

Steady-state modelling method for matrix-reactance frequency converter with boost topology*

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Abstract: This paper presents a method intended for calculation of steady-state processes in AC/AC three-phase converters that are described by nonstationary periodical differential equations. The method is based on the extension of nonstationary differential equations and the use of Galerkin's method. The results of calculations are presented in the form of a double Fourier series. As an example, a three-phase matrix-reactance frequency converter (MRFC) with boost topology is considered and the results of computation are compared with a numerical method.

Key words: AC/AC converters, matrix-reactance frequency converter, Galerkin's method, extension of differential equations

1. Introduction

Three-phase MRFCs [1, 2] based on a matrix-reactance chopper (MRC) are used for both frequency and voltage transformations. The processes in a MRFC system are described by nonstationary differential equations with periodical coefficients

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{A}(t)\mathbf{X}(t) + \mathbf{B}(t), \quad (1)$$

where $\mathbf{X}(t)$ is a vector of state variables, and $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are respectively a matrix and a vector of the converter. These equations can be solved using an approximation by the numerical-analytic method [3, 4]. The search for solutions leads to converting nonstationary differential equations into stationary equations and their solutions. The method is based on the use the Lapunov transformation. Then, in order to calculate the convolutions of functions the Laplace transform is applied. This method can be used to find transient and steady-state processes.

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A commonly used method for modelling MRFC is the averaged state-space method [5]. The method involves the generation of a replacement model of the circuit with averaged state variables. Other methods of solving nonstationary differential equations are Hill's method [6] and its modification of Taft's method [7]. In both methods, the solution can be obtained on the basis of infinite numbers of equations with infinite numbers of variables.

To solve the nonstationary differential equation it is also possible to use Galerkin's method [8, 9]. This method approximates the solution using a finite number of bases and weight functions, in which the bases functions for this method are the same. In order to obtain steady-state solutions bases and weight functions are chosen in the trigonometric form. The results of calculations are presented in the form of a Fourier series.

In the case of three-phase MRFCs the matrix $\mathbf{A}(t)$ and vector $\mathbf{B}(t)$ have different frequencies. In general, it makes the solution of equations (1) non periodical, especially when these frequencies are incommensurable.

The subject of this article is to present a numerical method used for the calculation of steady-state processes in the three-phase MRFCs. The method is based on the use of a state-space averaged mathematical model, an extension of nonstationary differential equations by an additional, independent variable of time and the use of Galerkin's method. The solution is described by a double Fourier series [10]. The results of the calculations are verified by a numerical method.

2. Extension of differential equations

The extension of differential equations for an additional independent variable of time [11] is used for obtaining periodic solutions of nonstationary periodic differential equations which contain different frequencies. Here, we suppose that the frequencies are incommensurable.

Let us consider the possibility of extending a differential equation using the example of a simple circuit shown in Fig. 1.

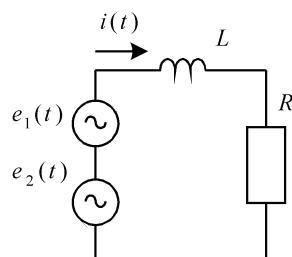


Fig. 1. Electric circuit with independent supply voltages

The electrical circuit has two independent supply voltages with periods T and Θ

$$e_1(t) = e_1(t + T),$$

$$e_2(t) = e_2(t + \Theta).$$

Solving the differential equation in a such circuit

$$L \frac{di(t)}{dt} + Ri(t) = e_1(t) + e_2(t), \quad (2)$$

yields a current $i(t)$ which is not periodical.

We can use the superposition of solutions of two differential equations

$$L \frac{di_1(t)}{dt} + Ri_1(t) = e_1(t),$$

$$L \frac{di_2(\tau)}{d\tau} + Ri_2(\tau) = e_2(\tau)$$

and then calculate the sum

$$i(t, \tau) = i_1(t) + i_2(\tau).$$

In that case the result will be periodical $i(t, \tau) = i(t + T, \tau + \Theta)$ with periods T and Θ . We extend (2) in such a way

$$L \left(\frac{\partial i(t, \tau)}{\partial t} + \frac{\partial i(t, \tau)}{\partial \tau} \right) + Ri(t, \tau) = e_1(t) + e_2(\tau).$$

The steady-state solution of this equation is periodical with periods T and Θ . The current $i(t)$ in the space of one variable of time is obtained by simply equating $t = \tau$, i.e. $i(t) = i(t, t)$.

Further, such an approach will be used for solving nonstationary differential equations. Then the equations (1) described by two independent variables of time take the form

$$\frac{\partial \mathbf{X}(t, \tau)}{\partial t} + \frac{\partial \mathbf{X}(t, \tau)}{\partial \tau} = \mathbf{A}(t, \tau) \mathbf{X}(t, \tau) + \mathbf{B}(t), \quad (3)$$

where $\mathbf{X}(t, \tau)$ is the vector of state variables.

Formation of the equation (3) leads to a replacement time variable t for τ in some places of the matrix $A(t)$ or vector $B(t)$ and introduces partial derivatives. If the matrix $A(t)$ has different frequencies then we replace t for τ at one frequency and keep t at an other frequency. If the matrix $A(t)$ and vector $B(t)$ have different frequencies then we replace t for τ in the matrix $A(t)$ or vector $B(t)$.

3. Converter general description

Let's consider the AC/AC three-phase MRFC with boost topology shown in Fig. 2 [1, 12]. A control strategy in a general form is illustrated in Fig. 3.

The switching functions are defined as

$$s_{jK} = \begin{cases} 1, & \text{switch } S_{jK} \text{ closed} \\ 0, & \text{switch } S_{jK} \text{ open,} \end{cases}$$

where $j = \{a, b, c\}, K = \{A, B, C\}$.

The input filter L_{S1}, L_{S2}, L_{S3} is used in order to raise the magnitude of output voltages, whereas C_{L1}, C_{L2}, C_{L3} are used as storage elements of the MRFC. In each sequence period T_{Seq} there are two time intervals, t_S and t_L . In the interval t_S the synchronous connected switches (SCS) are on whereas the matrix connected switches (MCS) are off.

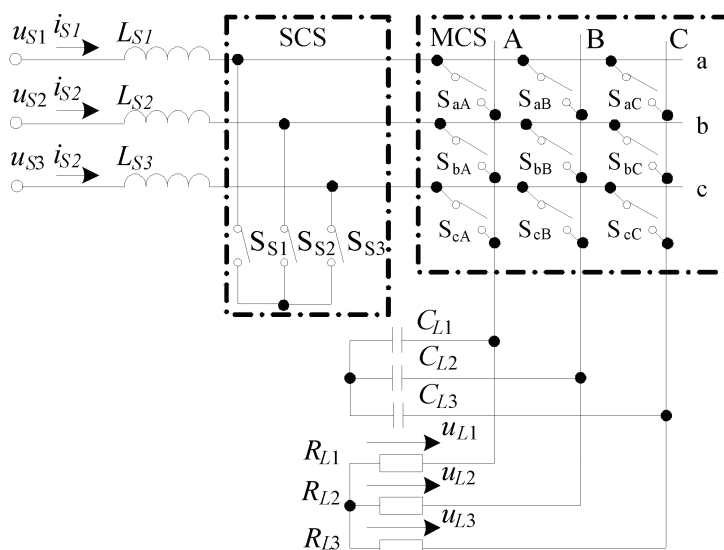


Fig. 2. MRFC based on boost MRC; SCS – synchronous connected switches, MCS – matrix connected switches

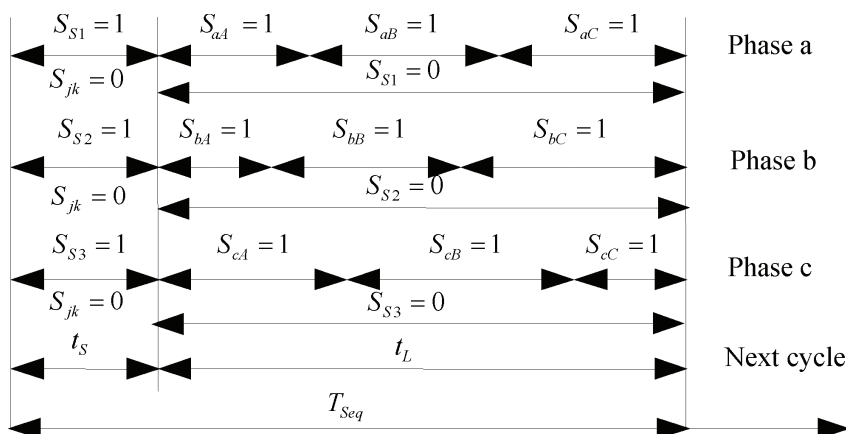


Fig. 3. General form of control strategy description

In the interval t_L the SCS are off whereas the MCS are switched over in accordance with the classical Venturini control strategy [13], i.e., MCS output voltages u_a, u_b, u_c and input currents i_A, i_B, i_C are formed as follows

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \mathbf{M}(t) \begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix},$$

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \mathbf{M}^T(t) \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix},$$
(4)

where: $\mathbf{M}(t)$ is a low frequency transfer matrix; T is the symbol of the transposition.

It should be noted that, as in the case of the Venturini control strategy, the transformation (4) gives the possibility of load voltage frequency change in the MRFC.

The low-frequency transfer matrix has the form

$$\mathbf{M}(t) = \begin{bmatrix} d_{aA} & d_{aB} & d_{aC} \\ d_{bA} & d_{bB} & d_{bC} \\ d_{cA} & d_{cB} & d_{cC} \end{bmatrix},$$

where:

$$d_{aA} = d_{bB} = d_{cC} = (1 - D_S)(1 + 2q \cos(\omega_m t)) / 3,$$

$$d_{aB} = d_{cA} = d_{bC} = (1 - D_S)(1 + 2q \cos(\omega_m t - 2\pi / 3)) / 3,$$

$$d_{aC} = d_{bA} = d_{cB} = (1 - D_S)(1 + 2q \cos(\omega_m t - 4\pi / 3)) / 3,$$

d_{jK} are the low frequency components of the MCS switching functions, $D_S = t_S / T_{Seq}$ is a sequence pulse duty factor, $\omega_m = \omega_L - \omega$, ω, ω_L are pulsations of the supply and load voltages respectively, q is a voltage gain.

4. Mathematical model

On the basis of the averaged state-space method [5] we can describe processes in such a system by the nonstationary differential equation (1), in which the vector of state variables has the form

$$\mathbf{X} = [i_{S1} \ i_{S2} \ i_{S3} \ u_{L1} \ u_{L2} \ u_{L3}]^T$$

and the vector $\mathbf{B}(t)$ and matrix $\mathbf{A}(t)$ are as follows

$$\mathbf{B}(t) = \left[\frac{U_m}{L_{S1}} \cos(\omega t), \frac{U_m}{L_{S2}} \cos\left(\omega t + \frac{2\pi}{3}\right), \frac{U_m}{L_{S3}} \cos\left(\omega t + \frac{4\pi}{3}\right), 0, 0, 0 \right]^T,$$

$$\mathbf{A}(t) = \begin{bmatrix} -\frac{R_{LS1}}{L_{S1}} & 0 & 0 & -\frac{d_{aA}}{L_{S1}} & -\frac{d_{bA}}{L_{S1}} & -\frac{d_{cA}}{L_{S1}} \\ 0 & -\frac{R_{LS2}}{L_{S2}} & 0 & -\frac{d_{aB}}{L_{S2}} & -\frac{d_{bB}}{L_{S2}} & -\frac{d_{cB}}{L_{S2}} \\ 0 & 0 & -\frac{R_{LS3}}{L_{S3}} & -\frac{d_{aC}}{L_{S3}} & -\frac{d_{bC}}{L_{S3}} & -\frac{d_{cC}}{L_{S3}} \\ \frac{d_{aA}}{C_{L1}} & \frac{d_{aB}}{C_{L1}} & \frac{d_{aC}}{C_{L1}} & -1 & 0 & 0 \\ \frac{d_{bA}}{C_{L2}} & \frac{d_{bB}}{C_{L2}} & \frac{d_{bC}}{C_{L2}} & 0 & -1 & 0 \\ \frac{d_{cA}}{C_{L3}} & \frac{d_{cB}}{C_{L3}} & \frac{d_{cC}}{C_{L3}} & 0 & 0 & -1 \end{bmatrix},$$

where U_m is an amplitude of the supply source, R_{LS} are resistors of the inductors L_S , R_L are resistors of a load.

5. Method of calculation

The approximation of the solution by Galerkin's method is based on the introduction of the residue defined as a system of differential equations

$$\mathbf{R}_{\mathbf{X}(t)} = \frac{d\mathbf{X}(t)}{dt} - \mathbf{A}(t)\mathbf{X}(t) - \mathbf{B}(t) \quad (5)$$

over the interval $0 \leq t \leq T$, where $T = 2\pi / \omega$.

The proposed method involves the extension of differential equations from (1) to (3). Then the residue (5) can be written as

$$\mathbf{R}_{\mathbf{X}(t,\tau)} = \frac{\partial \mathbf{X}(t,\tau)}{\partial t} + \frac{\partial \mathbf{X}(t,\tau)}{\partial \tau} - \mathbf{A}(t,\tau)\mathbf{X}(t,\tau) - \mathbf{B}(t) \quad (6)$$

and the elements d_{jK} of the matrix $\mathbf{M}(t)$ take the form

$$\begin{aligned} d_{aA} &= d_{bB} = d_{cC} = (1 - D_S)(1 + 2q \cos(\omega_L t - \omega \tau)) / 3, \\ d_{aB} &= d_{cA} = d_{bC} = (1 - D_S)(1 + 2q \cos(\omega_L t - \omega \tau - 2\pi/3)) / 3, \\ d_{aC} &= d_{bA} = d_{cB} = (1 - D_S)(1 + 2q \cos(\omega_L t - \omega \tau - 4\pi/3)) / 3. \end{aligned}$$

Taking base functions in the trigonometric form, we replace the vector $\mathbf{X}(t, \tau)$ in (6) by the vector $\mathbf{W}(t, \tau)$ whose components are

$$w^{(z)}(t, \tau) = \sum_{k=0}^N \sum_{n=0}^N [a_{kn}^{(z)} \phi_{kn}(t, \tau) + b_{kn}^{(z)} \psi_{kn}(t, \tau) + c_{kn}^{(z)} \theta_{kn}(t, \tau) + d_{kn}^{(z)} \xi_{kn}(t, \tau)]. \quad (7)$$

The superscript $z = 1, \dots, 6$ corresponds to the position of state variables in the vector $\mathbf{X}(t, \tau)$. The weight functions are described as follows

$$\begin{aligned}
 \phi_{nk}(t, \tau) &= \sin(n\omega t) \sin(k\omega_L \tau), \\
 \psi_{nk}(t, \tau) &= \sin(n\omega t) \cos(k\omega_L \tau), \\
 \theta_{nk}(t, \tau) &= \cos(n\omega t) \sin(k\omega_L \tau), \\
 \xi_{nk}(t, \tau) &= \cos(n\omega t) \cos(k\omega_L \tau).
 \end{aligned} \tag{8}$$

Then the residue (6) takes the form

$$\mathbf{R}_{\mathbf{W}(t, \tau)} = \frac{\partial \mathbf{W}(t, \tau)}{\partial t} + \frac{\partial \mathbf{W}(t, \tau)}{\partial \tau} - \mathbf{A}(t, \tau) \mathbf{W}(t, \tau) - \mathbf{B}(t). \tag{9}$$

The method is based on determining the coefficients

$$a_{kn}^{(z)}, b_{kn}^{(z)}, c_{kn}^{(z)}, d_{kn}^{(z)},$$

so that the residue (9) takes the minimum value. For this purpose we multiply (9) by weight functions (8) and integrate the obtained expressions over the region defined by $0 \leq t \leq T$, $0 \leq \tau \leq \Theta$,

$$\begin{aligned}
 \int_0^T \int_0^\Theta \phi_{kn}(t, \tau) \mathbf{R}_{\mathbf{W}(t, \tau)} d\tau dt &= 0, \\
 \int_0^T \int_0^\Theta \psi_{kn}(t, \tau) \mathbf{R}_{\mathbf{W}(t, \tau)} d\tau dt &= 0, \\
 \int_0^T \int_0^\Theta \theta_{kn}(t, \tau) \mathbf{R}_{\mathbf{W}(t, \tau)} d\tau dt &= 0, \\
 \int_0^T \int_0^\Theta \xi_{kn}(t, \tau) \mathbf{R}_{\mathbf{W}(t, \tau)} d\tau dt &= 0,
 \end{aligned}$$

where $\Theta = 2\pi / \omega_L$.

After the calculation of these integrals the result is presented in the form of linear equations. The solutions of such equations give the coefficients of the double Fourier series (9).

6. Calculation and simulation test results

The calculation of processes in the MRFC with boost topology is realized with the help of the program Mathematica. The steady-state solution is obtained taking into account the following parameters of the circuit

$$R_{LS1} = R_{LS2} = R_{LS3} = 0.01 \Omega, \quad R_{L1} = R_{L2} = R_{L3} = 10 \Omega,$$

$$C_{L1} = C_{L2} = C_{L3} = 0.1 \mu\text{F}, \quad L_{S1} = L_{S2} = L_{S3} = 1 \text{ mH},$$

$$U_m = 310 \text{ V}, \quad \omega_L = 2\pi/\Theta = 400 \text{ rad/S},$$

$$\omega = 2\pi/T = 250 \text{ rad/S}, \quad D_S = 0.7, \quad q = 0.4.$$

The steady-state solution of the equations (3) is obtained in the form of coefficients of the double Fourier series for the given parameters. The vector of steady-state variables is determined using base and weight functions for $N = 1$. It should be noted that calculations done for $N = 2, 3$ have the same results.

The exemplary steady-state current in the inductor L_{S1} is shown in Fig. 4, and the exemplary steady-state voltage across the capacitor C_{L1} is shown in Fig. 5, both in the space of two variables of time

One can see that these solutions are periodical with periods T and Θ .

To verify the obtained results let us calculate the steady-state process for $t = \tau$ and then compare it with the results obtained by the numerical method. The exemplary currents in inductors and voltages across capacitors in the steady-state obtained by the described method are shown in Fig. 6 and 7 respectively. The same results are also obtained by the numerical method embedded in Mathematica using the function NDSolve.

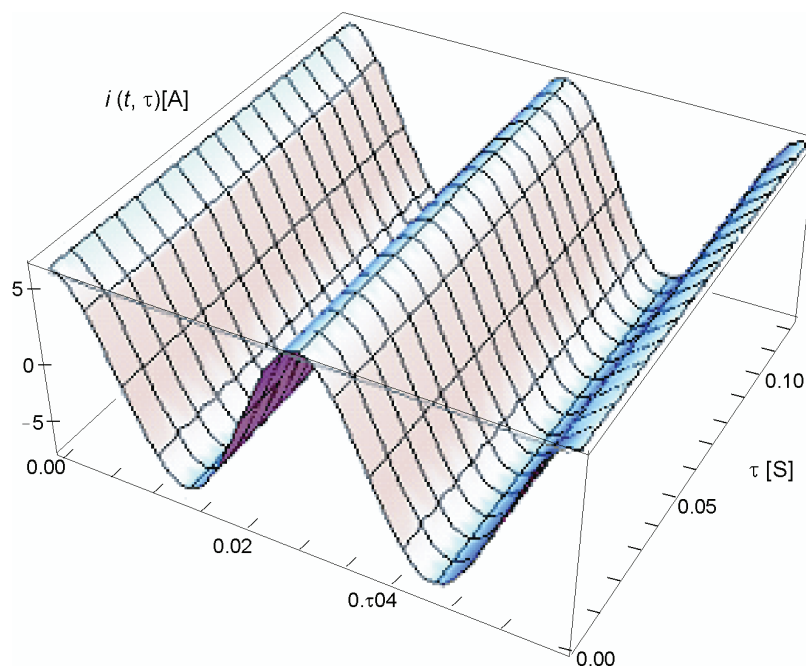


Fig. 4. Steady-state exemplary current in the inductor L_{S1}

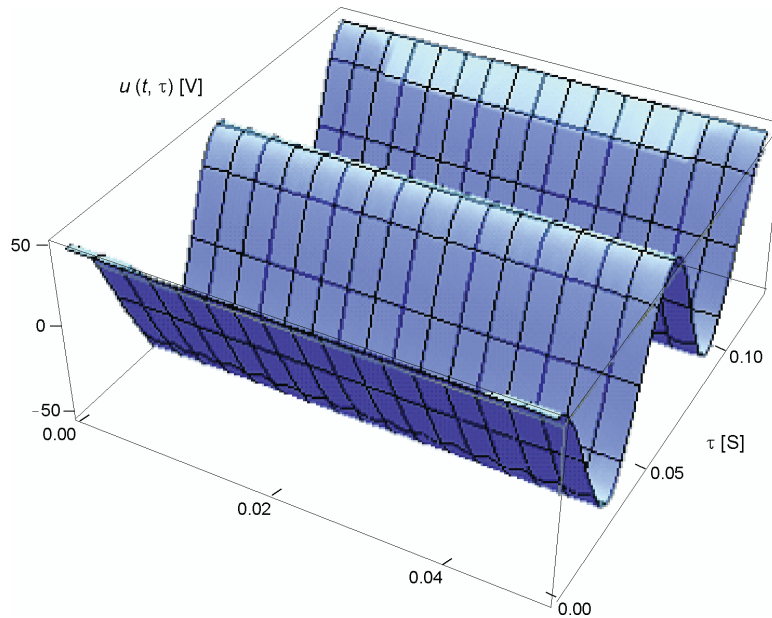


Fig. 5. Steady-state exemplary voltage across the capacitor C_{L1}

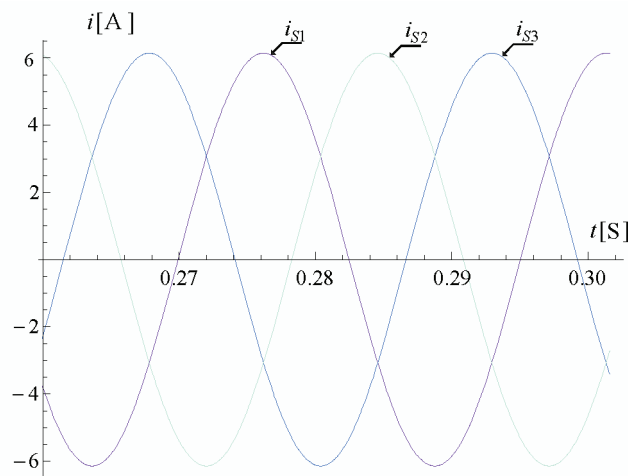


Fig. 6. Steady-state exemplary currents in the inductors: red – i_{S1} , blue – i_{S2} , green – i_{S3}

For the given parameters of the circuit the solutions of currents and voltages in steady-state obtained by the described method and numerical method are identical.

It should be noted that the steady-state solutions obtained by the numerical method can be determined only after the calculation of transient processes. In Figs. 8 and 9 are shown exemplaries of transient processes for current in the inductor i_{S1} and voltage across the capacitor u_{L1} obtained by using the numerical method and of the results obtained by the proposed method.

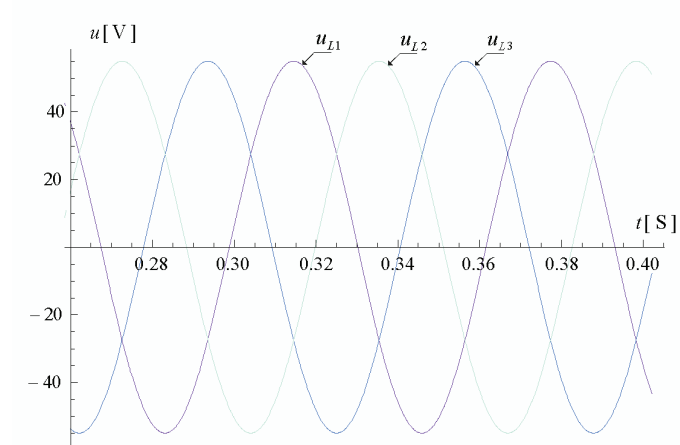


Fig. 7. Steady-state exemplary voltages across the capacitors: red – u_{L1} , blue – u_{L2} , green – u_{L3}

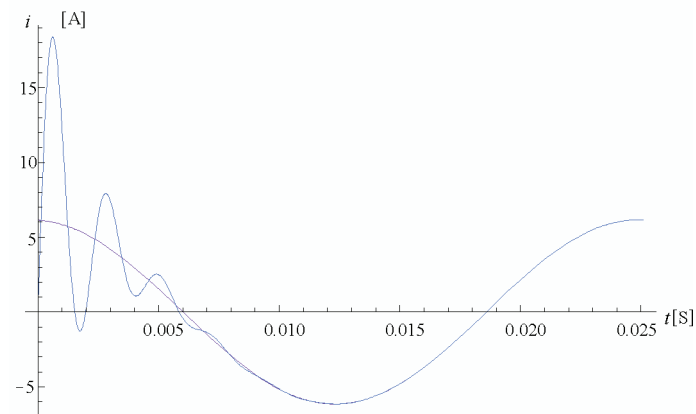


Fig. 8. Steady-state and transient exemplary currents in the inductor i_{S1} : red – the described method, blue – the numerical method

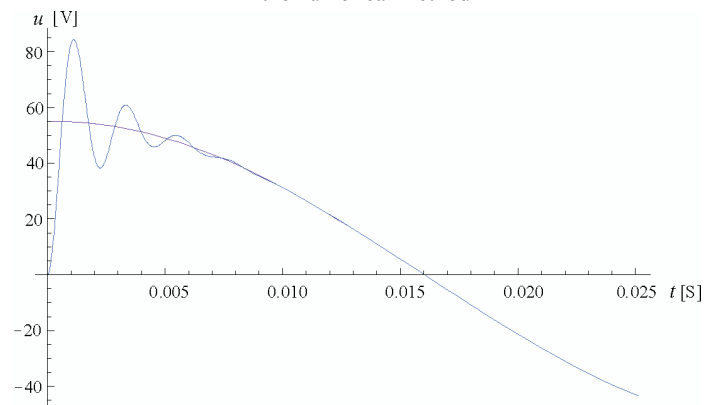


Fig. 9. Steady-state and transient exemplary voltages across capacitor: red – the described method; blue – the numerical method

The transient process is depended on values of parameters of the circuit and could be longer than some periods of the supply voltage. Moreover the differential equations could be stiff, which also increases the time of calculations.

7. Conclusions

The considered method is intended for the calculation of steady-state processes in three-phase MRFCs. Differential equations have been extended by the introduction of the additional independent variable of time. A solution has been obtained by Galerkin's method assuming aliquant relation between the periods of the supply source and the control signal. The results of calculation are presented in the form of a double Fourier series. Calculations have been realised for steady-state processes in the three-phase MRFC with boost topology. The same results have also been obtained by using the numerical method.

The proposed new method makes it possible to find steady-state processes in the three-phase AC frequency converters working with aliquant frequencies of supply voltages and control signals. Calculations of steady-state processes are obtained bypassing the procedure of computation of transient processes.

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