ARCHIVES OF ELECTRICAL ENGINEERING VOL. 60(1), pp. 15-21 (2011)

DOI 10.2478/v10171-011-0002-y

An inductance lookup table application for analysis of reluctance stepper motor model

JAKUB BERNAT, JAKUB KOŁOTA,
SŁAWOMIR STĘPIEŃ, GRZEGORZ SZYMAŃSKI

Chair of Computer Engineering, Poznań University of Technology
Piotrowo 3a, 60-965 Poznań, Poland
e-mail: Jakub.Kolota@put.poznan.pl

(Received: 02.07.2010, revised: 19.11.2010)

Abstract: This research presents a method of modeling and numerical simulation of a reluctance stepper motor using reduced finite-element time-stepping technique. In presented model, the circuit equations are reduced to non-stationary differential equations, i.e. the inductance mapping technique is used to find relationship between coil inductance and rotor position. A strongly coupled field-circuit model of the stepper motor is presented. In analyzed model the magnetostatic field partial differential equations are coupled with rotor motion equation and solved simultaneously in each iterative step. The nonlinearity problem is solved using Newton-Raphson method with spline approximation of the B-H curve.

Key words: stepper motor, finite element method, control, inductance lookup table

1. Introduction

A stepper motor, as its name suggests, moves one step at a time, unlike those conventional motors, which spin continuously. If we command a stepper motor to move some specific number of steps, it rotates incrementally that many number of steps and tops. Because of this basic nature of a stepper motor, it is widely used in low cost, open loop position control systems. Open loop control means no feedback information about the position is needed. This eliminates the need for expensive sensing and feedback devices, such as optical encoders. Motor position is known simply by keeping track of the number of input step pulses. They are used in consumer products, computer technology, complex drives and machines etc.

Classically, the analysis of field-circuit systems is composed of two parts: the analysis of electric circuits described by differential equations, where magnetic flux is a function of field potentials and magnetic field described by boundary value problem [1, 6]. Many researchers propose numerical procedure to investigate the coupled system of equations [5, 8]. Unfortunately, strongly coupled field-circuit models have disadvantages related with good definite of global matrices of equations system. Sometimes they lead to stability loss of numerical solution.

In the work a standard model with electric circuit equations considered as a function of field potentials is compared with a surrogate model, where the circuit equations are functions of non-stationary inductances.

In this paper the modeling method and its numerical approach is examined using the fou-phased variable reluctance stepper motor. The time stepping finite element approximation including inductance maps is applied to calculate the coupled field-circuit problem also considering mechanical motion. The validity of this method is also verified by the experiments.

2. Modelling technique

Standard analysis of the electromagnetic systems is composed of three parts: analysis of n-phased electric circuit [3]:

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{dt}}\oint_{\mathrm{l}_1} \mathbf{A}(\mathrm{t})\mathbf{dl} + \mathrm{R}_1\mathrm{i}_1(\mathrm{t}) = \mathrm{u}_1(\mathrm{t}) \\ &\vdots \\ &\frac{\mathrm{d}}{\mathrm{dt}}\oint_{\mathrm{l}_n} \mathbf{A}(\mathrm{t})\mathbf{dl} + \mathrm{R}_n\mathrm{i}_n(\mathrm{t}) = \mathrm{u}_n(\mathrm{t}) \end{aligned} \tag{1}$$

An above equations are expressed by impedance matrix, where $n = 1 \ldots 4$ (four-phased motor model). The analysis of electromagnetic field constructed as the following boundary value problem [6]:

$$\int_{\Omega} \nabla\times\left(\frac{1}{\mu}\nabla\times\mathbf{A}\right)\mathrm{d}\Omega = \int_{\Omega}\mathbf{j}\,\mathrm{d}\Omega. \tag{2}$$

The magnetic vector potential **A** is the magnetic field variable, μ is a permeability and **j** is a current density of the winding. In this model the eddy current problem is ignored, because the stator and rotor are laminated.

The second order differential motion equation (3) consists of magnetic moment M which is a mechanical excitation, rotor inertia J, friction b and rotor angle Θ [1, 3].

$$J\frac{d^2\Theta_\varphi}{dt^2} + b\frac{d\Theta_\varphi}{dt} = M_\varphi. \tag{3}$$

Analysis of the mechanical motion is following [1, 3]:

$$\frac{d}{dt}\begin{bmatrix}\Theta_\varphi \\ \omega_\varphi\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & -\frac{b}{J}\end{bmatrix}\begin{bmatrix}\Theta_\varphi \\ \omega_\varphi\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{J}\end{bmatrix}M_\varphi. \tag{4}$$

In case of the rotational system, the displacement of the rotor can be assumed as a one degree of freedom motion problem which is performed along axis φ.

Assuming that the magnetic circuit is linear and the coil inductances are independent of the stator current excitation, a relationship between the rotor position and windings inductance can be determined and written in lookup tables which are exactly called maps of inductance. In this fact creating a surrogate non-stationary circuit model, including inductance maps can be very helpful and useful in simulation.

In many electromechanical systems, the torque generation can be explained using the principle of electromechanical energy conversion in a solenoid. The solenoid has N turns, and when it is excited with a current i, the coils sets up a flux Φ. The incremental mechanical energy in terms of the electromagnetic torque and change in rotor position is written as [2, 5]:

$$\partial W_m = T_e \, \partial \theta, \tag{5}$$

where W_m is energy converted into mechanical work, T_e is the electromagnetic torque, and θ is the incremental rotor angle. If the inductance is linear vs. the rotor position (i.e. inductance does not depend on the current), then the torque can be derived as [1, 2]:

$$T_e = \frac{dL(i,\theta)}{d\theta} \frac{i^2}{2} \tag{6}$$

and numerically:

$$\frac{d\,L(i,\theta)}{d\theta} \approx \left. \frac{L(\theta_2, i) - L(\theta_1, i)}{\theta_2 - \theta_1} \right|_{i=\mathrm{const}} . \tag{7}$$

The differentiation of the inductance can be considered in evaluated model with quasi-constant torque expressed in $\mathrm{Nm/A^2}$. It means, the surrogate model of reluctance stepper motors is equipped with non-stationary equivalent circuit [1, 2, 4, 7]. In this way the circuit equation (1) is simplified to non-stationary initial value problem:

$$\frac{d}{dt}\left(L(\Theta(t)) \cdot i(t)\right) = -R_n \, i_n(t) + u_n(t). \tag{8}$$

In these types of motors the mutual inductances are small enough (Fig. 4) to be excluded from calculation [1].

3. Numerical experiment

A variable reluctance stepper motor is considered in numerical experiment. The device consists of four-phase windings with eight slots. There is examined 2D problem using 3D first order approach, in which the eddy current problem is neglected. The data of the motor are presented in the Table 1.

The simulation is performed for the model with 145 440 nodes. The mesh of the coupled model is shown in Fig. 1 and Fig. 2.

Table 1. Motor parameters

Quantity	Value	Unit
Outer diameter of stator	39	mm
Inner diameter of stator	34	mm
Outer diameter of rotor	26.96	mm
Inner diameter of rotor	5	mm
Air gap width	0.02	mm
Motor length	30	mm
Resistance/phase	15	Ω
Rotor inertia	8.5	gcm^2
Friction	10^{-5}	Nms

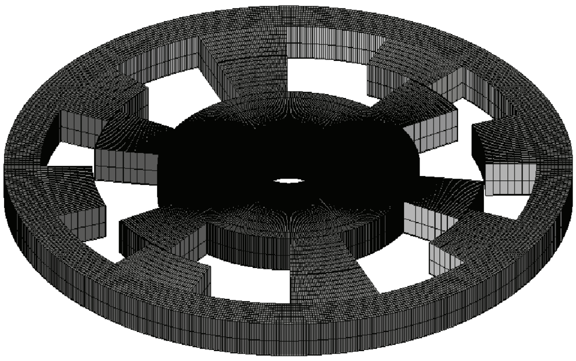

Fig. 1. 3D mesh of the reluctance stepper motor

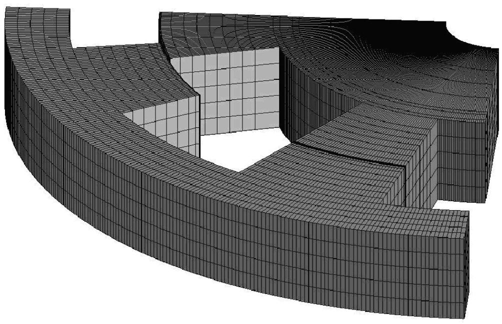

Fig. 2. Quarter part of the model mesh of solid magnetic parts

The stepper motor consists of four windings with 108 turns each excited from pulse current source. The mechanical parameters of the rotor are: moment of inertia $J = 8.5$ gcm^2 and friction $b = 10^{-5}$ Nms. In the model the rotor as well as the stator are considered as magnetic material characterized by B-H curve.

The significant changes of inductance profile are determined in terms of the stator and rotor pole arcs and number of rotor poles. Phase inductances versus rotor position are shown in Fig. 3 for a constant phase current. There also exists relatively small mutual inductance between phase windings (Fig. 3). So, in the modelled problem they can be excepted in simulations.

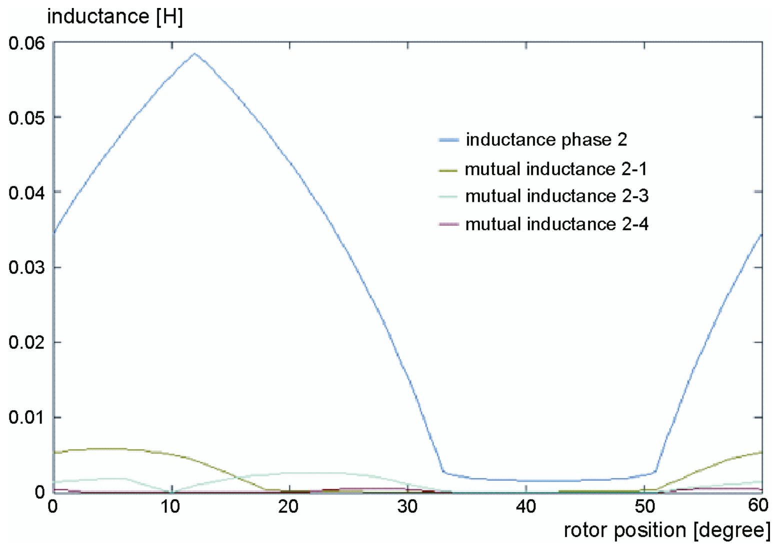

Fig. 3. Inductance vs. rotor position for research model

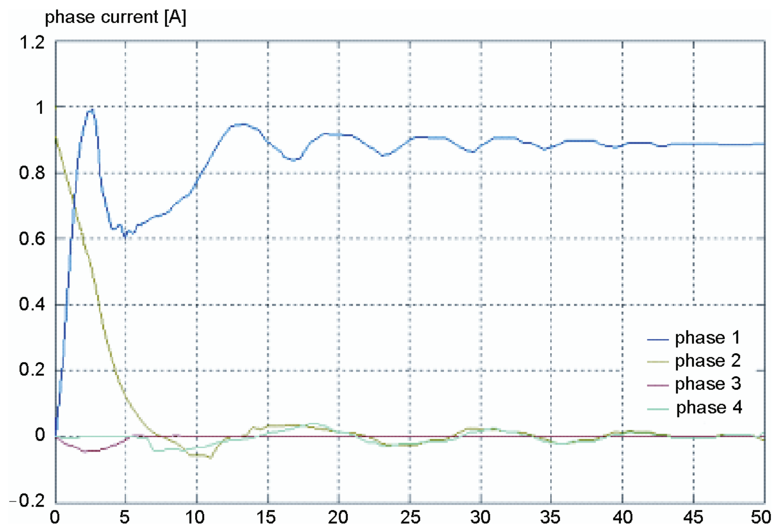

Fig. 4. An input current in motor winding $f(t)$

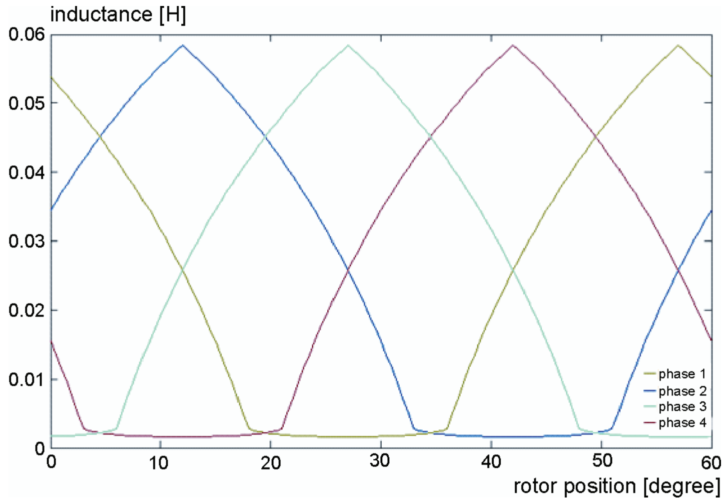

Fig. 5. Inductance vs. rotor position for research model

In analyzed stepper motor model, the inductance depends on rotor position such as is presented in Figure 5. When the functions are placed in lookup tables as maps, then motor circuits are strongly simplified and makes the numerical solution easier and faster.

The displacement responses presented in Fig. 6 proves that the accepted technique is correct. Step response signals are identically and the time consumption of the calculation is significantly reduced. For computation one step displacement which is presented in Fig. 6 the gain is about 70 percent.

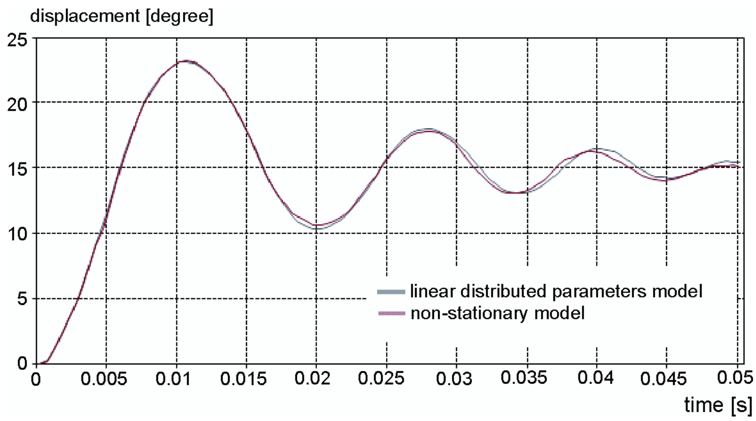

Fig. 6. Comparison of linear distributed model and simplified non-stationary model using inductance lookup table method

4. Conclusion

This paper shows how to create useful and simplified stepper motor model based on reduced field-circuit description. The simulation experiment shows that all properties of stepper motor are obtained. Step response has oscillations and steady-state position is equaled to basic step. This approach is helpful, when the equivalent circuits are built with the condition of the magnetic circuit linearity (without saturation). An advantage of this approach is reduced time consumption during simulation, especially in case of closed-loop analysis with controller or optimization needs. The proposed method can be used to design electromagnetic drives and may contribute to the improvement of the drives dynamics. The phase inductance L of a SRM is determined by the phase current I and the rotor position θ and is a highly nonlinear function of both rotor position and phase current. It is well known that the ANFIS (Adaptive Neuro-Fuzzy Inference System) is a very powerful approach for building a complex and nonlinear relationship between a set of input and output data. For this reason, in the next work, the phase inductance of a SRM will be calculated by using a method based on the ANFIS.

References

[1] Krishnan R., *Switched reluctance motor drives Modeling, Simulation, Analysis, Design, and Applications.* CRC Press (2001).

[2] Lee B.S., Bae H.K., Vijayraghavan P., Krishnan R., *Design of a linear switched reluctance machine.* IEEE Industry Appl. Conf. (IAS '99) 4, Oct. 3-7, Phoenix, AZ: 2267-2274 (1999).

[3] Bernat J., Kołota J., Stępień S., *Dynamic of Reluctance Stepper Motor Depending on Different Air Gap.* Monograph: *Information Systems Architecture and Technology – Model-based Decision.* pp. 179-191 (2008).

[4] Bae H.K., Krishnan R., *A novel approach to control of switched reluctance motors considering mutual inductance.* IEEE Ind. Electronics Conf., Nagoya, Japan: 369-374 (2000).

[5] de Oliveira A., Antunes R., Kuo-Peng P. et al., *Electrical machine analysis considering field – circuit – movement and skewing effects.* Compel: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering 23(4): 1080-1091 (2004).

[6] Stępień S., Patecki A., *Modeling and Position Control of Voltage Forced Electromechanical Actuator.* Compel: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering 25(2): 412-426 (2006).

[7] Omekanda A.M., *A new technique for multidimensional performance optimization of switched reluctance motors for vehicle propulsion.* IEEE Trans. Ind. Appl. 39(3): 672-676 (2003).

[8] Dunlop Wu W., Stephen J.B., Collocott J., Kalan B., *Design optimization of a switched reluctance motor by electromagnetic and thermal finite-element analysis.* IEEE Trans. Magn. 39(5): 3334-3336 (2003).