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Overloss coefficient in magnetic laminations during PWM supply voltage

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Abstract: Author presents an analytical method of calculation of unit power losses in magnetic laminations used in electrical machines and transformers. The idea of this method, based on the solution of Maxwell's equations in the lamination material, was described by the author in the previous work [3], taking into account approximation of constitutive static hysteresis loop by elliptic form of the function $B = f(H)$ depending on magnetic saturation. In the previous formula for new isotropic and anisotropic materials it is needed to introduce so called "anomaly coefficient" deduced from the comparison of measured and calculated value of power losses in arbitrary excitation frequency for assumed induction. The method was tested by comparison with the results of experiments presented in commercial catalogues [1, 2]. Assuming superposition of harmonic power losses it is possible to enlarge this method for the estimation of overloss coefficient in dynamo sheet during axial magnetization with nonsinusoidal flux generated e.g. by PWM voltage supply.

Key words: magnetic laminations, power losses, PWM voltage

1. Introduction

The unit power losses in isotropic dynamo laminations used in electrical machines and in the past in transformers and reactors may be calculated in a relatively small range of frequencies (0-500) Hz with the application of known traditional formula [4, 5]

$$P^* = \frac{A}{\rho} f + \frac{1}{6}\pi^2 d^2 \frac{\sigma}{\rho} f^2 B_{m\,av}^2 , \tag{1}$$

where: A – area of static hysteresis loop [J/m^3], $B_{m\,av}$ – average value of magnetic flux density in a lamination [T], d – lamination width [m], f – frequency [Hz], σ – conductivity [S/m], ρ – specific mass [kg/m^3]. The two part losses are equal to the sum of hysteresis and eddy-current losses calculated without skin effect in lamination. The dependence as above is not observed in new isotropic and anisotropic laminations, widely used for example in transformer cores. The difference between measured and calculated losses observed in this case and named anomalous losses, depends on the structure of a lamination material. Another name used in different papers about losses in ferromagnetic materials is “excess losses” depending on the similar difference between measured and calculated losses by respective formulae defined by special assumptions.

In this work the combined formula for calculation of unit power losses in isotropic and anisotropic laminations will be discussed.

2. Combined formula in a case of sinusoidal magnetic flux

The approximation connected with application of equivalent elliptical hysteresis loop always evokes a sinusoidal magnetic field strength H forced by sinusoidal induction B in cross-section of lamination. The elliptical hysteresis is described by angular displacement of sinusoidal functions B and H, which assures the equivalence of physical and elliptical hysteresis loop area and enables introduction of complex magnetic permeability to Maxwell’s equations describing electromagnetic field in lamination material simultaneously with hysteresis and skin effect. The active power may be deduced from Poyting’s vector theorem after analytical solution of Maxwell’s equations. Physically, the magnetic field strength H generated by sinusoidal induction B is in general not sinusoidal and unit reactive power should be correct by relation of first harmonic H_1 to maximal value H_m of the magnetic strength. The correction of active power is needed taking into account the relation between measured and calculated values by coefficient A_n .

According to [5] it is possible to write the formula for unit power losses

$$P = \frac{k^3}{2\sigma\mu_m^2 \rho d}\varphi_m^2 \xi_\Phi A_n , \tag{2}$$

where:

$$\mu = \mu_m e^{-j\delta}, \quad \mu_m = \frac{B_m}{H_m}, \quad \sin\delta = \frac{A}{\pi B_m H_m}, \tag{3}$$

$$\varphi_m = B_{mav}(d \cdot 1), \tag{4}$$

$$\xi_\varphi = \frac{a\sinh(akd) - b\sin(bkd)}{\cosh(akd) - \cos(bkd)}, \tag{5}$$

$$a = \cos\frac{\delta}{2} + \sin\frac{\delta}{2}; \quad b = \cos\frac{\delta}{2} - \sin\frac{\delta}{2}, \tag{6}$$

$$k = \sqrt{\pi f \mu \sigma}\,, \tag{7}$$

3. Verification of calculation method

The verification of the method was made by comparison of calculation with measurement results for isotropic lamination type EP 23 0.5 mm (made in Poland) and anisotropic lamination UNISIL 56 0.33 mm (made in UK).

Table 1 presents the respective comparison prepared for isotropic lamination EP 23 used in electrical machines. The hysteresis angles were calculated from static hysteresis loops given in catalogue [1] for mixed sample assembled in a half from laminations stamped along in a half perpendicular to rolling axis. In this case the "anomaly losses coefficient" $A_n = 1$ in formula (2) must be assumed.

Table 1. Calculation and measurement results for lamination EP23 0.5 mm, $\sigma = 2.09\ 10^6$ S/m (mixed sample), f =50 Hz

Order number		1	2
induction B $_{\text{mav}}$	T	1.5	1.0
magnetic field strength H_{mav}	A/m	800	192
magnetic permeability μ_m	10^{-3}H/m	1.88	5.2
hysteresis angle δ	degree	10	32.5
unit power losses P_c (calculated)	W/kg	4.73	2.4
unit power losses P_m (measured)	W/kg	4.52	2.1
P_c/P_m	–	1.05	1.14
unit reactive power Q_c (calculated)	VA/kg	12.82	3.33
unit reactive power Q_m (measured)	VA/kg	14.03	2.83
Q_c/Q_m	–	0.92	1.18
H_l/H_m	–	0.55	1.0

The formula for unit power losses (2) was applied for anisotropic laminations UNISIL 56 which magnetic and power characteristics are given in catalogue [2]. The input data prepared for calculation are presented in Table 2.

Table 2. The calculation input data for lamination UNISIL56 0.33 mm, $\sigma = 2.174\ 10^6$ S/m, f = 50 Hz

	Induction	Magnetic permeability	Hysteresis angle	Unit power losses	Anomaly coefficient
	B_{mav}	μ_m	δ	P_c	An
	T	10^{-3}H/m	degree	W/kg	–
1	0.4	42.0	19.0	0.045	2.14
2	0.6	53.6	21.0	0.094	1.90
3	1.0	56.8	23.0	0.267	2.07
4	1.5	23.4	12.2	0.7	1.86

In this paper the results of calculations and measurements concerning the unit power losses in the sample composed of laminations stamped along the main rolling axis are published only. The power loss comparison in a wide range change of induction and frequency is drawn in Fig. 1.

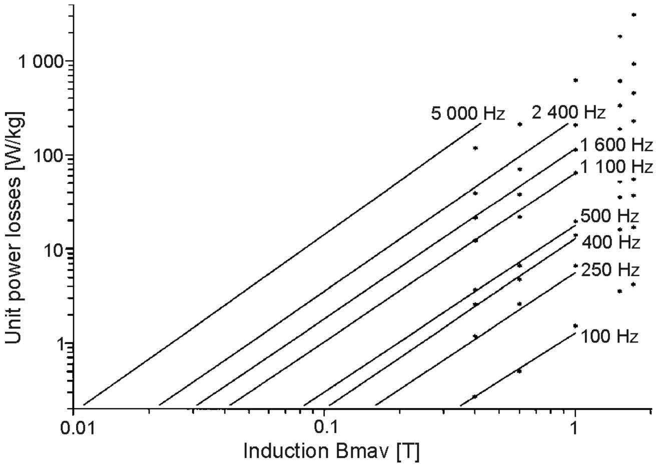

Fig. 1. Unit power losses for lamination UNISIL 56. Continued lines--measurement, points-calculation

4. Losses in a case of nonsinusoidal magnetic flux. PWM supply voltage

The output voltage of recent PWM converters being applied for wide range control of angular speed of induction motors is very much different from sinusoidal form (Fig. 2). In such cases, the sinusoidal wave forms of magnetic flux in a core of real machine are modulated by higher order harmonics of the flux (Fig. 3) [5].

As a first step of overloss coefficient investigation the example of axial magnetization of the lamination will be described. Axial magnetization in the case of nonsinusoidal magnetic flux may be presented as follows. The basic magnetization is defined by first order magnetic

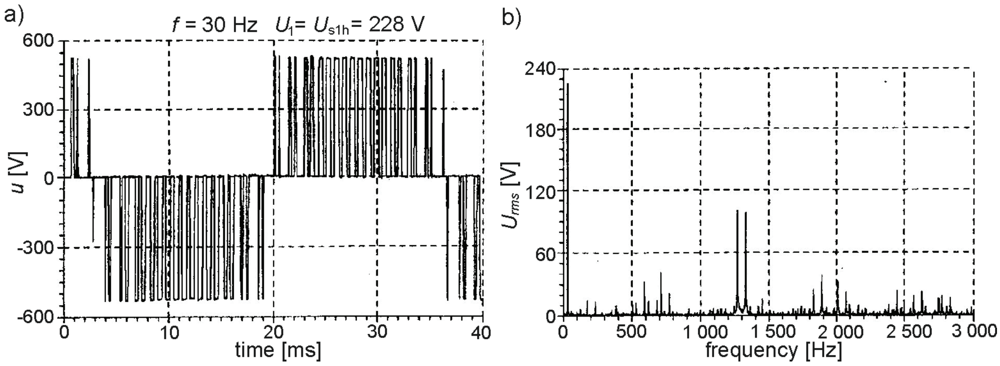

Fig. 2. Example of the PWM voltage waveform (a) and frequency spectrum (b) (f_1 = 30 Hz)

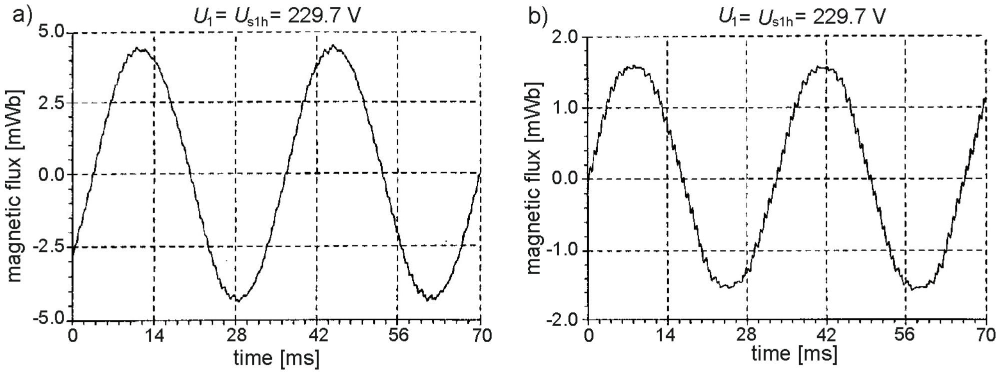

Fig. 3. Examples of magnetic flux in an induction motor fed by PWM converter ($f_1 = 30$ Hz). Yoke flux (a), tooth flux (b)

flux component along main hysteresis loop. Each higher order magnetic flux component generates small differential hysteresis loop. The number of differential loops in a period of the first harmonic component is equal to the order of higher component. Very similar forms of hysteresis loops as an example for PWM voltage supply, have been measured in static state and presented in [5].

Small hysteresis loops have elliptical form depending on the eigen position in relation to main loop of magnetization. It is possible to approximate the main hysteresis loop by an ellipse and to use the formula for the power losses in the case of nonsinusoidal magnetic flux based on the expression.

$$\sum P = k_p P_1, \tag{8}$$

where: P_1 – power losses for first order harmonic component, k_p – overloss coefficient

$$k_p = 1 + \sum_{\upsilon.1}^{\infty} \frac{P_\upsilon}{P_1} = 1 + \sum_{\upsilon.1}^{\infty} \sqrt{\frac{\mu_{m1}}{\mu_{m\upsilon}}} \sqrt{\frac{f_1}{f_\upsilon}} \left(\frac{U_\upsilon}{U_1}\right)^2 \frac{\xi_{\varphi\upsilon}}{\xi_{\varphi 1}}. \tag{9}$$

Assuming similar shapes and slopes of differential loops for all higher components, we obtain

$$k_p = 1 + \sqrt{\frac{\mu_{m1}}{\mu_{m\upsilon}}} \frac{\xi_{\varphi\upsilon}}{\xi_{\varphi 1}} \sum_{\upsilon.1}^{\infty} \sqrt{\frac{f_1}{f_\upsilon}} \left(\frac{U_\upsilon}{U_1}\right)^2. \tag{10}$$

5. Estimation of overloss coefficient

As an example, the magnetic lamination EP 26 will be considered for overloss coefficient estimation ($d = 0.5$ mm, $\sigma = 2.09\ 10^6$ S/m). It is not possible to make investigations in real cases of dynamic magnetization with full spectrum of harmonic components of magnetic flux density. There are also difficulties with registration of all differential loops along the main hysteresis loop in static state. Therefore, another way was assumed for realization of magnetic measurements which enables the estimation of the overloss coefficient. The main and differential loops were measured in Laboratory of Electrical Metrology, Technical University of Lodz, in static conditions along a peak characteristics of the lamination material (Fig. 4).

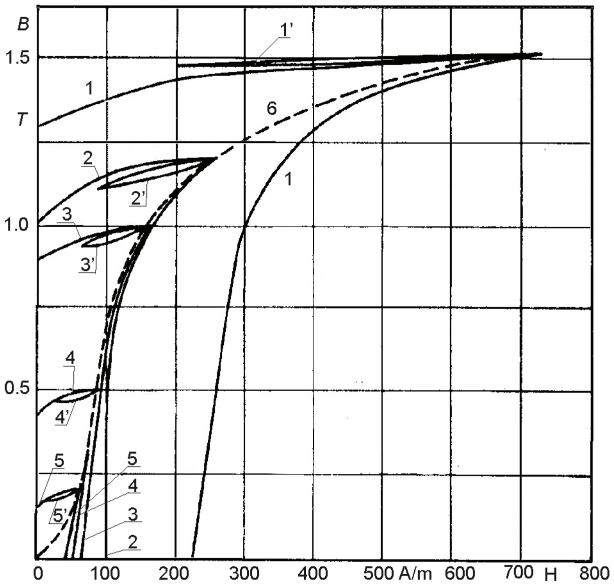

Fig. 4. Peak characteristics (6), main (1-5) and differential (1' – 5') hysteresis loops for dynamo lamination type EP (by Z. Kusmierek and S. Derlecki)

Table 3 presents the data worked out after magnetic measurements. Assuming the modulus of first order component (f_1 = 50 Hz) of the flux density B = 1.5 T we obtain representative values for full cycle of magnetization:

The average value of coefficients from Table 4 is equal to 0.74. The assumed average value of differential permeability μ_{mv} = 0.0003 H/m.

The estimated overloss coefficient:

$$\begin{gathered} k_p = 1+\sqrt{\frac{0.0015}{0.0003}\,\frac{0.74}{1.55}}\sum_{\upsilon.1}^{\infty}\sqrt{\frac{f_1}{f_\upsilon}}\left(\frac{U_\upsilon}{U_1}\right)^2 = 1+1.06\sum_{\upsilon.1}^{\infty}\sqrt{\frac{f_1}{f_\upsilon}}\left(\frac{U_\upsilon}{U_1}\right)^2, \\ k_p \approx 1+\sum_{\upsilon.1}^{\infty}\sqrt{\frac{f_1}{f_\upsilon}}\left(\frac{U_\upsilon}{U_1}\right)^2. \end{gathered} \tag{11}$$

In reference [5] some results of experimental investigations of overloss coefficient under PWM supply of the sample have been described. The ring sample of the core was made of the magnetic lamination type EP. The measured overloss coefficient varies from 1.4 to 1.45 and practically does not depend on the magnetic flux density.

Table 3. Magnetic permeabilities and angles of elliptical hysteresis for lamination EP26

Magnetic flux density B	*T*	0.1	0.5	1.2	1.5
Static permeability μ_{m1}	H/m	0.002	0.006	0.005	0.0015
Angle of hysteresis δ	deg	15	37	26	8.6
Differential permeability μ_{mv}	H/m	0.0006	0.0006	0.0003	0.0001
Angle of hysteresis δ	deg	15	11	7	6

The parameters kd_υ and coefficients $\xi_{\varphi\upsilon}$ as a function of frequency are presented in Table 4.

Table 4. Parameters kd_ν and coefficients $\xi_{\varphi\nu}$ as functions of frequency

Frequency *f* [Hz]	Parameter kd_υ	Coefficient $\xi_{\varphi\upsilon}$
250	0.36	1.5
500	0.52	0.78
1000	0.70	0.74
2000	1.03	0.7
4000	1.45	0.74

The estimated coefficient, taking into account formula (11), as a result of harmonic analysis is equal to 1.35 (a few percent lower than the measured one).

5. Conclusions

The application of elliptical model of hysteresis enables using the combined formula for calculation of unit power losses in a wide range of frequency. It is needed to introduce the so called "coefficient of anomalous losses" A_n which will be obtained from comparison of calculated and measured losses only for one frequency (e.g. 50 Hz). This coefficient is depended on the average value of induction in a cross-section of the lamination. The method was checked for sinusoidal form of induction in time.

The elliptical magnetization is also interesting for estimation of overloss coefficient k_p in relation to unit power losses for the first order harmonic as a result of nonsinusoidal form of induction in a lamination.

The described means has been applied successfully for nonsinusoidal form of induction generated in laminations by PWM supply voltage.

References

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