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Robust output feedback Model Predictive Control design

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The paper addresses the problem of designing a robust output/state model predictive control for linear polytopic systems without constraints. The new robust BMI stability condition for given predictive and control horizon is derived which guarantees the parameter dependent quadratic stability and guaranteed cost.The proposed condition is appropriate for centralized and decentralized control design, as illustrated on example.

Key words: model predictive control, robust control, parameter dependent quadratic stability, Lyapunov function, polytopic system, decentralized control

1. Introduction

Model predictive control (MPC) has attracted notable attention in control of dynamic systems. The idea of MPC can be summarized as follows, Camacho and Bordons [2], Maciejovski [13], Rositer [19], and [4], [5], [6], [8], [20]:

- Predict the future behaviour of the process state/output over the finite time horizon.
- Compute the future input signals on line at each step by minimizing a cost function under inequality constraints on the manipulated (control) and/or controlled variables.
- Apply on the controlled plant only the first vector of control variables computed for chosen control horizon and repeat the previous step with new measured input/state/output variables.

Therefore, the presence of the plant model is a necessary condition for the development of the predictive control. The success of MPC depends on the degree of precision of the plant model. In the most references the principal shortcoming of existing MPC-based

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control techniques is their inability to explicitly incorporate plant model uncertainty, Kothare et al. [9]. Thus, the present state of robustness problem in MPC can be summarized as follows:

- Analysis of robustness properties of MPC.
  Zafiriou nad Marchal [22] have used the contraction properties of MPC to develop necessary-sufficient conditions for robust stability of MPC with input and output constraints for SISO systems and impulse response model. Polak and Yang [17] have analyzed robust stability of MPC using a contraction constraint on the state.

- MPC with explicit uncertainty description.
  Zheng and Morari [23], have presented robust MPC schemes for SISO FIR plants, given uncertainty bounds on the impulse response coefficients. Some MPC consider additive type of uncertainty, de la Pena et al [16] or parametric (structured) type uncertainty using CARIMA model and linear matrix inequality, Bouzouita et al. [1]. In Lovaas et al. [11], the unstructured uncertainty is used for open-loop stable systems having input constraints. The robust stability can be established by choosing the large value for the control input weighting matrix R in the cost function. The authors proposed a new less conservative stability test for determining a sufficiently large control penalty R using bilinear matrix inequality (BMI). The other technique- constrained tightening to design of robust MPC have been proposed in Kuwata et al. [10]. The above approaches are based on idea of increasing the robustness of the controller by tightening the constraints on the predicted states. The mixed H_2/H_∞ control approach to design of MPC has been proposed by Orukpe et al [14]. Robust constrained MPC using linear matrix inequality (LMI) have been proposed by Kothare et al. [9], [3], [7], [21] where the polytopic model or structured feedback uncertainty model have been used. The main idea of Kothare et al. [9] is to use the infinite horizon control laws which for state feedback guarantee nominal stability.

In this paper, the necessary and sufficient robust stability conditions for MPC have been developed for a given predictive horizon and control horizon, using formulation through the polytopic system with output feedback, the generalized parameter dependent Lyapunov matrix $P(\alpha)$ is used to achieve robust stability. The proposed robust MPC ensures parameter dependent quadratic stability (PDQS) and guaranteed cost. The developed necessary and sufficient robust stability conditions for specific parameter dependent Lyapunov function reduce to sufficient ones and for robust stability analysis of MPC they are in the form of the set of LMIs. For robust MPC design which guarantees PDQS with guaranteed cost, the developed necessary and sufficient robust stability conditions for specific parameter dependent Lyapunov function reduce to sufficient ones in the form of bilinear matrix inequality.

The paper is organized as follows. Sec. 2 presents preliminaries and problem formulation. In sec. 3, the main results are given and finally, in sec. 4 the simple example using Yalmip BMI [12] solvers shows the effectiveness of the proposed method.

Hereafter, the following notational conventions will be adopted: given a symmetric matrix $P = P^T \in R^{n\times n}$, the inequality $P > 0 (P \geqslant 0)$ denotes matrix positive definiteness (semi-definiteness). Given two symmetric matrices P, Q, the inequality $P > Q$ indicates that $P - Q > 0$. The notation $x(t + k)$ will be used to define at time t k-steps ahead prediction of a system variable x from time t onwards under specified initial state and input scenario. I denotes the unity matrix of corresponding dimensions.

2. Problem formulation and preliminaries

Consider a time invariant linear discrete-time system

$$\begin{aligned} x(t+1) &= A(\alpha)x(t) + B(\alpha)u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n, u(t) \in R^m, y(t) \in R^l$ are state, control and output variables of the system, respectively; $A(\alpha), B(\alpha)$ belong to the convex set

$$\begin{aligned} S = \Bigg\{ A(\alpha) \in R^{n\times n},\ B(\alpha) \in R^{n\times m},\ A(\alpha) = \sum_{j=1}^{N} A_j\alpha_j \\ B(\alpha) = \sum_{j=1}^{N} B_j\alpha_j, \alpha_j \geqslant 0 \Bigg\} \\ \sum_{j=1}^{N} \alpha_j = 1. \end{aligned} \tag{2}$$

Matrix C is known matrix of corresponding dimension.

The cost function to be minimized is

$$J = \sum_{t=0}^{\infty} J(t) \tag{3}$$

where

$$J(t) = \sum_{k=1}^{N_2+1} x^T(t+k-1)Q_k x(t+k-1) + \sum_{k=1}^{N_u} u^T(t+k-1)R_k u(t+k-1),$$

$Q_k \in R^{n\times n}, R_k \in R^{m\times m}$ are positive semidefinite (definite) and definite matrices, respectively for all k; N_2, N_u are prediction and control horizon, respectively.

Let us denote

$$
\begin{aligned}
&Q = diag\{Q_i\}_{i=1,2\ldots N_2+1} \in R^{n(N_2+1)\times n(N_2+1)},\\
&R = diag\{R_i\}_{i=1,2\ldots N_u} \in R^{mN_u\times mN_u},\\
&v(t)^T = [u(t)^T \ldots u(t+N_u-1)^T] \in R^{1\times mN_u},\\
&z(t)^T = [x(t)^T \ldots x(t+N_2)^T] \in R^{1\times n(N_2+1)},
\end{aligned}
\tag{4}
$$

then for (3) one obtains

$$
J(t) = z(t)^T Q z(t) + v(t)^T R v(t). \tag{5}
$$

Predictive control algorithm with output feedback is given as follows

$$
u(t+i-1) = \sum_{j=1}^{N_2+1} F_{ij} y(t+j-1) \quad i = 1,2\ldots N_u \tag{6}
$$

where F_{ij} is gain matrix of corresponding dimension. Recall, that $y(t)$ is real system output in time t and $y(t+k)$ for $k \geqslant 1$ is predicted system output for time $t+k$. Assume that in (6) matrix $F_{ij} = 0$ for $N_u < i \leqslant N_2$. Note that for decentralized control structure the gain matrices F_{ij} have to be structured in corresponding way.

The problem studied in this paper is to design a robust output feedback model predictive control for a given horizon which ensures guaranteed cost and parameter dependent quadratic stability for the system (1) with control algorithm (6).

Definition 1 *Consider the system* (1). *If there exists a control law* u^* *and a positive scalar* J^* *such that the closed loop system* ((1) and (6)) *is stable and the closed loop cost function value* (3) *satisfies* $J \leqslant J^*$, *then* J^* *is said to be the guaranteed cost and* u^* *is said to be the guaranteed cost control law for system* (1).

3. Robust Model Predictive Control

For the given linear discrete-time system define the parameter dependent Lyapunov function in the form

$$
V(t) = [x(t)^T \ldots x^T(t+N_2-1)]\widetilde{P(\alpha)}[x(t)^T \ldots x^T(t+N_2-1)]^T \tag{7}
$$

where

$$
\widetilde{P(\alpha)} = \begin{bmatrix} P(\alpha) & 0 & \ldots & 0 \\ 0 & P(\alpha) & \ldots & 0 \\ 0 & 0 & \ldots & P(\alpha) \end{bmatrix}.
$$

Using denotation (4), the first difference of Lyapunov function is

$$\Delta V(t) = z(t)^T D(\alpha) z(t) \tag{8}$$

where

$$D(\alpha) = \begin{bmatrix} -P(\alpha) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & P(\alpha) \end{bmatrix} \in R^{n(N_2+1)\times n(N_2+1)}$$

and $P(\alpha) \in R^{n\times n}$ is a parameter dependent Lyapunov matrix [15]. Let us introduce the following denotation for (6)

$$K_i = [F_{i1}C \quad \dots \quad F_{iN_2+1}C] \in R^{m\times n(N_2+1)} \qquad i = 1,\dots,N_u \tag{9}$$

and

$$K = \begin{bmatrix} K_1 \\ K_2 \\ \dots \\ K_{N_u} \end{bmatrix} \in R^{N_u m\times n(N_2+1)} \tag{10}$$

then (6) can be rewritten as

$$v(t) = Kz(t) \tag{11}$$

and

$$J(t) = z^T(t)(Q + K^T RK)z(t). \tag{12}$$

From LQ theory, see e.i. [18], the following lemma holds.

Lemma 1 *Consider the closed-loop system* (1) *with control algorithm* (11). *Control algorithm* (11) *is the guaranteed cost control law for the closed-loop system if and only if there exist matrices $P(\alpha), K$ such that the following condition holds*

$$B_e = \Delta V(t) + J(t) = z^T(t)(D(\alpha) + Q + K^T RK)z(t) \leqslant 0. \tag{13}$$

Moreover, summarizing (13) *from initial time t_0 to $t \to \infty$, the following inequality is obtained*

$$-V(t_0) + J \leqslant 0. \tag{14}$$

Definition 1 and inequality (14) imply

$$J^* = V(t_0).$$

The main results of this paper are summarized in the following theorem.

Theorem 1 *Consider the discrete-time system* (1) *with model predictive control algorithm* (11) *and parameter dependent Lyapunov function* (7) *then the following statements are equivalent:*

- *Control algorithm* (11) *is the guaranteed cost control law with guaranteed cost* $J \leqslant J^*$.
- *There exist matrices*

$$P(\alpha) = P(\alpha)^T > 0, N \in R^{(N_2+1)n \times nN_2}, F_{i1}, F_{i2}, ..., F_{iN_2+1}, i = 1, 2, ..., N_u$$

such that the following bilinear matrix inequality (BMI) holds

$$B_{em} = NG + G^T N^T + D(\alpha) + Q + K^T RK \leqslant 0 \tag{15}$$

where

$$G = \begin{bmatrix} G_1 \\ G_2 \\ \dots \\ G_{N_2} \end{bmatrix} \in R^{N_2 n \times n(N_2+1)},$$

$$G_i = \begin{bmatrix} M_{i1} & \dots & M_{ii-1} & A_{cii} & M_{ii+1} & \dots & M_{iN_2+1} \end{bmatrix} \in R^{n \times n(N_2+1)},$$

$$M_{ij} = B(\alpha)F_{ij}C \quad j < i \quad or \quad j = i+2, \dots, N_2+1 \quad M_{ii+1} = B(\alpha)F_{ii+1}C - I,$$

$$A_{cii} = A(\alpha) + B(\alpha)F_{ii}C \quad i = 1, 2, \dots, N_2,$$

$$N = \begin{bmatrix} N_1 & \dots & N_{N_2} \end{bmatrix} \in R^{n(N_2+1) \times n(N_2)},$$

$$N_i = \begin{bmatrix} N_{i1}^T \\ N_{i2}^T \\ \dots \\ N_{iN_2+1}^T \end{bmatrix} \in R^{(N_2+1)n \times n)},$$

$N_{ij} \in R^{n \times n}$ *is unknown matrix with constant entries* $i = 1, 2, ..., N_2, j = 1, 2, ...N_2 + 1.$

As an example we show how concrete robust stability conditions can be obtained from (15) with guaranteed cost for one and two steps ahead prediction control. For one-step ahead model predictive control, inequality (15) can be rewritten as follows: $N_2 = 1$, $N_u = 1$,

$$N_1 = \begin{bmatrix} N_{11}^T \\ N_{12}^T \end{bmatrix} \in R^{2n \times n}, \quad G_1 = \begin{bmatrix} A_{c11} & M_{12} \end{bmatrix} \in R^{n \times 2n},$$

$N = N_1, \; G = G_1, \; A_{c11} = A(\alpha) + B(\alpha)F_{11}C, \; M_{12} = B(\alpha)F_{12}C - I,$

$$D(\alpha) = \begin{bmatrix} -P(\alpha) & 0 \\ 0 & P(\alpha) \end{bmatrix} \in R^{2n \times 2n}, \quad Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \in R^{2n \times 2n},$$

and $R = R_1$. Robust stability condition with guaranteed cost is given by following BMI:

$$\begin{bmatrix} N_{11}^T A_{c11} + A_{c11}^T N_{11} + Q_1 + C^T F_{11}^T R F_{11} C - P(\alpha) & (*)^T \\ M_{12}^T N_{11} + N_{12}^T A_{c11} + C^T F_{12}^T R F_{11} C & K_{22} \end{bmatrix} \leqslant 0 \tag{16}$$

where

$$K_{22} = N_{12}^T M_{12} + M_{12}^T N_{12} + P(\alpha) + Q_2 + C^T F_{12}^T R F_{12} C.$$

Control algorithm is given as follows

$$u(t) = F_{11} C x(t) + F_{12} C x(t+1) = F_{11} y(t) + F_{12} y(t+1).$$

For two step-ahead horizon $N_2 = N_u = 2$ and for the case $Q_i = 0,\ i = 1,2,3,\ R_1 = R_2 = 0$ the following equations and inequality are obtained

$$N_1 = \begin{bmatrix} N_{11}^T \\ N_{12}^T \\ N_{13}^T \end{bmatrix}, \quad N_2 = \begin{bmatrix} N_{21}^T \\ N_{22}^T \\ N_{23}^T \end{bmatrix}, \quad N = \begin{bmatrix} N_1 & N_2 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} A_{c11} & M_{12} & M_{13} \end{bmatrix}, \quad G_2 = \begin{bmatrix} M_{21} & A_{c22} & M_{23} \end{bmatrix}, \quad G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix},$$

$$A_{c11} = A(\alpha) + B(\alpha) F_{11} C, \quad M_{12} = B(\alpha) F_{12} C - I, \quad M_{13} = B(\alpha) F_{13} C,$$

$$A_{c22} = A(\alpha) + B(\alpha) F_{22} C, \quad M_{23} = B(\alpha) F_{23} C - I, \quad M_{21} = B(\alpha) F_{21} C.$$

The respective control algorithm is

$$u(t) = F_{11} C x(t) + F_{12} C x(t+1) + F_{13} C x(t+2),$$

$$u(t+1) = F_{21} C x(t) + F_{22} C x(t+1) + F_{23} C x(t+2).$$

Note, that as a receding horizon strategy is used, only $u(t)$ is sent to the real plant control, $u(t+1)$ is used for output model prediction $y(t+2)$. Note, that for model prediction [2] one can use any $(A_o, B_o, C) \in S$ from the uncertainty domain (2) for model described by (1); model $(A_o, B_o, C) \in S$ can be obtained by on-line identification, while the change of model parameters is slower than system dynamics. Robust stability condition is given by the following BMI

$$\begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{12}^T & g_{22} & g_{23} \\ g_{13}^T & g_{23}^T & g_{33} \end{bmatrix} \leqslant 0 \tag{17}$$

where

$$g_{11} = N_{11}^T A_{c11} + A_{c11}^T N_{11} + N_{21}^T M_{21} + M_{21}^T N_{21} - P(\alpha),$$

$$
\begin{aligned}
g_{12} &= N_{11}^T M_{12} + A_{c11}^T N_{12} + N_{21}^T A_{c22} + M_{21}^T N_{22},\\
g_{13} &= N_{11}^T M_{13} + A_{c11}^T N_{13} + N_{21}^T M_{23} + M_{21}^T N_{23},\\
g_{23} &= N_{12}^T M_{13} + M_{12}^T N_{13} + N_{21}^T M_{23} + A_{c22}^T N_{23},\\
g_{22} &= N_{12}^T M_{12} + M_{12}^T N_{12} + N_{21}^T A_{c22} + A_{c22}^T N_{21},\\
g_{33} &= N_{13}^T M_{13} + M_{13}^T N_{13} + N_{23}^T M_{23} + M_{23}^T N_{23} + P(\alpha).
\end{aligned}
$$

Proof The proof is based on the fact that first three terms in (15) correspond to the first difference od Lyapunov function (7) $\Delta V(t)$ on the solution of (1) with predictive control law (6) and last two terms of (15) are connected with cost function to be minimized. Then Lemma 1 implies that solution of (15) is guaranteed cost control law respective to (14) with parameter dependent Lyapunov function $P(\alpha)$. Since Lemma 1 provides necessary and sufficient condition for guaranteed cost control law for LQ problem (1), (11), (12), Theorem 1 and namely BMI (15) is to provide necessary and sufficient condition for guaranteed cost control law for robust predictive control (11). The proof follows the same lines of argument of Lemma 1 [18]. For illustration we show that Lemma 1 and inequality (15) are equivalent for the case $N_2 = N_u = 1$.

Sufficiency. The proof is provided for the case of robust stability conditions. Since the matrix $L_{11} = [I \quad -M_{12}^{-1}A_{c11}^T]$ has full row rank, (16) implies that

$$
L_{11}\{LHS(eq.(16))\}L_{11}^T = -P(\alpha) + A_{c11}^T (M_{12}^{-1})^T P(\alpha) M_{12}^{-1} A_{c11} \leqslant 0. \tag{18}
$$

Because the closed-loop system with one step-ahead control law is

$$
x(t+1) = -M_{12}^{-1} A_{c11} x(t)
$$

the obtained result (18) proves the sufficient robust stability conditions.

Necessity. Suppose that there exists symmetric positive definite matrix $P(\alpha)$ that robust stability condition (18) holds. Necessarily, there exists a scalar $\beta > 0$ such that

$$
-P(\alpha) + A_{c11}^T (M_{12}^{-1})^T P M_{12}^{-1} A_{c11} \leqslant -\beta (A_{c11}^T M_{12}^{-1})^T M_{12}^{-1} A_{c11}.
$$

The above inequality may be rewritten as follows

$$
A_{c11}^T (M_{12}^{-1})^T (P(\alpha) + \beta I) M_{12}^{-1} A_{c11} - P(\alpha) \leqslant 0. \tag{19}
$$

Applying Schur complement formula to (19)

$$
\begin{bmatrix} -P(\alpha) & * \\ (-A_{c11}^T (M_{12}^{-1})^T (P(\alpha)+\beta I))^T & -(P(\alpha)+\beta I) \end{bmatrix} \leqslant 0. \tag{20}
$$

Taking $N_{12} = -(M_{12})^T (P(\alpha) + \beta/2I)$ and $N_{11}^T = -A_{c11}^T (M_{12}^{-1})^T M_{12}^{-1} \beta/2$ after some manipulation one obtains

$$
\begin{bmatrix} S_{11\beta} & (*)^T \\ (N_{11}^T M_{12} + A_{c11}^T N_{12})^T & S_{22\beta} \end{bmatrix} \leqslant 0 \tag{21}
$$

where

$$S_{11\beta} = N_{11}^T A_{c11} + A_{c11}^T N_{11} - P(\alpha) + \beta(A_{c11}^T (M_{12}^{-1})^T M_{12}^{-1} A_{c11},$$

$$S_{22\beta} = N_{12}^T M_{12} + M_{12}^T N_{12} + P(\alpha).$$

When $\beta \to 0$ the inequality (16) is got, which proves the necessity. For guaranteed cost the proof of theorem goes the same way as given above. □

According to robust control references, there is no general and systematic way to formally determine $P(\alpha)$ as a function of A_{cii}. Such a matrix $P(\alpha)$ is called the parameter dependent Lyapunov matrix (PDLM) and for a particular structure of $P(\alpha)$ defines the parameter dependent quadratic stability (PDQS). For the case of $P(\alpha) = P$ the quadratic stability conditions are obtained. Formal approach to determine $P(\alpha)$ for real convex polytopic uncertainty can be found in many references e.i Peaucelle et al. [15] and references cited therein. In the existing studies, however, the PDLFs employed are restricted to those affine in the uncertain parameters in the form

$$\begin{gathered} P(\alpha) = \sum_{i=1}^{N} P_i \alpha_i, \qquad \sum_{i=1}^{N} \alpha_i = 1, \\ P_i = P_i^T > 0, \quad i = 1, 2, \ldots, N. \end{gathered} \tag{22}$$

To decrease the conservatism of (22) arising from affine PDLF, more recently, the use of polynomial PDLF has been proposed in different forms. In this paper we use the PDLM given by (22). Note that above BMIs (15), (16) and (17) are affine with respected to α. Substituting (2) and (22) to (15), (16) and (17) for the polytopic model predictive control system, the robust parameter dependent quadratic stability with guaranteed cost conditions are obtained in the form of corresponding BMIs. Note that for concrete $P(\alpha)$ necessary and sufficient conditions in Theorem 1 reduces to sufficient ones.

4. Example

The proposed MPC design procedure based on Theorem 1 is illustrated in this section: one step-ahead predictive controller respective to decentralized PI structure is designed for the uncertain system. The considered system is of 3rd order with 2 inputs and 2 outputs (i.e. decentralized controller with 2 PI loops). The nominal 3rd order system is augmented into 5th order to include integration parts of both PI controllers and finally the 5th order system is converted for a chosen sampling period to discrete one:

$$A_0 = \begin{bmatrix} .6 & .0097 & .0143 & 0 & 0 \\ 0.012 & 0.9754 & 0.0049 & 0 & 0 \\ -.0047 & 0.0101 & 0.46 & 0 & 0 \\ 0.0488 & 0.0002 & .0004 & 1 & 0 \\ -.0001 & 0.0003 & 0.0488 & 0 & 1 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0.0425 & 0.0053 \\ 0.0052 & 0.010 \\ 0.0024 & 0.0474 \\ 0.0011 & 0.00010 \\ 0 & 0.0012 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

uncertainty matrices

$$A_u = \begin{bmatrix} 0.0012 & 0 & 0 & 0 & 0 \\ 0 & 0.0023 & 0 & 0 & 0 \\ 0 & 0.0001 & 0.0032 & 0 & 0 \\ 0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0.21 & 0 \\ 0 & 0.21 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} * 0.001.$$

The corresponding convex polytope (2) is given by its vertices:

$$A_1 = A_0 + A_u, \quad A_2 = A_0 - A_u, \quad B_1 = B_0 + B_u, \quad B_2 = B_0 - B_u.$$

The cost function is

$$J(t) = x^T(t)Q_1x(t) + x(t+1)^T Q_2x(t+1) + u(t)^T Ru(t).$$

The one-step ahead predictive control law is :

$$u(t) = F_{11}y(t) + F_{12}y(t+1) = F_{11}Cx(t) + F_{12}Cx(t+1)$$

where control gain matrices F_{11} and F_{12} correspond to output feedback for present and one-step ahead predicted future outputs respectively. Matrix F_{11} has decentralized control structure: two PI control loops for two real proces output feedback; F_{12} is full matrix respective to one step-ahead predictive output feedback taken from predictive model.

Two forms of Lyapunov function are considered and compared:

- quadratic stability case with Lyapunov function matrix $P(\alpha) = P$, where BMI (16) is solved simultaneously for both vertices with the same matrix P,

- affine parameter dependent Lyapunov function

$$P(\alpha) = \alpha_1 P_1 + \alpha_2 P_2, \alpha_1 + \alpha_2 = 1$$

where (16) is solved simultaneously for matrices P_1 and P_2 respective to vertices A_1 and A_2.

To solve BMI (16), Yalmip with PENBMI solver has been used. Computed results for both cases are summarized as follows: Case $Q_1 = 0.5I,\ \ Q_2 = 5I,\ \ R = 0.1I_r$.

Quadratic stability:

$$F_{11} = \begin{bmatrix} -1.217 & 0 & -2.4052 & 0 \\ 0 & -1.1441 & 0 & -0.5007 \end{bmatrix}.$$

$$F_{12} = \begin{bmatrix} 0.1531 & -0.4077 & -3.1907 & 1.2205 \\ -3.0652 & -1.3169 & -1.2004 & -3.25 \end{bmatrix}.$$

Maximal absolute value of eigenvalue for polytopic system is $\max_i Eig(A_{cii}) = 0.9798$ and guaranteed cost is $\max_i Eig(P_i) = 494.35$.

Parameter dependent quadratic stability:

$$F_{11} = \begin{bmatrix} -8.8007 & 0 & -1.2016 & 0 \\ 0 & -16.4102 & 0 & -20.6045 \end{bmatrix},$$

$$F_{12} = \begin{bmatrix} 39.0631 & 10.0902 & 13.1678 & 2.7348 \\ -23.8798 & 19.8237 & -14.1435 & 20.2016 \end{bmatrix}.$$

Maximal absolute value of eigenvalue for polytopic system is $\max_i Eig(A_{cii}) = 0.9854$ and guaranteed cost is $\max_i Eig(P_i) = 490.9$. Dynamic behavior of closed-loop nominal system is in Fig 1 and Fig 2.

5. Conclusion

The paper addresses the problem of designing a parameter dependent quadratic stability static output/state feedback for N_2 step ahead model predictive control for linear polytopic systems without constraints. The new robust stability conditions for N_2 step ahead model predictive control are given in Theorem 1. The simple example using Yalmip BMI solvers shows the effectiveness of the proposed method.

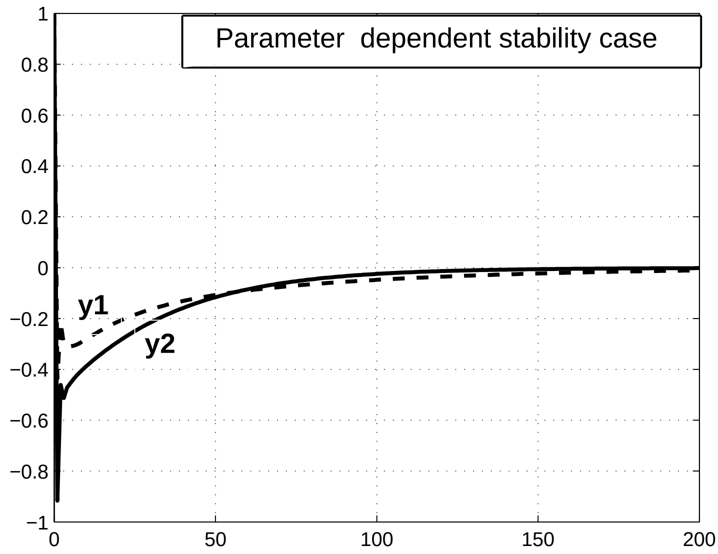

Figure 1. Outputs versus time in [s] for quadratic stability case.

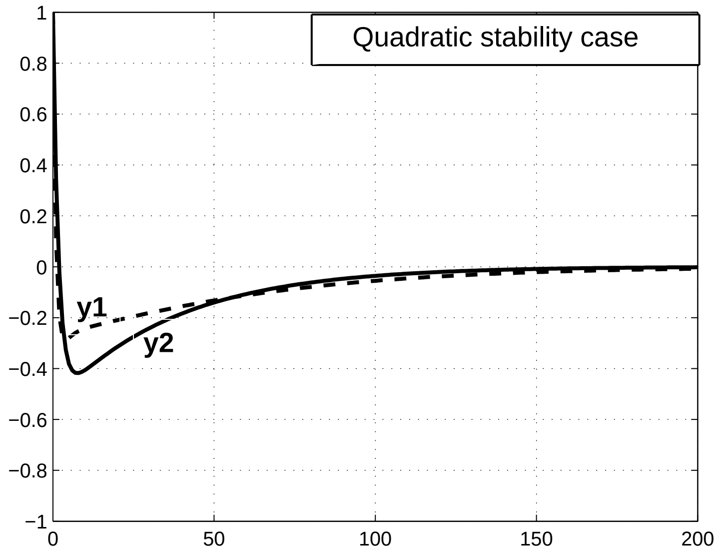

Figure 2. Outputs versus time in [s] for parameter dependent quadratic stability.

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