetic coils. Thus, the modeling of AMB system uncertainty was divided into model uncertainty and parametric uncertainty. The model uncertainty in the system dynamics is described by unknown, structured, norm-bounded perturbations. This perturbation acts on the nominal model via linear fractional transformation (LFT) and is represented as feedback gains connected to the plant anywhere inside. The magnetic bearing model error was described with an uncertainty weighting function. At low frequency, below 96 [rad/s], the variation can reach 10% of the nominal value. Over 96 [rad/s] the uncertainty (percentage variation) starts to increase and reaches 140% at about 3200 [rad/s]. The frequencies 96 [rad/s] and 3200 [rad/s] are the values of 2nd rigid and 1st flexible mode of AMBs system. Also some parameters of AMBs system can vary from the nominal values during operation. Thus, the uncertainties in the current stiffness k_i and displacement stiffness k_s for the two AMBs were modeled by scalar real models representing uncertainty of 5%. The actuator gain uncertainty was modeled as 1%. If robust controller is designed for rotational speed equal zero, there are no guarantee that the system will be stable for other values of rotational speed. Thus, the nominal value of rotational speed Ω was equal 10 000 [rpm], with uncertainty of 100%, such as rotational speed could change from 0 to 20 000 [rpm]. The model of AMB system uncertainty with external signals is presented in Fig. 8.

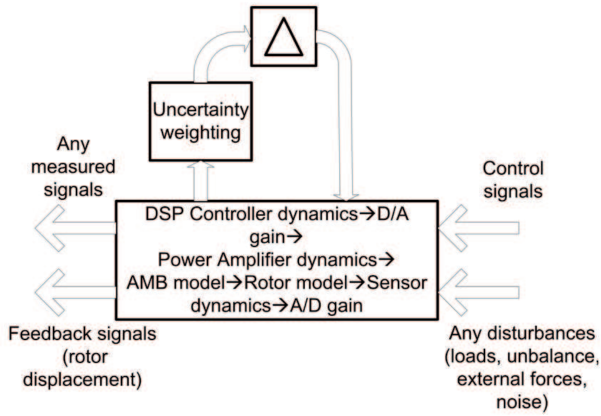

Figure 8. Model of AMB system uncertainty and Bode plots of μ-controllers.

The μ-controller was computed using function `dksyn` of The Robust Control Toolbox of MATLAB [8]. Fig. 9 presents the Bode plot of designed μ-controllers computed for AMB system with uncertainties and weighting functions with one of them being optimized for AMB rotor vibrations compensation. The optimized μ-controller was computed for properly selected weighting functions. The magnitude of the optimized μ-controller is minimized due to maximal value of the power amplifier current.

4. Experimental results

The μ-controllers used to stabilize the rotor in two directions of two radial AMBs were experimentally verified. The four independent μ-controllers were applied for rotor model with couplings between the vertical and horizontal axes of the rotor known as gyroscopic effect. The open-loop AMB model includes: rotor model, model of actuator dynamics, sensor dynamics, A/C and D/C converters gains. The control algorithm was implemented in DSP as a discrete time state space system. Real Time Interface and Real Time Workshop of MATLAB was used. Since the μ-controller cannot levitate the rotor alone, the program firstly brings the rotor into support under a low performance PID control algorithm. After that, the program switches to the μ-control algorithm. Both, the μ-synthesis and PID control algorithms are implemented in DSP with a sampling rate of 10 [kHz]. In experimental tests the disturbances as mass unbalance, gravity loads and sensor noise were considered. The aerodynamic loads and the nonlinear phenomena like eddy-currents loses and hysteresis were neglected for the model simplification. Tests

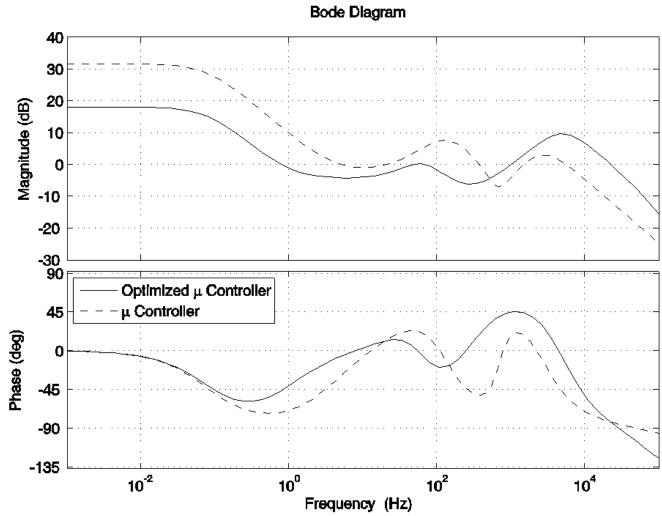

Figure 9. Bode plot of μ-controllers.

were performed for the range of angular speed 0 ÷ 21 000 [rpm]. For experimental investigations the AMB rotor ring was built. The experimental set-up is presented in Fig. 10. Orbit plot of rotor operation at 21 000 [rpm] is presented in Fig. 11.

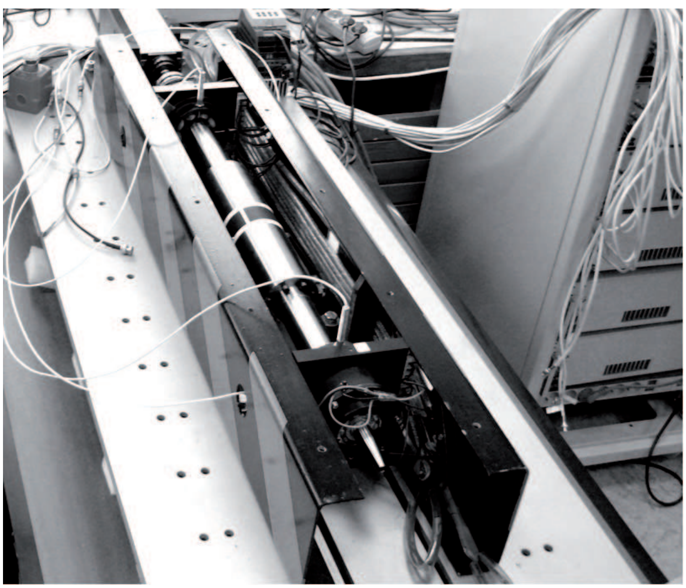

Figure 10. Experimental test ring and orbit plot for rotor operation speed at 21 000 [rpm].

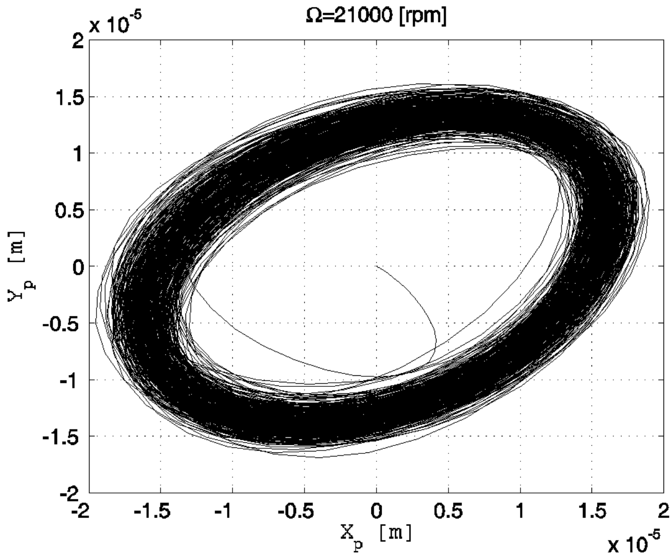

Figure 11. Orbit plot of rotor for rotational speed at 21 000 [rpm].

5. Summary

The presented simulation and experimental results show the capability of μ-synthesis control method of AMBs to improve vibrations control performance. For proper design of the control law, the FEM rotor model was obtained and the modal analysis of flexible rotor supported by AMBs was performed. The vibrations compensation in wide range of rotation speed changes was performed successfully. The μ-controllers have good vibrations damping, disturbance rejection and robustness to the plant structural uncertainty. All designed control systems with μ-controllers were stable and able to realize a high bandwidth. The general disadvantage of μ-synthesis control is requirement of a detailed model of control plant i.e. model of rotor, magnetic bearings, actuators and sensors. Therefore, the order of computed controller is large, thus order reduction is necessary before the model is implemented in a real processor.

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