

Probabilistic model of fibrillation currents created by superposition of two shocking currents with different frequencies

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Abstract: The paper presents the probabilistic model of fibrillation currents containing two components with different frequencies. An analysis was conducted of the threat of ventricular fibrillation which occurs in consequence of the electric shock with the highest permissible contact shocking voltage of the network frequency (50 Hz), taking into account the threat caused by the second component of the voltage which has the frequency higher than the network frequency. The sample results of calculations apply to the probability of the ventricular fibrillation in case of a shock caused by the highest permissible contact shocking voltage, for the defined time of shock duration, without and with the participation of an additional voltage component with higher frequency. The formula has been presented for the calculation of the highest permissible contact shock voltages with taking into account the voltage component of the frequency higher than the network frequency. The results of calculations indicate that a considerable reduction of the highest permissible contact shock voltage is necessary in order to compensate for a growth of the ventricular fibrillation threat caused by the presence of an additional component with the frequency other the network frequency. This applies in particular to the long shock duration times and low frequencies (up to 500 Hz) of an additional component of the shocking voltage.

Key words: fibrillation current, superposition of shocking currents, frequency 50 Hz and higher, threat of ventricular fibrillation, highest contact shock voltages

1. Introduction

In various threat situations the shocking voltages contain two components, usually one with the network frequency 50(60) Hz and the second with the higher frequency. Therefore it is necessary to elaborate a method of the assessment of the threat of ventricular fibrillation in case of the superposition of two shocking currents with different frequencies.

The paper presents a description of the elaborated probabilistic model of the fibrillation currents containing two components with different frequencies as well as the results of the analysis of the ventricular fibrillation threat caused by the voltage being a superposition of two components, one having the 50 Hz frequency and the second having the higher frequency. The principles were formulated for the calculation of the permissible contact shock voltages in the vicinity of the grounded electrical equipment, taking into account the presence of the higher frequency component in the short-circuit current.

2. Description of probabilistic model

A description of probabilistic model of the fibrillation currents containing two components with different frequencies has been elaborated [3] basing on the analysis of the research results by Azhibaev [1], taking into account the findings of Knickerbocker published in [2].

Azhibaev has conducted a research of the ventricular fibrillation in dogs (10 dogs with a mass of a dozen kg, lengthwise shock path, shock duration time 3 s), applying to them the electric shock being a superposition of two sinusoidal currents: one with a frequency of 50 Hz and the other with the frequencies of: 100, 150, 200, 500, 1000, 2000, 5000 and 10000 Hz. The selected results of Azhibaev research are presented at the Fig. 1. Basing on these results, the average values of the effective fibrillation currents $I_{fsr}(50, f)$ as well as of their components $I_{fsr}(50/f)$ and $I_{fsr}(f/50)$ have been calculated and presented in Table 1. A condition $I_{fsr}(50/f) = I_{fsr}(f/50)$ was held during the described research.

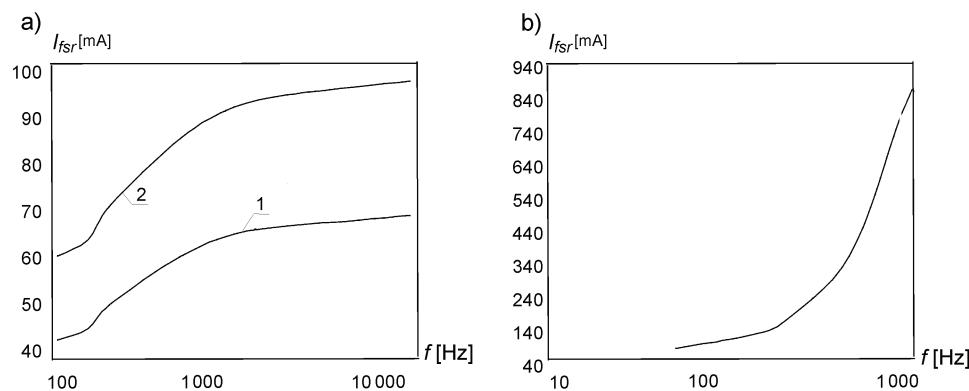


Fig. 1. Dependence of average values of the effective fibrillation currents on the frequency, in presence of two components of the current, with the frequencies of 50 Hz and higher, published by Azhibaev [2]:

a) for the components of 50 Hz frequency and the frequency marked at the scale $(1 - I_{fsr}(50/f)) = I_{fsr}(f/50)$) and the effective currents ($2 - I_{fsr}(50, f)$), and b) only for the “pure” current $I_{fsr}(f)$ with the frequency marked at the scale

The results of Azhibaev (Fig. 1a) indicate that the effective values of the fibrillation current having two components $I_{fsr}(50, f)$ – one $I_{fsr}(50/f)$ with the frequency of 50 Hz and the other $I_{fsr}(f/50)$ with the frequency from the range of 100-170 Hz, are lower than the effective

values of the “pure” fibrillation current $I_{fsr}(50)$ with the frequency of 50 Hz ($I_{fsr}(50) = 69$ mA – Fig. 1b).

Table 1. Average values of the effective fibrillation currents having two components, with the frequencies of 50 Hz and higher (calculated basing on the results published by Azhibaev – Fig. 1a)

f [Hz]	100	150	200	500	1000	2000	5000	10000
$I_{fsr}(50/f) = I_{fsr}(f/50)$ [mA]	42.56	45.0	50.0	60.0	64.6	66.3	67.5	68.5
$I_{fsr}(50,f)$ [mA]	60	63.45	70.5	84.6	91.1	93.5	95.2	96.5

In order to reflect the shares of the individual components in the fibrillation current it is practical to use the appropriately defined relative values of these components [3]. The following relative values may be applied for the results of Azhibaev research:

$$i_{wsr}(50/f) = \frac{I_{fsr}(50/f)}{I_{fsr}(50)}, i_{wsr}(f/50) = \frac{I_{fsr}(f/50)}{I_{fsr}(f)} = \frac{I_{fsr}(f/50)}{I_{fsr}(50) \cdot k_{fsr}}, \quad (1)$$

where: $I_{fsr}(50/f)$ and $I_{fsr}(f/50)$ is the average effective values of the fibrillation current components, with the frequencies of 50Hz and f , $I_{fsr}(50)$ and $I_{fsr}(f)$ is the average effective values of the “pure” fibrillation currents, with the frequencies of 50 Hz and f ; k_{fsr} is the coefficient expressing a dependence of the average values of “pure” fibrillation currents on frequency, expressed by the quotient of $I_{fsr}(f)$ and $I_{fsr}(50)$.

The relative values of the fibrillation current components $i_{wsr}(50/f)$ and $i_{wsr}(f/50)$, calculated on the base of the function presented at Fig. 1, are shown in Tab. 2, and the values of k_{fsr} coefficient are shown in Tab. 3.

Table 2. Average relative values of two components of the fibrillation current, with the frequencies of 50 Hz and higher, for dogs with a mass of a dozen kg, at the lengthwise shock path and the shock duration time 3s (computed basing on the results published by Azhibaev – Fig. 1)

f [Hz]	100	150	200	500	1000	2000	5000	10000
$i_{wsr}(50/f)$	0.617	0.652	0.725	0.870	0.936	0.961	0.978	0.993
$i_{wsr}(f/50)$	0.425	0.372	0.333	0.158	0.075	–	–	–

Table 3. Values of k_{fsr} coefficient expressing a dependence of the values of “pure” fibrillation currents on the frequency, computed basing on the results of Azhibaev research on dogs with a mass of a dozen kg, at the lengthwise shock path and the shock duration time 3s

F [Hz]	50	100	150	200	500	1000
k_{fsr}	1	1.450	1.754	2.174	5.500	12.536

The analysis of the dependence of the average relative values of two components of the fibrillation current $i_{wsr}(50/f)$ and $i_{wsr}(f/50)$ on the frequency f (Tab. 2) has demonstrated that the following conditions may hold:

$$i_{wsr}(50/f) + i_{wsr}(f/50) \geq 1 \quad \text{or} \quad [i_{wsr}(50/f)]^2 + [i_{wsr}(f/50)]^2 = 1. \quad (2)$$

It was found that if one of the current components has a frequency of 50 Hz and the other over 700 Hz, then the second of (2) conditions may be applied for the description of the component shares in the fibrillation current.

In case of fibrillation current containing the direct component and the sinusoidal component, the following formula is valid, according to Knickerbocker:

$$[i_{wsr}(0/f)]^2 + [i_{wsr}(f/0)]^2 = 1. \quad (3)$$

Having in mind the above, the following formula has been proposed for the reflection of shares of the individual relative values of components in the fibrillation current, averages found by Azhibaev:

$$[i_{wsr}(50/f)]^2 + 2k_\phi \cdot i_{wsr}(50/f) \cdot i_{wsr}(f/50) + [i_{wsr}(f/50)]^2 = 1, \quad (4)$$

while

$$k_\phi = \frac{2f}{\pi 50} \sin \frac{\pi 50}{2f}, \quad (5)$$

where: k_ϕ is the coefficient taking into account a random coincidence in creating fibrillation by the current components with the frequencies of 50 Hz and f , the other notations as above.

The proposed formula (4) is universal because it may, depending on the value of k_ϕ coefficient, assume any of (2) versions, and because it fulfills the condition (3), found by Knickerbocker.

Table 4 shows the values of k_ϕ coefficient for the components of the fibrillation current, with one frequency of 50 Hz and the second as in the Table, computed on the base of (5) formula. The value of the coefficient for $f = 50$ Hz has been calculated with the assumption that a phase shift of two components with the same frequency has a random character.

Table 4. Values of k_ϕ coefficient taking into account a coincidence of two current components (one with the frequency of 50 Hz and the other as in the Table) in creating the ventricular fibrillation

f [Hz]	0 (direct current)	50	100	150	200	500	1 000
k_ϕ	0	0.636	0.900	0.955	0.975	0.996	≈ 1

The application of (4) and (5) formulae to the approximation of Azhibaev results has shown the correctness of these formulae. The inaccuracies of the average values of individual components (Tab. 1) did not exceed a few percent.

The formulae (4) and (5) may be applied also to the relative median values of individual components of the fibrillation current. Such approach is possible because a standard deviation of the logarithmic normal distribution of the fibrillation current values is in practice constant [4]. In such case a formula defining the relative median value of the fibrillation current component with a frequency of 50 Hz, derived from the condition (4), is the following:

$$i_{wm}(50/f) = \sqrt{[i_{wm}(f/50)]^2 \cdot (k_\varphi^2 - 1) + 1} - k_\varphi \cdot i_{wm}(f/50), \quad (6)$$

while

$$i_{wm}(50/f) = \frac{I_{fm}(50/f)}{I_{fm}(50)}, \quad i_{wm}(f/50) = \frac{I_{fm}(f/50)}{I_{fm}(f)} = \frac{I_{fm}(f/50)}{I_{fm}(50) \cdot k_f}, \quad (7)$$

where: $i_{wm}(50/f)$ and $i_{wm}(f/50)$ is the relative median values of individual components of fibrillation current, with the frequencies of 50 Hz and f , $I_{fm}(50/f)$ and $I_{fm}(f/50)$ is the median values of individual components of fibrillation current, with the frequencies of 50 Hz and f , $I_{fm}(50)$ and $I_{fm}(f)$ is the median values of “pure” fibrillation currents, with the frequencies of 50 Hz and f .

A value $i_{wm}(50/f)$ may be treated as a relative measure of decreasing a median value of the fibrillation current component with the frequency 50 Hz, due to the presence of an additional component with the other frequency in the shocking circuit. After the transformation of formula (6) the following formula for the calculation of the median value of this current is obtained:

$$I_{mf}(50/f, t) = \sqrt{\left[\frac{I_{rm}(f/50, t)}{k_{f,t}} \right]^2 \cdot (k_\varphi^2 - 1) + I_{mf}^2(50, t) - k_\varphi \frac{I_{rm}(f/50, t)}{k_{f,t}}}, \quad (8)$$

while

$$k_{f,t} \frac{I_{mf}(f, t)}{I_{mf}(50, t)}, \quad (9)$$

where: $I_{rm}(f/50, t)$ is the median value of the shocking current component with the frequency f (taking into account a presence of a component with the frequency 50 Hz) $k_{f,t}$ is the coefficient applicable for all shock duration times t , $I_{mf}(f, t)$ and $I_{mf}(50, t)$ is the median values of “pure” fibrillation currents, for the frequencies f and 50 Hz.

Median values of “pure” fibrillation currents $I_{mf}(f, t)$ and $I_{mf}(50, t)$ are calculated with the use of the following formula [4]:

$$I_{mf}(f, t) = \frac{1}{k_d} \sqrt{\frac{m}{m_m} \left\{ \frac{(I_{f1\max} + I_{f2\max} \cdot (f/50) - I_{f\min} k_f)F}{(I_{f1\max} + I_{f2\max} \cdot (f/50) - I_{f\min} k_f)t^2 + F} + I_{f\min} k_f \right\}}, \quad (10)$$

while the coefficient $I_{f\min}$ expressing a dependence of a median value of fibrillation current on the frequency in case of long duration shock with $t > 10$ s is expressed by the formula:

$$k_f = \sqrt{\left(\frac{41.1}{f + 8.5} \right)^2 + 2 \cdot 10^{-4} f^2}, \quad (11)$$

where: k_d is the constant coefficient dependent on the shock path d , m and m_m is the mass of human body and median mass of tested animals in kg ($m = 75$ kg, $m_m = 12$ kg), $I_{f1\max}$, $I_{f2\max}$ is the median values of fibrillation current in mA (for t from 5 to 50 ms), $I_{f\min}$ is the median

value of fibrillation current in mA (minimum for $f = 50$ Hz and $t \geq 10$ s), k_f is the coefficient expressing a dependence of the value of I_{fmin} on the frequency f , F is the constant coefficient in mAs², t is the shock duration time in s.

The formulae (8-11) are universal. They may be applied in case when only one component of the shocking current is present and they may also be adapted to be used in case when the shocking current has two components with any frequencies.

3. Analysis of the threat of ventricular fibrillation

A probability of the occurrence of ventricular fibrillation in the defined electric shock conditions is a good measure of fibrillation threat. In case of a shock caused by superposition of two currents, one with the frequency of 50 Hz and the other with higher frequency, this probability is equal to the distribution function of normal distribution, expressed by the following formula [3]:

$$q_f = \Phi\left(\frac{\ln[I_{rm}(50/f, t)/I_{mf}(50/f, t)]}{\sigma_{\ln U_f}}\right) = \Phi(u_\alpha), \quad (12)$$

while a random variable u_α of the standardized normal distribution and a standard deviation $\sigma_{\ln U_f}$ are calculated from the condition:

$$u_\alpha = \frac{\ln \frac{I_{rm}(50/f, t)}{I_{mf}(50/f, t)}}{\sigma_{\ln U_f}}, \quad (13)$$

$$\sigma_{\ln U_f} = \sqrt{\sigma_{\ln I_f}^2 + 2\rho\sigma_{\ln I_f}\sigma_{\ln I_r} + \sigma_{\ln I_r}^2}, \quad (14)$$

where: $I_{rm}(50/f, t)$ is the median value of the 50 Hz component in case of presence of the second component of the shocking current having the frequency f , $I_{mf}(50/f, t)$ is the median value of fibrillation current with the frequency 50 Hz in case of presence of an additional component of the shocking current, having the frequency f , calculated from the formula (8), $\sigma_{\ln U_f}$, $\sigma_{\ln I_f}$, and $\sigma_{\ln I_r}$ is the standard deviations of the logarithmic normal distribution of the components with the frequency 50 Hz, subsequently for the voltage, fibrillation current and the 50 Hz component of the shocking current; ρ is the coefficient of correlation between the random values of $\ln I_r$ and $\ln I_f$.

The probability q_f should be read from the normal distribution tables, basing on the calculated random variable u_α of the standardized normal distribution. In order to make the calculation of probability more efficient, the approximation of the distribution function of normal distribution may be used, with the application of the appropriate expression [4].

The value $I_{rm}(50/f, t)$ is calculated with the following formula:

$$I_{rm}(50/f, t) = \frac{U_r(50)}{Z_m(U_r, 50)}, \quad (15)$$

while

$$U_r = \sqrt{U_r^2(50) + U_r^2(f)}, \quad (16)$$

where: U_r , $U_r(50)$ and $U_r(f)$ is the effective values of shocking voltage and its components with the frequencies of 50 Hz and f , $Z_m(U_r, 50)$ is the median value of human body impedance, for the shocking voltage U_r and the frequency 50 Hz, computed with the use of the expressions published in [5].

The value $I_{mf}(50/f, t)$ is computed using the formula (8), after the previous calculation of the median values of “pure” fibrillation currents $I_{mf}(f, t)$ and $I_{mf}(50 \text{ Hz}, t)$ with the use of formula (10) and after the calculation of median value of the shocking current component with the frequency f — $I_{rm}(f/50, t)$.

The value $I_{rm}(f/50, t)$ is computed with the use of the formula:

$$I_{rm}(f/50, t) = \frac{U_r(f)}{Z_m(U_r, f)}, \quad (17)$$

where: $Z_m(U_r, f)$ is the median value of human body impedance, computed for the shocking voltage U_r and for the frequency f with the use of the expressions published in [5]. The other notations as in case of formulae (15) and (16).

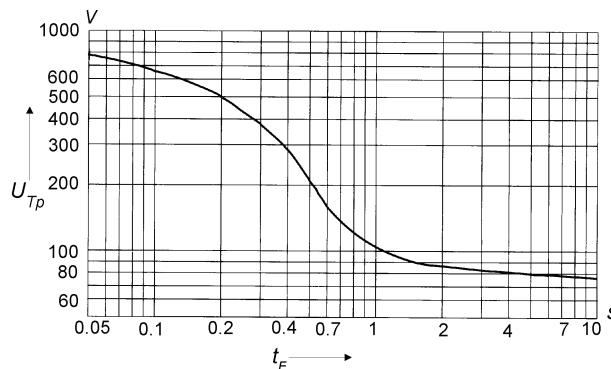


Fig. 2. Dependence of the highest permissible contact shock voltages U_{Tp} on the damage time t_F (duration of shocking current flow), according to the requirements of the standard [6]

Calculation of the standard deviation $\sigma_{\ln U_f}$ with the use of formula (14) requires the previous knowledge of the standard deviations $\sigma_{\ln I_f}$ and $\sigma_{\ln I_r}$ as well as the coefficient of correlation ρ . The values of $\sigma_{\ln I_f}$ and ρ are considered to be constant [4] and the standard deviations are considered to be equal $\sigma_{\ln I_r} = \sigma_{\ln Z}$. Value of $\sigma_{\ln Z}$ is computed with the use of the formula published in [5].

An analysis of the threat of ventricular fibrillation has been conducted [3] for two major cases of the electric shock:

- 1) current with the frequency of 50 Hz, voltage equal to the highest permissible contact shock voltage U_{Tp} and shock duration time t_F (Fig. 2),
- 2) current with two components, one with the frequency 50 Hz and voltage U_{Tp} and the second with the frequency higher $U_r(f)$ than 50 Hz and voltage being a multiple of U_{Tp} , both with shock duration time t_F .

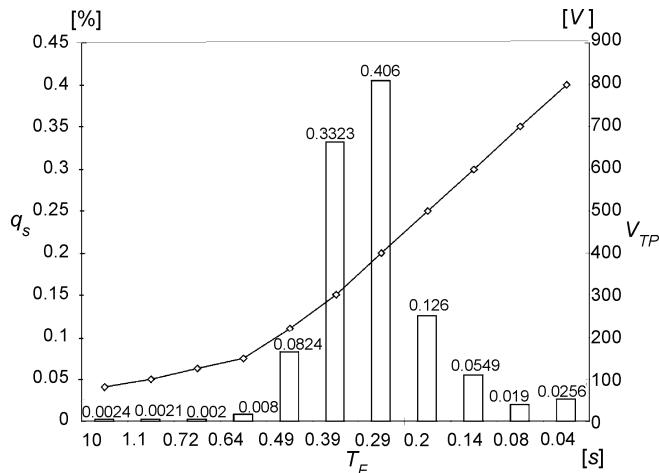


Fig. 3. Probabilities q_f of the occurrence of ventricular fibrillation at the shock caused by the permissible values of the contact shock voltages U_{Tp} with the frequency of 50 Hz and the shock duration time t_F (epidermis wet, shock path left hand – left foot)

In both cases the probabilities of ventricular fibrillation have been calculated for the following conditions of electric shock: shock path – left hand-left foot, ambient temperature – 20°C, area of contact between electrodes and human body – $S_1 = S_2 = 100 \text{ cm}^2$ human body mass $m = 75 \text{ kg}$.

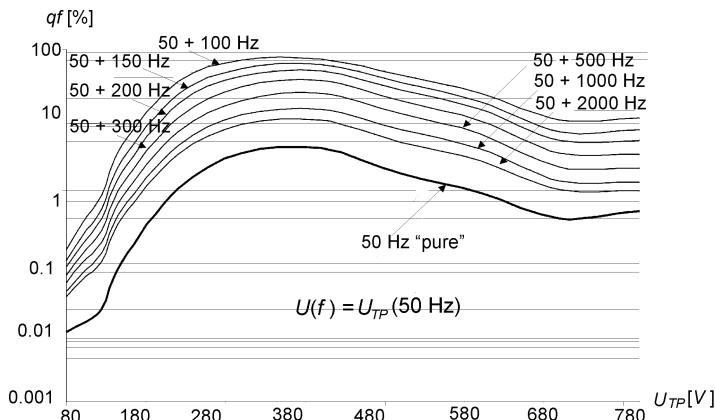


Fig. 4. Dependence of the probability of ventricular fibrillation q_f on the shock voltages, “pure” U_{Tp} with the frequency of 50 Hz (bold curve), and containing two components, with the frequencies of 50 Hz and higher $U_r(f) = U_{Tp}(f)$, shock duration time t_F (epidermis wet, shock path left hand – left foot)

The analysis of the computed probabilities of ventricular fibrillation q_s for the first case of electric shock has shown [3] that the applied criteria of the effectiveness of anti-shock protection for the high voltage equipment [6] do not ensure the equal level of safety for all voltages $U_{Tp} = f(t_F)$. Fig. 3 presents the selected results of such calculations for the case of wet epidermis. The results are not substantially different in cases of dry epidermis and saltwater wet epidermis.

The analysis of the computed probabilities of ventricular fibrillation q_f for the second case of electric shock has shown [3] that a level of threat, resulting from a presence of the second component of the shocking voltage $U_r(f)$ is considerably higher. For example:

- with $U_r(f) = U_{Tp}$ and $f = 100$ Hz the level of threat grows 10-fold and with $f = 2000$ Hz only 2-3-fold;
- with $U_r(f) = 2U_{Tp}$ and $f = 100$ Hz the level of threat grows 100 times and with $f = 2000$ Hz only 10 times;
- with $U_r(f) = 3U_{Tp}$ and $f = 100$ Hz a probability of fibrillation is in practice equal to 100% and with $f = 2000$ Hz it is only few percent.

Figure 4 shows the selected results of calculations for $U_r(f) = U_{Tp}$ the case of wet epidermis. The results are not substantially different in cases of dry epidermis and saltwater wet epidermis.

4. Highest permissible contact shock voltages with the frequency of 50 Hz and an additional component with the higher frequency

The highest permissible contact shock voltages should be set for the frequency of 50 Hz at such level that the presence of an additional component with the frequency higher than the network frequency will not increase a threat of the ventricular fibrillation.

The formula for the calculation of such corrected values of the highest permissible contact shock voltages with the frequency of 50 Hz and with an additional component of higher frequency $U_{Tp}(50\text{ Hz}/f)$ has been derived on the base of the formula (8) which defines $I_{mf}(50\text{ Hz}/f)$. The following formula was obtained:

$$U_{TP}(50/f) = \sqrt{\frac{U_{TP}^2(f) \cdot \left[\frac{Z_m(U_r, 50)}{Z_m(U_r, f)} \right]^2}{k_{f,t}^2} \left(k_\phi^2 - 1 \right) + U_{TP}^2 - k_\phi \cdot \frac{U_r(f)}{k_{f,t}} \cdot \frac{Z_m(U_r, 50)}{Z_m(U_r, f)}}, \quad (18)$$

where: U_{Tp} – highest permissible contact shock voltage for the frequency 50 Hz; $U_r(f)$ – additional component of the shock voltage with the frequency f , $Z_m(U_r, 50)$ and $Z_m(U_r, f)$ – median values of human body impedance for the shock voltage U_r and two frequencies of 50 Hz and f , calculated with the use of expressions published in [4], $k_{f,t}$ – quotient of median values of “pure” fibrillation currents with the frequencies of f and 50 Hz, computed according to the formula (9), k_ϕ – coefficient taking into account a coincidence of the current components in creating the fibrillation, computed with the use of formula (5).

Sample results of calculations of the highest permissible contact shock voltages of the frequency of 50 Hz, with the presence of an additional component of the higher frequency are presented at Fig. 5. The calculations have been conducted under the assumption $U_r(f) = 0.5U_{Tp}$.

The results of calculations indicate that the considerable reduction of the highest permissible contact shock voltage U_{Tp} is necessary in order to reduce the threat of ventricular fibrilla-

tion in cases when the additional component with the frequency other than the network frequency may occur in the shocking voltage (of the frequency 50 Hz).

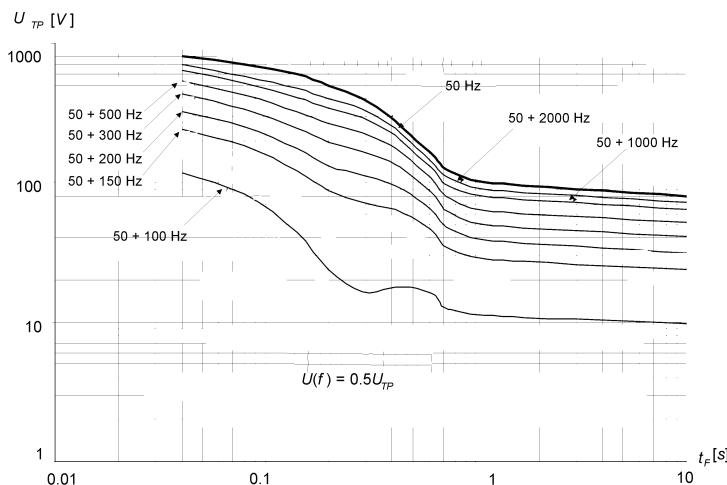


Fig. 5. Dependence of the highest permissible contact shock voltages U_{Tp} on the shock duration time t_F , bold curve without, and the other curves with the presence of the additional component of shock voltage $U_r(f) = U_{Tp}$, with the frequency higher than the network frequency (epidermis wet, shock path left hand– feet)

This applies in particular to the cases of long duration times of electric shock and to the cases of low frequencies of the additional components of shocking voltage (for example in case of 100 Hz frequency the highest permissible voltage should be reduced from 80 V to a dozen of Volts).

5. Final conclusions

The elaborated probabilistic model of the fibrillation current created by superposition of two shocking currents with different frequencies enables to calculate the reduced median value of the fibrillation current component with the network frequency, due to the presence of an additional component of the shocking current with the different frequency.

The calculations of the probability of ventricular fibrillation occurrence have indicated that:

- the highest permissible contact shock voltages $U_{Tp} = f(t_F)$ (at the frequency of 50 Hz), currently applied for the high voltage electrical equipment, do not ensure the equal levels of the protection effectiveness for all shock duration times t_F ,
- in case of the electric shock with the duration time t_F , and with the voltage containing two components, one being the highest permissible contact shock voltage U_{Tp} (at the frequency of 50 Hz) and the second equal to the multiple of U_{Tp} (and having the frequency

$f > 50$ Hz), a threat grows, particularly if the frequency of the second component is within the range of 100-500 Hz,

- in case when two components of the shocking voltage are present (with the frequencies of 50 Hz and higher), the assurance of the effectiveness of anti-shock protection makes it necessary to reduce the highest permissible contact shock voltages, which are applied for the frequency of 50 Hz.

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