10.2478/v10170-011-0018-y

Archives of Control Sciences
Volume 22(LVIII), 2012
No. 2*, pages 161–174*

An active electromagnetic stabilization of the Leipholz column

TOMASZ SZMIDT and PIOTR PRZYBYŁOWICZ

We study the application of electromagnetic actuators for the active stabilization of the Leipholz column. The cases of the compressive and tensional load of the column placed in air and in water are considered. The partial differential equation of the column is discretized by Galerkin's procedure, and the stability of the obtained control system is evaluated by the eigenvalues of its linearization. Four different methods of active stabilization are investigated. They incorporate control systems based on feedback proportional to the transverse displacement of the column, its velocity and the current in the electromagnets. Conditions in which these strategies are effective in securing safe operation of the column are discussed in detail.

Key words: follower load, flutter, closed-loop control, active stabilization, electromagnetic actuators

1. Introduction

The classical example of a nonconservative elastic system with follower load is the Leipholz column, in which the acting force is distributed uniformly along the deflected beam. A sufficiently high value of the load yields destabilization of the column. Then a small disturbance results in self-excited lateral flutter vibrations. The magnitude of load at which such a situation happens is called the critical.

A considerable effect on the stability of this system is exerted by the internal and external damping, see for example [5], the external support of the column [1], the foundation on which it is laid [8] and also by the action of piezoelectric elements [6].

An application of electromagnetic actuators for stabilization of rotating shafts was proposed in [3]. It was shown that the magnetic force passively generated by actuators can bring an increase of the rotational velocity at which a shaft loses stability. The efficiency of passive stabilization was also confirmed in systems of columns subjected to a follower load [7], [13] and fluid-conveying pipes [12].

The Authors are with Institute of Machine Design Fundamentals, Faculty of Automotive and Construction Machinery Engineering, Warsaw University of Technology, Narbutta 84, 02-524 Warsaw, Poland. E-mails: tomasz.szmidt@gmail.com, piotrp@ipbm.simr.pw.edu.pl

This research has been supported by the National Science Centre in Kraków, Poland, under the predoctoral grant N N501 047740.

Received 12.02.2012. Revised 30.04.2012.

It was proved that the active stabilization of shafts can be even more effective [4]. The goal of this work is to examine the active magnetic stabilization of the Leipholz column. The voltage applied to actuators is controlled in a closed loop feedback. We analyze various types of control – depending on the transverse displacement and velocity of the column as well as depending on the electric current in coils of the electromagnets.

The Galerkin two-mode discretisation based on cantilever beam eigenfunctions is applied to the partial differential equation governing the motion of the column. The resultant ordinary equations are coupled with ordinary equations of electro-magnetodynamics of actuators. Then, the stability is determined by numerical calculations of eigenvalues of the linear system.

2. Examined system

Consider a Leipholz column subjected to the follower load of density q, with electromagnetic actuators (electromagnets) attached at a distance x_e from the column support, Fig. 1. Lateral vibrations in the plane of symmetry $x-w$ are investigated. We assume

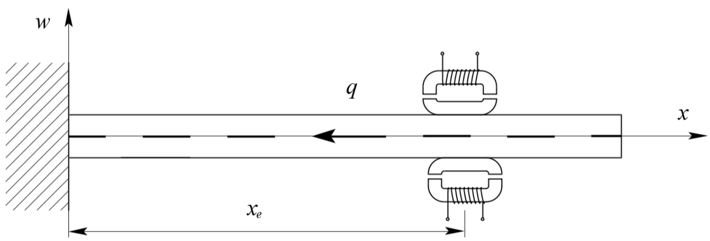

Figure 1. Analyzed system – Leipholz column with electromagnetic actuators

small deflections $w = w(x,t)$, so a linear model is applicable. The column is made of the viscoelastic Kelvin–Voigt material and subjected to external viscous damping. It is slender enough to make Bernoulli–Euler theory valid, and its dimensions do not change under the load. The effect of gravitation is neglected.

The dynamics of the column is governed by the following equation (see for example [5])

$$(m+M_a)\frac{\partial^2 w}{\partial t^2} = -\gamma^\star\frac{\partial w}{\partial t} - \left(1+\beta^\star\frac{\partial}{\partial t}\right)EI\frac{\partial^4 w}{\partial x^4} + q\,(x-L)\,\frac{\partial^2 w}{\partial x^2} + F_m\delta_e, \qquad (1)$$

where L and EI denote the length and flexural rigidity of the beam, m, M_a – column mass and mass of the fluid which surrounds the column and is accelerated by its motion, per unit length, $\beta^\star, \gamma^\star$ – coefficients of the internal and external damping, F_m – resultant magnetic force generated by actuators, in the direction of w axis, δ_e – Dirac's delta function concentrated at x_e.

The „+” sign at the load density q corresponds to compression, in the case of tension we change the sign to „−”, so the load value is always positive.

We shall analyse the approximate solution $\widetilde{w}(x,t)$ to partial equation (1). This approximation is obtained by the Galerkin procedure based on the first two cantilever eigenfunctions (scaled to the column length and normalized with respect to integrals of their squares) $W_j(x),\ 0 \leqslant x \leqslant L,\ j = 1,2$

$$\widetilde{w}(x,t) = W_1(x)T_1(t) + W_2(x)T_2(t). \tag{2}$$

Galerkin's method yields two second-order ordinary equations with unknown functions $T_j(t),\ t \geqslant 0,\ j = 1,2$. These functions determine deflection of the first and second modes of the column. We reduce the order of obtained ordinary equations by introduction of new functions $u_1 = T_1$, $u_2 = \dot{T}_1$, $u_3 = T_2$, $u_4 = \dot{T}_2$.

In our previous studies, we found it convenient to determine the state of actuators by the value of magnetic induction in their cores. However, in engineering practice, the measurement of induction is cumbersome, so we have decided to set electric currents which flow in the coils of electromagnets as the state variables. It implies that we have to neglect the effect of magnetic hysteresis and assume a linear primary magnetization curve in a form $B = \mu^\star \mu_0 H$, where $\mu^\star = 1989$ is the relative magnetic permeability of steel, and μ_0 is the permeability of vacuum. The dissipative effect of hysteresis is not significant (but noticeable) [10], and the BH curve can be approximated by a linear one if the voltage applied to electromagnets is not too high (this is because it does not take the effect of magnetic saturation into account) [9], thus both simplifications are acceptable.

Let us define $u_{5,6} = i_{1,2} - U_0/R$, where U_0 is a constant voltage on each of the two electromagnets when the column is in the middle – initial – equilibrium position, and $i_{1,2}$ – electric currents in the coils of the bottom and upper electromagnet, respectively. The control is the deviation of the voltage, $y_{1,2} = U_{1,2} - U_0$. We obtain the following law of motion

$$\begin{aligned}
\dot{u}_1 &= u_2 \\
\dot{u}_2 &= \frac{1}{m+M_a}\Big(-\gamma^\star u_2 + EI\big(a_{11}u_1 + a_{21}u_3\big) + q\big(b_{11}u_1 + b_{21}u_3\big) + \\
&\qquad\qquad + \beta^\star EI\big(a_{11}u_2 + a_{21}u_4\big) + F_m W_1(x_e)\Big) \\
\dot{u}_3 &= u_4 \\
\dot{u}_4 &= \frac{1}{m+M_a}\Big(-\gamma^\star u_4 + EI\big(a_{12}u_1 + a_{22}u_3\big) + q\big(b_{12}u_1 + b_{22}u_3\big) + \\
&\qquad\qquad + \beta^\star EI\big(a_{12}u_2 + a_{22}u_4\big) + F_m W_2(x_e)\Big) \\
\dot{u}_5 &= \left(\frac{2\dot{w}_e}{2\big(z+w_e\big)+\frac{l}{\mu^\star}} - \frac{2\big(z+w_e\big)+\frac{l}{\mu^\star}}{\mu_0 A N^2}R\right)\left(u_5 + \frac{U_0}{R}\right) + \frac{2\big(z+w_e\big)+\frac{l}{\mu^\star}}{\mu_0 A N^2}\big(y_1 + U_0\big) \\
\dot{u}_6 &= \left(-\frac{2\dot{w}_e}{2\big(z-w_e\big)+\frac{l}{\mu^\star}} - \frac{2\big(z-w_e\big)+\frac{l}{\mu^\star}}{\mu_0 A N^2}R\right)\left(u_6 + \frac{U_0}{R}\right) + \frac{2\big(z-w_e\big)+\frac{l}{\mu^\star}}{\mu_0 A N^2}\big(y_2 + U_0\big),
\end{aligned} \tag{3}$$

where w_e and $\dot{w}_e$ denote the deflection and velocity of the column at the point where the actuators are attached

$$w_e = W_1(x_e)u_1 + W_2(x_e)u_3, \qquad \dot{w}_e = W_1(x_e)u_2 + W_2(x_e)u_4 \tag{4}$$

and the resultant magnetic force F_m amounts to

$$F_m = \mu_0 A N^2 \left(\left(\frac{u_6 + \frac{U_0}{R}}{2(z - w_e) + \frac{l}{\mu^\star}} \right)^2 - \left(\frac{u_5 + \frac{U_0}{R}}{2(z + w_e) + \frac{l}{\mu^\star}} \right)^2 \right). \tag{5}$$

In the equations above, the coefficients a_{ij}, b_{ij} depend only on the base functions and their values are given below:

$$a_{11} = -0.77265 \qquad a_{12} = 0 \qquad b_{11} = -0.21456 \qquad b_{12} = -0.59089 \tag{6}$$

$$a_{21} = 0 \qquad a_{22} = -30.3449 \qquad b_{21} = 2.16857 \qquad b_{22} = 3.32357 \tag{7}$$

The column considered is a cantilever pipe of length $L = 2000$ mm, external and internal diameter $D = 11.11$ mm and $d = 10$ mm. It is made of steel. The system is surrounded by air ($\gamma^\star = 0.00097$ kg m^{-1}s^{-1}, $M_a = 0.00018$ kg m^{-1}) or water ($\gamma^\star = 0.17086$ kg m^{-1}s^{-1}, $M_a = 0.11158$ kg m^{-1}). The internal damping coefficient equals $\beta^\star = 0.00019$. The values of $\gamma^\star$ and M_a have been obtained by the method described in [2]. The value of $\beta^\star$ has been calculated with our own procedure, based on known amount of the energy dissipated in one cycle of vibration of the first bending mode. Each magnetic circuit is $l = 300$ mm long, with cross-sectional radius $a = 2$ mm and area $A = \pi a^2$. On the electromagnets cores, $N = 1000$ copper wire coils of resistance $R = 2.38\ \Omega$ (copper) are wound. The mass of cores embedded in the column accounts for approx. 4 % of the total mass of the system, thus their effect on dynamics can be neglected. The gap in magnetic circuits in the middle equilibrium of the column (i.e. between the cores embedded in the column and electromagnets) amounts to $z = 4$ mm. The constant σ denotes electric conductivity of the steel cores.

3. Closed–loop control

Let us introduce the vector variables of state $\boldsymbol{u} = [u_1, \ldots, u_6]^{\mathrm{T}}$ and control $\boldsymbol{y} = [y_1, y_2]^{\mathrm{T}}$. Additionally, let $\boldsymbol{f} = [f_1, \ldots, f_6]^{\mathrm{T}}$, where $f_i = f_i(\boldsymbol{u}, \boldsymbol{y}, q)$, $i = 1, \ldots, 6$, denote right-hand sides of equations (3). Then the dynamical system can be written in a vector form

$$\dot{\boldsymbol{u}} = \boldsymbol{f}(\boldsymbol{u}, \boldsymbol{y}, q). \tag{8}$$

The zero values of state and control $(\boldsymbol{u}, \boldsymbol{y}) = (\mathbf{0}, \mathbf{0})$ constitute the equilibrium of the system for all values of load q, i.e. $\boldsymbol{f}(\mathbf{0}, \mathbf{0}, q) = 0$, and $q = q_{cr}$ is the critical load value, for which this steady equilibrium loses stability.

Let $\boldsymbol{\psi}$ be a differentiable function on the state space $\mathcal{U} \subset \mathbb{R}^6$ with values in $\mathbb{R}^2$, satisfying $\boldsymbol{\psi}(\mathbf{0}) = \mathbf{0}$. We look for a control $\boldsymbol{y} = \boldsymbol{\psi}(\boldsymbol{u})$, so that system (8) is stable at $\boldsymbol{u} = \mathbf{0}$ for a given load q. Let us denote $\widetilde{\boldsymbol{f}}(\boldsymbol{u}, q) = \boldsymbol{f}(\boldsymbol{u}, \boldsymbol{\psi}(\boldsymbol{u}), q)$. The asymptotical stability of system (8) is equivalent to stability of the Jacobian matrix $\widetilde{\boldsymbol{A}}$ of function $\widetilde{\boldsymbol{f}}$, evaluated at $\boldsymbol{u} = \mathbf{0}$

$$\widetilde{\boldsymbol{A}} = \frac{\partial \widetilde{\boldsymbol{f}}}{\partial \boldsymbol{u}}(\mathbf{0}, q) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}(\mathbf{0}, \mathbf{0}, q) + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\psi}}(\mathbf{0}, \mathbf{0}, q) \frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{u}}(\mathbf{0}). \tag{9}$$

So without loss of generality we can confine ourselves to the analysis of linear control laws

$$\boldsymbol{y} = \boldsymbol{K}\boldsymbol{u}, \tag{10}$$

where $\boldsymbol{K} = (K_{ij})_{2\times 6}$ is the control matrix.

Because the assumed model is a good approximation of a real system providing that the induction in cores is not too high, so the norms of considered matrices have to be bounded.

The observed motion is dominated by the first base function. It means that the control should not depend on the displacement u_3 and velocity u_4 of W_2. Thus we assume $K_{13} = K_{14} = K_{23} = K_{24} = 0$. We shall investigate the efficiency of the following simplest methods of control:

1. ***Control depending on the transverse displacement of the column***

   In this case, the only non-zero elements of control matrix $\boldsymbol{K}$ are K_{11} and K_{21}. Each coefficient of the characteristic polynomial of the matrix $\widetilde{\boldsymbol{A}}$ takes the form

$$C(K_{11} - K_{21}) + D, \tag{11}$$

   where C and D do not depend on the control, and one of them may be equal to 0. From this, we get that K_{11} and K_{21} should have opposite signs, because otherwise the controls of both electromagnets would act against each other. Moreover, any given values of the coefficients of this polynomial can be obtained if $K_{11} = -K_{21} = \kappa_w$. Thus, the control matrix takes the form

$$\boldsymbol{K} = \boldsymbol{K_w} = \begin{bmatrix} \kappa_w & 0 & 0 & 0 & 0 & 0 \\ -\kappa_w & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12}$$

2. ***Control depending on the velocity***

   Similarly as above, we obtain the control matrix for the differential case

$$\boldsymbol{K} = \boldsymbol{K_v} = \begin{bmatrix} 0 & \kappa_v & 0 & 0 & 0 & 0 \\ 0 & -\kappa_v & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{13}$$

3. ***Control depending on the electric current***

   In practice, this method of control is the most important one, because it is easier to measure the current in the coils of electromagnets than the deflection or velocity of the column. The system is symmetric, so the simple control of voltage applied to the lower electromagnet in a function of the current flowing through its coil is the same as the control of voltage on the upper electromagnet in a function of the current in the other coil, i.e. $K_{15} = K_{26} = \kappa_s$. It is the same in the case of the cross control of one actuator in a function of the state of the other one, $K_{25} = K_{16} = \kappa_c$.

   The coefficients of the characteristic polynomial of the Jacobian matrix $\widetilde{\boldsymbol{A}}$ are in the form of

$$E\kappa_s + F\kappa_c + G(\kappa_s^2 - \kappa_c^2), \tag{14}$$

   where the constants E, F and G do not depend on the control. From this, we can see that a combination of both methods of control ($\kappa_s \neq 0$, $\kappa_c \neq 0$) might be more efficient than an application of ***simple*** ($\kappa_c = 0$) and ***cross*** ($\kappa_s = 0$) control individually. However, for simplicity, we shall analyze them separately, assuming

$$\boldsymbol{K} = \boldsymbol{K_s} = \begin{bmatrix} 0 & 0 & 0 & 0 & \kappa_s & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_s \end{bmatrix}, \quad \boldsymbol{K} = \boldsymbol{K_c} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \kappa_c \\ 0 & 0 & 0 & 0 & \kappa_c & 0 \end{bmatrix}. \tag{15}$$

In next sections, the stability of actively controlled column shall be evaluated by numerical calculations of eigenvalues of Jacobian matrix $\widetilde{\boldsymbol{A}}$, for various gain factors. The eigenvalue with the highest real part is decisive for the stability of the system, and will be used for calculations of the critical load.

4. Stabilization of the column subjected to compression

Firstly, we will study the case when passive action of the magnetic force improves stability of the column. The actuators are attached at 33% of the column length (measuring from its support), and the whole system is put in air.

Assume the control which depends on the transverse displacement of the column. Figure 2 (on the left) depicts the relation between the critical load and voltage on the actuators: without active control, with constant control $\kappa_w = -300$ V and with optimal control (stabilization) $\kappa_w = \kappa_{opt}$. The optimal control is a function of voltage, and for every given U_0 is defined as the value of gain factor (here: κ_w) for which q_{cr} is as high as possible. The active control can significantly improve stability of the system.

In the figure beside, presenting the relation between the critical load and the control for $U_0 = 1$ V, one can see that the maximum load is reached for the control that yields instability of the unloaded column. The sign of κ_w is negative, thus the control increases the voltage on the electromagnet towards whom the column is deflected, which is not an intuitive result.

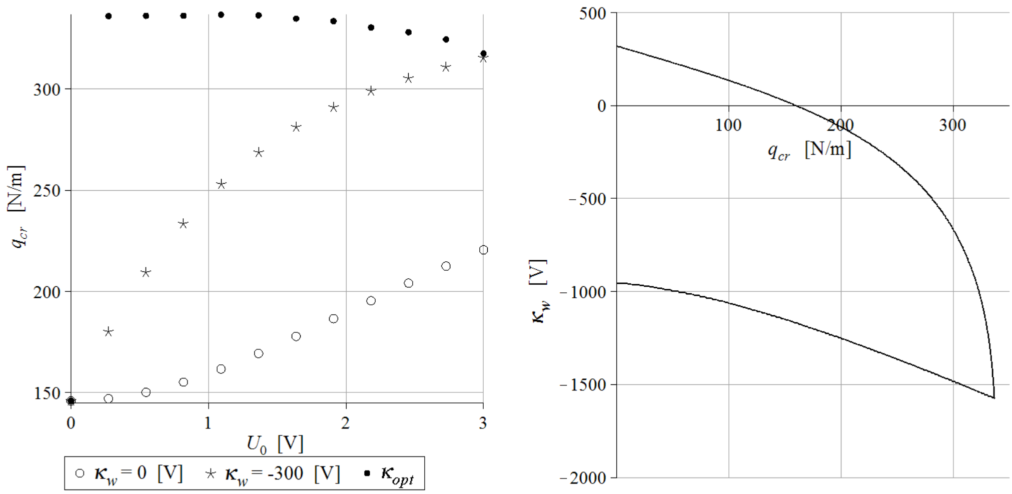

Figure 2. Critical compressive load for the control depending on the displacement and without the active control (on the left) and the relation between this load and the control for $U_0 = 1$ V (right), $x_e = 0.33L$, air.

Figure 3 regards the active control which depends on the velocity of the column. Its efficiency is smaller than the proportional control, but the optimal control does not destabilize the unloaded column. In this case, the coefficient $\kappa_v > 0$. That meets expectations – the motion of the column towards one of the actuators implies a growth of voltage on the opposite side.

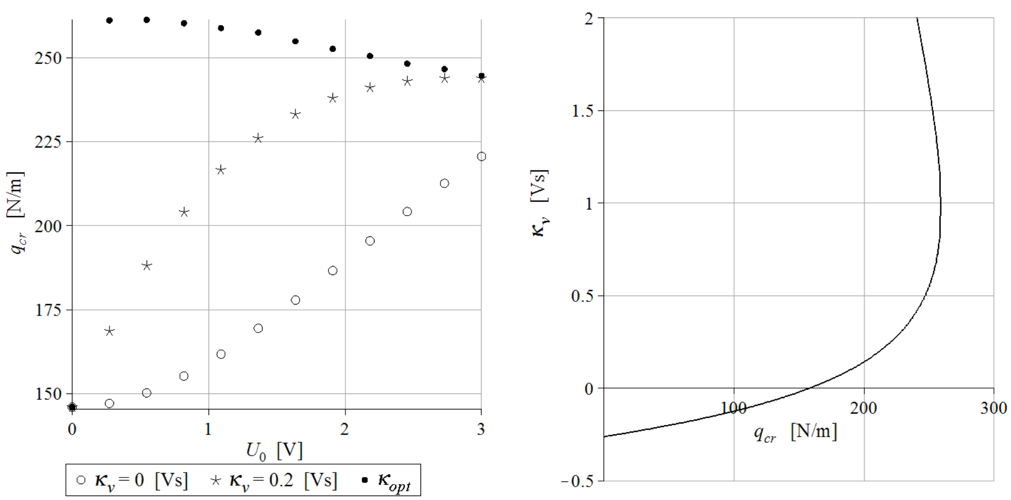

Figure 3. Critical compressive load for the control depending on velocity and without active control (on the left) and the relation between this load and the control for $U_0 = 1$ V (right), $x_e = 0.33L$, air.

As for the simple control depending on the current, it is the least efficient of all methods if the voltage applied to actuators is low, and stabilizes the column to a similar extent as the control depending on the velocity if the voltage is greater, Fig. 4. The motion of the column in a direction of one of the actuators results in an increase of the current on the opposite side (the Lenz rule), a stabilizing control should emphasize this effect, thus $\kappa_s > 0$.

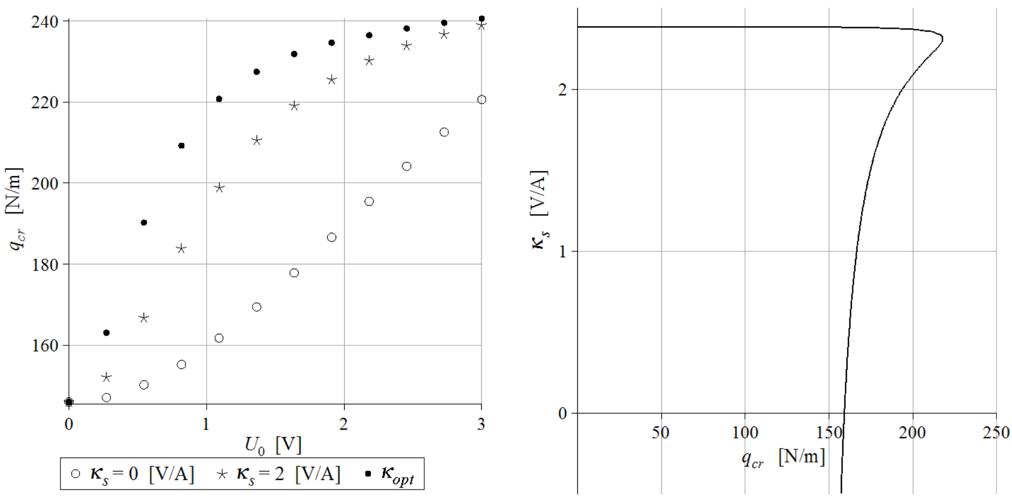

Figure 4. Critical compressive load for the simple control depending on the current and without active control (on the left) and the relation between this load and the control for $U_0 = 1$ V (right), $x_e = 0.33L$, air.

The cross control depending on the current stabilizes the system similarly to the simple one.

If the same system is immersed in water, then the passive electromagnetic damping destabilizes the column. Still, the proportional and differential controls can increase the critical load above the value for the system without actuators (the latter – in much less extent and only when the voltage is not too high), Fig. 5.

In this case, both methods of control depending on the current are only able to diminish the destabilising effect of the passive action of actuators. Moreover, the relation between the efficiency of control and the values of coefficients κ_s, κ_c for various levels of voltage turns out to be interesting. For example, when the voltage applied to the actuators is low, the simple control improves stability if κ_s is negative. However, if a positive value of κ_s is sufficiently high, then the same growth of the critical load will occur. On the other hand, greater levels of voltage reverse this relation, with such a difference that the negative values of κ_s always deteriorate stability.

If we move the actuators away from the column support to the position $x_e = 0.67L$, then their passive action is more effective and stabilizes both the system put in air and in water. An application of active control yields qualitatively the same conclusions as in the case of $x_e = 0.33L$ and low damping of the external surrounding (air).

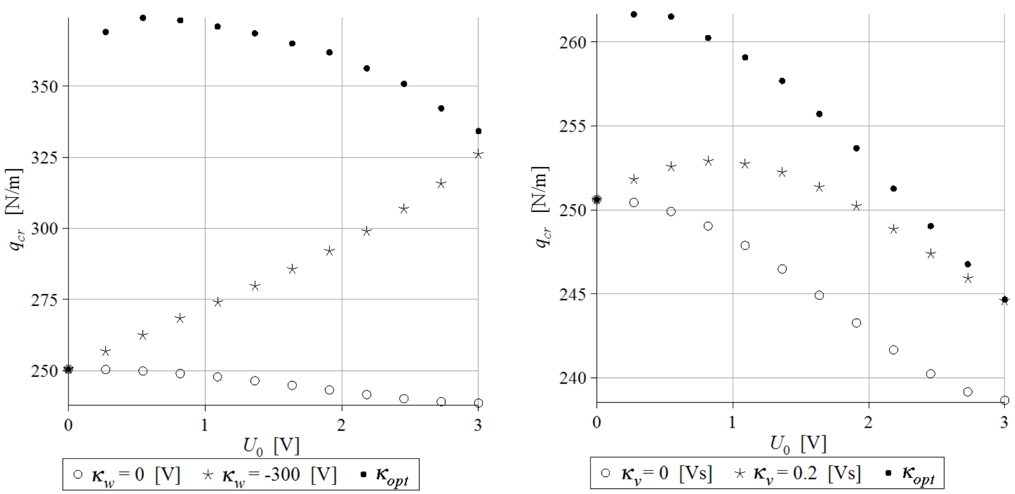

Figure 5. Critical compressive load for control depending on the transverse displacement (on the left) and on the velocity (right), $x_e = 0.33L$, water.

5. Active control in the case of follower tension

We shall study the system immersed in air only, because one can prove that if the external damping is sufficiently high (here: in water), the critical follower tension does not exist and the column remains stable for all values of the load.

As in the previous section, we are beginning with the situation when the passive magnetic damping raises the critical follower force. Unlike in the compression case, we need to move the actuators further away from the support. Let us assume $x_e = 0.50L$. Both the control depending on the deflection and the simple control depending on the current remarkably intensify the passive action of the actuators. The latter enables reaching a greater value of the critical load, but a higher voltage is required. The proportional control is more efficient also for low U_0, Fig. 6.

As far as the differential feedback is concerned, it is – unlike in the case of compression – slightly more efficient than the proportional one. The cross control depending on the current results in the same q_{cr} as the simple one.

The same as in the compression case, the optimal stabilization depending on the transverse displacement destabilizes the column with zero load. Here, the sign of κ_w is opposite to the sign in the compression case, i.e. it is positive. The differential feedback and both types of the control depending on the current are safer, which means that if $q_{cr} = 0$ then the actively controlled system is stable. In such circumstances, the control coefficients have the same sign as for the compression.

Now let us move to the situation when the passive action of the actuators reduces the critical load, assuming $x_e = 0.33L$. In Figs. 7, 8 and 9, relations between the critical tension and the gains corresponding, respectively, to the proportional, velocity and

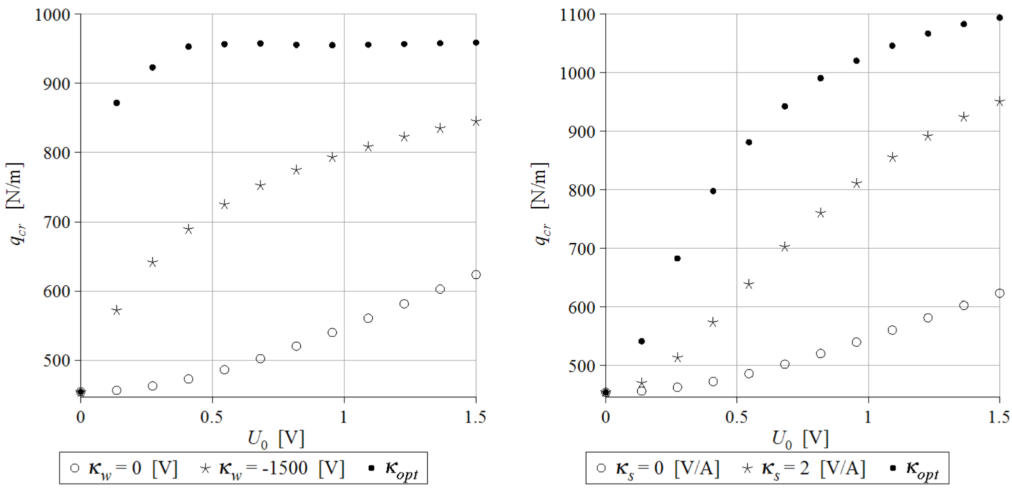

Figure 6. Critical tensional load for control depending on the transverse displacement (on the left) and on the current (right), $x_e = 0.50L$, air.

current (a simple one) strategies, for various levels of voltage, are presented. In the first

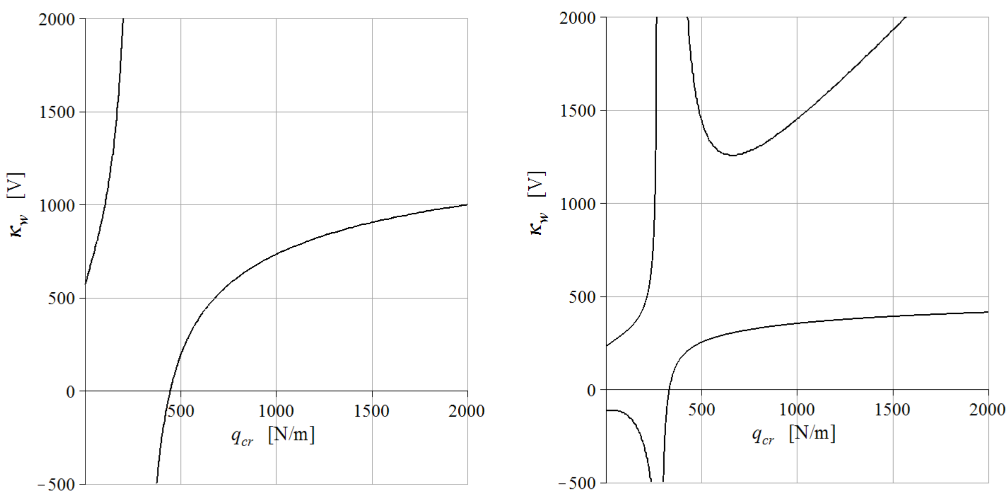

Figure 7. Relation between the critical tension and the control depending on the displacement for $U_0 = 0.5$ V (on the left) and $U_0 = 3$ V (right), $x_e = 0.33L$, air.

two cases, one can theoretically reach the stability for an arbitrarily large tensional force. Thus, for a given U_0 we define κ_{opt} in a different way than so far, i.e. as the maximum (if positive) or minimum (negative) value of κ_w, κ_v for which the unloaded column is still stable. As for the signs of the gain factors improving the stability, the value of κ_w is

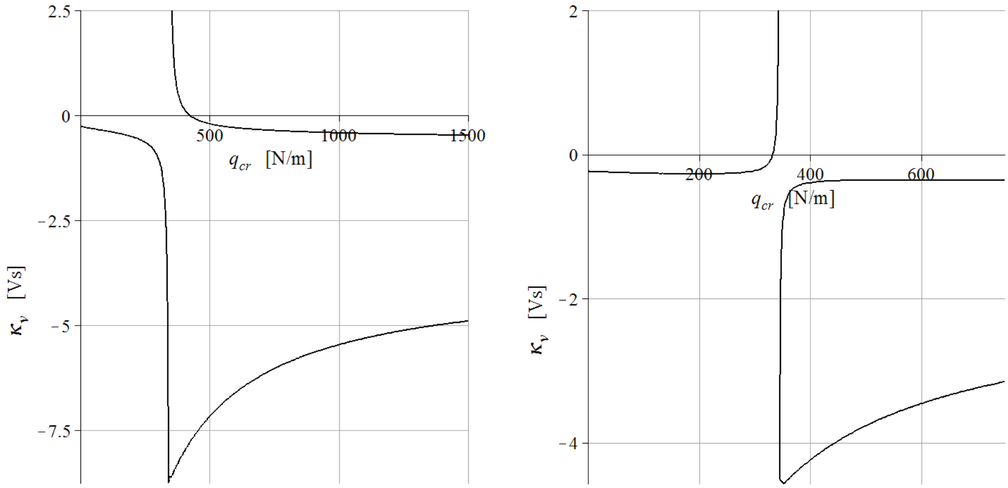

Figure 8. Relation between the critical tension and the active control depending on the velocity for $U_0 = 1$ V (on the left) and $U_0 = 3$ V (right), $x_e = 0.33L$, air.

always positive (once again – conversely to the compression case), whereas the signs of stabilizing κ_v, κ_s depend on the voltage U_0.

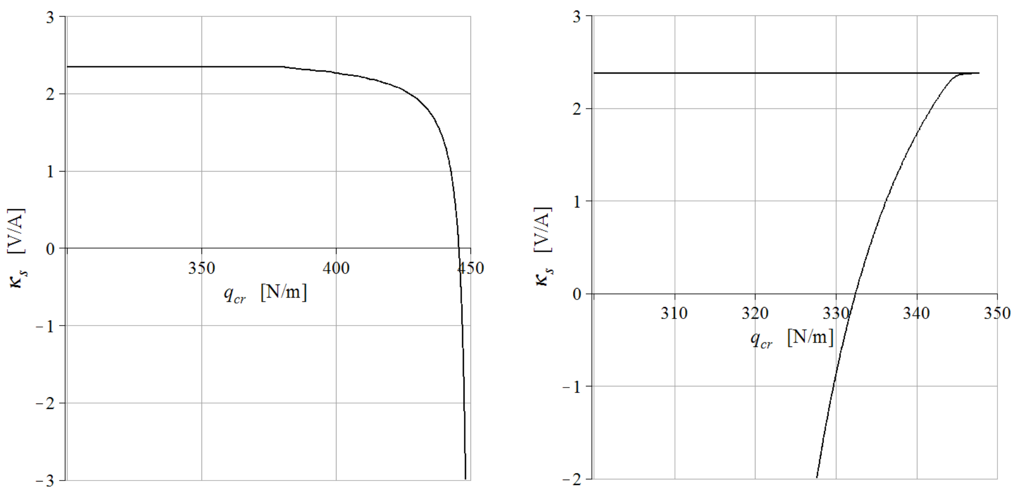

Figure 9. Relation between the critical tension and the simple control depending on the current for $U_0 = 0.5$ V (on the left) and $U_0 = 3$ V (right), $x_e = 0.33L$, air.

Figure 10 depicts reachable in this situation values of the critical follower tension for the active stabilization depending on the displacement and the simple active approach depending on the current. The differential control is slightly less effective than the proportional one, and the cross control – than the simple one. As in the compression case, only mechanical methods of control enable full compensation of the passive magnetic destabilization, and an increase of the critical load above the value without actuators.

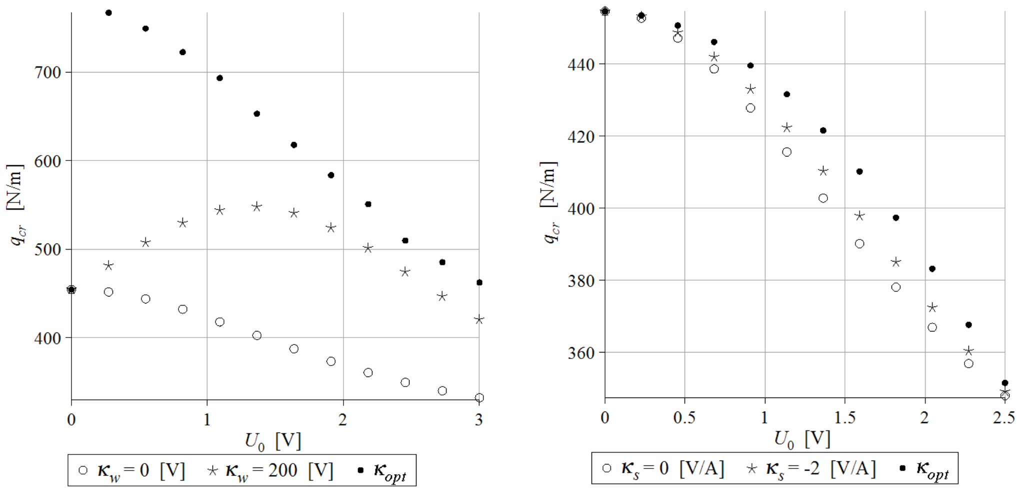

Figure 10. Critical tensional load for the control depending on the displacement (on the left) and on the current (simple one, right), $x_e = 0.33L$, air.

6. Conclusions

We analyzed an active closed-loop stabilization of the Leipholz column by means of electromagnetic actuators. All four investigated methods of control – depending on the displacement and velocity of the column, and both methods depending on the current in coils of electromagnets – turned out to be effective in increasing the critical follower compression as well as the critical tensional load of the column.

The effect of control significantly depends on the position at which the actuators are attached to the column, the voltage supplying actuators, the external damping and direction of the load.

The most effective are mechanical controls depending on current state of the column, i.e. on the displacement and velocity. They enable reaching the highest critical loads, which exceed the critical loads of the column without actuators also when the passively generated magnetic force destabilizes the system. Moreover, they work very well also under a low voltage applied to the actuators.

The simple and cross control depending on the current work similarly to each other. They also allow one to raise the critical load remarkably, however they should be used when the passive method stabilizes the system. They also need a higher voltage supplying the electromagnets.

References

[1] M.A. De Rosa and C. Franciosi: The Influence of an Intermediate Support on the Stability Behaviour of Cantilever beams Subjected to Follower Forces. *J. of Sound and Vibration*, **137** (1990), 107-115.

[2] S. Kirstein, M. Mertesdorf and M. Schönhoff: The influence of a viscous fluid on the vibration dynamics of scanning near-field optical microscopy fiber probes and atomic force microscopy cantilevers. *J. of Applied Physics*, **84**(4), (1998), 1782-1790.

[3] W. Kurnik: Magnetic Stabilization of a Rotor with Hydrodynamic Bearings. *Machine Dynamics Problems*, **7** (1994), 117-133.

[4] W. Kurnik: Active Magnetic Antiwhirl Control of a Rigid Rotor Supported on Hydrodynamic Bearings. *Machine Dynamics Problems*, **10** (1995), 21-36.

[5] W. Kurnik and M. Pękalak: Stability and Bifurcation Analysis of the Non-Linear Damped Leipholz Column. *J. of Sound and Vibration*, **152**(2), (1992), 285-294.

[6] W. Kurnik and P.M. Przybyłowicz: Nonlinear Behavior of the Leipholz Column Actively Stabilized by Piezoelements. *Proc. of the Int. Symp. on Active Control of Sound and Vibration ACTIVE'95*, (Eds. S. Sommerfeldt, H. Hamada), Newport Beach, CA, USA, (1995), 139-150.

[7] W. Kurnik, P. Przybyłowicz and T. Szmidt: An application of electromagnetic phenomena to damping of mechanical vibrations. A chapter in: A. Tylikowski, Influence of parameter modifications on structure vibrations. Warsaw University of Technology, 2009, 53-87.

[8] H.P. Lee: Dynamic Stability of a Tapered Cantilever Beam on an Elastic Foundation Subjected to a Follower Force. *Int. J. of Solids and Structures*, **33**(10), (1996), 1409-1424.

[9] P. Przybyłowicz and T. Szmidt: Magnetic damping of vibrations of a harmonic oscillator. *Modelowanie Inżynierskie*, **35** (2008), 101-106, (in Polish).

[10] P. Przybyłowicz and T. Szmidt: Electromagnetic damping of a mechanical harmonic oscillator with the effect of magnetic hysteresis. *J. of Theoretical and Applied Mechanics*, **47**(2), (2009), 259-273.

[11] P. Przybyłowicz and T. Szmidt: Nonlinear response of a harmonically driven oscillator in magnetic field. *Archives of Control Sciences*, **20**(1), (2010), 19-30.

[12] P. PRZYBYŁOWICZ and T. SZMIDT: Critical flow velocity in a pipe with electromagnetic actuators. *XVIIth PSCS Scientific Workshops*, Krynica Zdrój, Poland, (2010), (in Polish).

[13] T. SZMIDT and P. PRZYBYŁOWICZ: Bifurcation analysis of the Leipholz column with electromagnetic actuators. *IInd Congress of Polish Mechanics*, Poznań, Poland, (2011), (in Polish).