

Method for estimating probabilistic models of touch currents and impedance of human body relying on the effects of electric shock

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Abstract: The article provides a theoretical basis for a method allowing to calculate probability of effects of electric shock, as well as a method for determining probabilistic characteristics of random touch current values and of human body impedance in a person who suffered from specific effects of electric shock. Results of example calculations are presented, including probabilities of occurrence of sensory symptoms, exceeding the let-go threshold, and development of ventricular fibrillation, as well as probabilistic characteristics of random touch current values and of impedance of human body in people who experienced specific effects of electric shock.

Key words: probability of occurrence of electric shock effects, method for estimating, impedance of human body, touch currents, effects of electric shock

1. Introduction

Investigation of electric shock accidents is one of the tasks carried out in the process of shock safety management [1]. During the in-depth analysis of electric shock cases, one of the main problems involves establishing probable values of touch currents and of impedance of the victim's body.

The probability of occurrence of electric shock effects depends on the probability of remaining in the condition of electric shock and on the probability of manifestation of pathophysiological consequences in that condition. In particular, it relies on a series of circumstances and factors pertaining to the affected person, environmental conditions, and parameters of the electric shock circuit [2, 3].

The effects of electric shock manifested in a particular case allow us to establish for a given person (electric shock victim) probabilistic characteristics of the shock circuit parameters such as touch current values and body impedance.

The article presents theoretical foundations of a method for calculating probabilities of electric shock effects, and a method for determining probabilistic characteristics of random

touch current values and of impedance of human body in a person affected by specific consequences of electric shock. Example calculations concerning probabilities of occurrence of sensory effects, exceeding the let-go threshold, and development of ventricular fibrillation are offered, together with determination of probabilistic characteristics of random touch current values and of body impedance for a person who experienced specific electric shock effects.

2. Method for determining probability of occurrence of electric shock effects

By definition, the probability that specific effects will appear in the case of electric shock to human body is the probability of exceeding a random value of current I_s causing specific effects by the value of touch current I_r .

$$q_s = \Pr\{I_r > I_s\} = \Pr\{I_r / I_s > 1\}, \text{ for } I_r \text{ and } I_s \text{ in the range from } 0 \text{ to } +\infty. \quad (1)$$

The analytical formula describing q_s can be derived from the following dependency

$$q_s = \int_0^{+\infty} f(I_r)F(I_s)dI_r = \int_0^{+\infty} r(I_r)dI_r, \quad (2)$$

where: $f(I_r)$ – probability density function of touch current I_r values, $F(I_s)$ – probability distribution function of current I_s causing specific shock effects, $r(I_r)$ – risk function of occurring a particular shock effect for a given random value of touch current.

The idea of calculating probability q_s is presented in Figure 1. Derivation of analytical formula describing q_s requires the knowledge of statistical distribution types for both random values of touch current I_r and random values of current causing specific shock effect I_s .

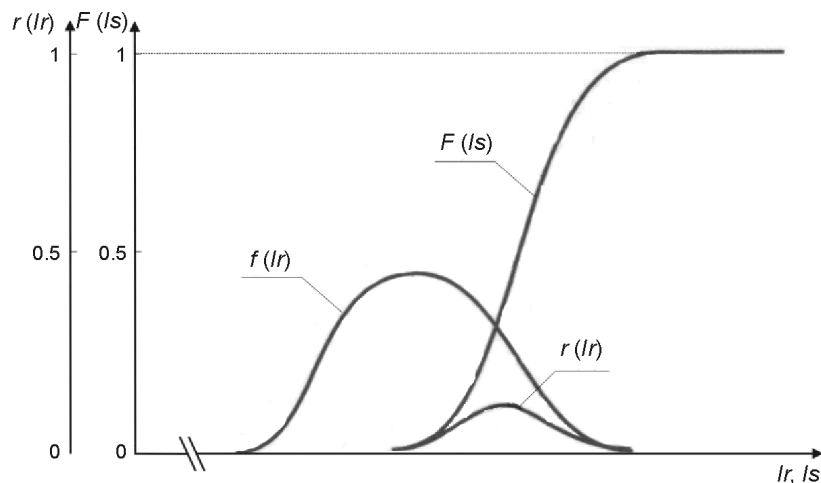


Fig. 1. Calculation of probability q_s

For a determined value of effective touch voltage, probability density function of touch current values is defined by the type of statistical distribution of human body impedance. To describe the distribution of random body impedance values under constant conditions of electric shock characterized by the value of touch voltage U_{dr} and its frequency f , time of exposure t , shock current path d , condition of epidermis s , equivalent surface of electrode contact S , and ambient temperature T , a conditional log-normal distribution can be used [4]. Parameters of the distribution, such as median Z_{cm} , and standard deviation of logarithm of random variable $\sigma_{\ln Z_c}$ are dependent on the abovementioned conditions of electric shock, and thus $Z_{cm}(U, f, t, d, s, S, T)$ and $\sigma_{\ln Z_c}(U, f, t, d, s, S, T)$.

The log-normal distribution has a property of stability with regard to division in the probabilistic sense, i.e. a random variable obtained by division of determined variable U_{dr} by a random log-normally distributed variable preserves the same type of distribution. For this reason, random touch current values under constant conditions of electric shock will have a log-normal distribution.

Also, to model statistical distribution of random touch current values causing in humans such symptoms as electrical sensation, muscular contraction (above the let-go threshold), and ventricular fibrillation, a conditional log-normal distribution can be used [5, 6]. As for the parameters of this conditional distribution of fibrillation, the following relationships take place: median $I_{fm}(f, t, d, m)$ – depends on the frequency f , time of exposure t , shock current path d , and weight of human body m , whereas standard deviation of the logarithm of random variable $\sigma_{\ln I_f}$ is constant.

Because values of random touch current I_r and of current I_s causing specific shock effects are distributed log-normally, relationship (2) allows us to derive a formula describing probability q_s of occurrence of specific shock effects in the following way:

$$q_s = \Phi\left(\frac{\ln I_{rm} - \ln I_{sm}}{\sigma_{\ln U_{dr}}}\right) = \Phi\left(\frac{\ln U_{dr} - \ln U_{sm}}{\sigma_{\ln U_{dr}}}\right) = \Phi(u), \quad (3)$$

where: Φ – standardized normal distribution function (values of u are given in statistical tables), I_{rm} – median of touch current I_r , I_{sm} – median of current I_s causing specific shock effects, U_{dr} – touch voltage for which probability of occurrence of specific shock effect is calculated, U_{sm} – median of voltage causing specific shock effects, $\sigma_{\ln U_s}$ – standard deviation of logarithm of voltage values causing specific shock effects, u – standardized normal random variable.

In calculation of q_s values, properties of the log-normal distribution allow us to take into account linear correlation between the considered electrical quantities such as e.g. impedance of human body Z_c and currents I_s causing specific shock effects. The correlation is accounted for in the expression describing standard deviation of logarithm of voltage values causing specific shock effects, according to the following dependency:

$$\sigma_{\ln U_{dr}} = \sqrt{\sigma_{\ln I_s}^2 + 2\rho\sigma_{\ln I_s}\sigma_{\ln I_r} + \sigma_{\ln I_r}^2}, \quad (4)$$

where: ρ – coefficient of correlation between $\ln I_r$ and $\ln I_s$, $\sigma_{\ln I_r} = \sigma_{\ln I_c}$ and $\sigma_{\ln I_s}$ – standard deviations of logarithms of I_r and I_s values.

To improve the effectiveness of determining probability q_s , approximation of cumulative normal distribution can be used with the accuracy to 10^{-7} order of magnitude, using the expression [7]:

$$\Phi(u) = 1 - z \cdot y \cdot \{a_1 + y \cdot [-a_2 + y \cdot (a_3 + y \cdot (-a_4 + y \cdot a_5))]\}, \quad (5)$$

with:

$$y = 1 / (1 + 0.2316419 \cdot |u|),$$

$$z = e^{-\frac{u^2}{2}} / \sqrt{2\pi},$$

$$a_1 = 0.31938153, \quad a_2 = 0.356563782, \quad a_3 = 1.781477937,$$

$$a_4 = 1.821255978, \quad a_5 = 1.330274429,$$

where: u – standardized normal random variable.

3. Method for estimating probabilistic models of touch currents and impedance of human body

Estimation of a probabilistic model of touch current in a person who experienced specific effects of electric shock, i.e. those random values of touch currents which might have caused appearance of the observed effects, involves determination of probability density function $f(I_r / I_r > I_s)$. The determination is possible when probability density functions of touch current $f(I_r)$ and of current causing specific shock effects $f(I_s)$ are known, and also when possible correlations between I_s and I_r concerning the total population of individuals under the considered electric shock circumstances is taken into account [3].

Probability density function $f(I_r / I_r > I_s)$ of occurrence of those values of touch current which exceed values of current causing specific shock effects is derived from the Bayesian formula:

$$f(I_r / I_r > I_s) = \frac{\int_0^{I_r} f(I_s) dI_r}{q_s}, \quad (6)$$

where q_s is determined from formula (3), and probability density function of touch current from the relationship:

$$f(I_r) = \frac{1}{\sqrt{2\pi} \cdot I_r \cdot \sigma_{\ln I_r}} \left[-\frac{1}{2} \left(\frac{\ln I_r - \ln I_{rm}}{\sigma_{\ln I_r}} \right)^2 \right], \quad I_r \geq 0, \quad (7)$$

whereas probability density function of current causing specific shock effects:

$$f(I_s) = \frac{1}{\sqrt{2\pi} \cdot I_s \cdot \sigma_{\ln I_s}} \left[-\frac{1}{2} \left(\frac{\ln I_s - \ln I_{sm}}{\sigma_{\ln I_s}} \right)^2 \right], \quad I_s \geq 0, \quad (8)$$

where: I_{rm} – median of touch current, I_{sm} – median of specific shock effect current, $\sigma_{\ln I_r}$ – standard deviation of log-normal distribution of touch current, $\sigma_{\ln I_s}$ – standard deviation of log-normal distribution of specific shock effect current.

Median values of touch current I_{rm} and specific shock effect current I_{sm} can be obtained using dependencies provided in [2, 5, 6].

Probability density function of appearance of human body impedance values $f(Z_c / I_r > I_s)$ is determined considering the fact that $Z_c / I_r > I_s = U_{dr} / (I_r / I_r > I_s)$.

The sought probabilistic characteristics of random touch currents $I_r / I_r > I_s$ and of impedance of human body $Z_c / I_r > I_s$ are determined relying on their probability density functions $f(I_r / I_r > I_s)$ and $f(Z_c / I_r > I_s)$. In the case of touch current, they can be determined according to the formulas:

- cumulative distribution function

$$F(I_r / I_r > I_s) = \int_0^{I_r} f(I_r / I_r > I_s) dI_r, \quad (9)$$

- mean value

$$\bar{I}_r / I_r > I_s = \int_0^{\infty} I_r \cdot f(I_r / I_r > I_s) dI_r, \quad (10)$$

- standard deviation

$$\sigma_{\bar{I}_r / I_r > I_s} = \sqrt{\int_0^{\infty} (I_r - \bar{I}_r / I_r > I_s)^2 \cdot f(I_r / I_r > I_s) dI_r}, \quad (11)$$

- coefficient of variation

$$\nu = \frac{\sigma_{\bar{I}_r / I_r > I_s}}{\bar{I}_r / I_r > I_s}. \quad (12)$$

If the value of coefficient of variation meets the $\nu < 0.25$ criterion, then the distribution of probability of random touch current values $I_r / I_r > I_s$ and of impedance of human body $Z_c / I_r > I_s$ can be described by a theoretical normal distribution.

4. Example probabilistic characteristics of touch currents and body impedance in electric shock victims

The presented here example probabilistic characteristics of touch currents and impedance of human body pertain to the cases of men who experienced electric shocks under given shock

circumstances (Tab. 1). In particular, the occurrence of three main effects was considered: sensory symptoms, exceeding the let-go threshold, ventricular fibrillation.

Table 1. Electric shock circumstances assumed for determination of probabilistic characteristics of touch currents

Pathophysiological effect	Condition of epidermis	Shock current path	Gender	U_{dr} [V]	t_r [s]	f [Hz]	S_1 [cm ²]	S_2 [cm ²]	m [kg]	T [°C]
Sensory symptoms	wet	hand-foot	male	20	1	50	82	200	75	20
Exceeding of the let-go threshold	wet	hand-foot	male	50	1	50	82	200	75	20
Ventricular fibrillation	wet	hand-foot	male	230	5	50	82	200	75	20

Symbols: U_{dr} , f – effective touch voltage and its frequency, t_r – exposure time, S_1 and S_2 – contact surfaces between electrodes and hand and feet m – body weight, T – ambient temperature

Table 2 presents the results of calculation which were carried out according to the methods given in [2], parameters of log-normal distribution (i.e., medians and standard deviations) of human body impedance values, touch currents, currents causing specific shock effects, as well as probabilities of occurrence of a particular effect for the electric shock circumstances offered in Table 1. The achieved results allowed for establishing (taking advantage of the method described above) probabilistic characteristics (mean values, medians, and quantiles) of touch current values and of body impedance of body for individuals with specific shock effects (Tab. 3).

Table 2. Calculation results of parameters of log-normal distribution of human body impedance, touch currents, currents causing specific shock effects, and probability of occurrence of a particular effect for the shock circumstances in Table 1

Pathophysiological effect	Z_{cm} [Ω]	$\sigma_{\ln Z_c} = \sigma_{\ln I_r}$	I_{rm} [A]	I_{sm} [A]	$\sigma_{\ln I_s}$	q_s [%]
Sensory symptoms	1812	0.319	0.011	0.002	0.391	95
Exceeding of the let-go threshold	1620	0.294	0.03	0.02	0.255	87
Ventricular fibrillation	1101	0.203	0.208	0.202	0.298	54

Symbols as explained in the text

The established probability density and distribution functions:

- in the case of sensory symptoms – values of touch current $f(I_r)$, current causing sensory symptoms $f(I_o)$, victim's touch current $f(I_r / I_r > I_o)$ and $F(I_r / I_r > I_o)$, as well as impedance values of the victim's body $f(Z_c / I_r > I_o)$ and $F(Z_c / I_r > I_o)$ are presented in Figure 2.
- in the case of exceeding the let-go threshold – values of touch current $f(I_r)$, let-go threshold current $f(I_{sa})$, victim's touch current $f(I_r / I_r > I_{sa})$ and $F(I_r / I_r > I_{sa})$, as well as impedance values of the victim's body $f(Z_c / I_r > I_{sa})$ and $F(Z_c / I_r > I_{sa})$ are presented in Figure 3.

Table 3. Calculation results of probabilistic characteristics of touch currents and body impedance in individuals who experienced specific shock effects under circumstances presented in Table 1

Pathophysiological effect	Probabilistic characteristics							
	$I_r / I_r > I_s$				$Z_c / I_r > I_s$			
	mean value \bar{I}_r	median I_{rm}	quantile $I_{r5\%}$	quantile $I_{r95\%}$	mean value \bar{Z}_c	median Z_{cm}	quantile $Z_{c5\%}$	quantile $Z_{c95\%}$
Sensory symptoms	9.28 mA	10.5 mA	6.5 mA	17 mA	1890 Ω	1980 Ω	1140 Ω	3300 Ω
Exceeding of the let-go threshold	31.8 mA	32 mA	21 mA	51 mA	1730 Ω	1610 Ω	1000 Ω	2380 Ω
Ventricular fibrillation	209 mA	226 mA	168 mA	308 mA	1100 Ω	1020 Ω	750 Ω	1370 Ω

Symbols as explained in the text

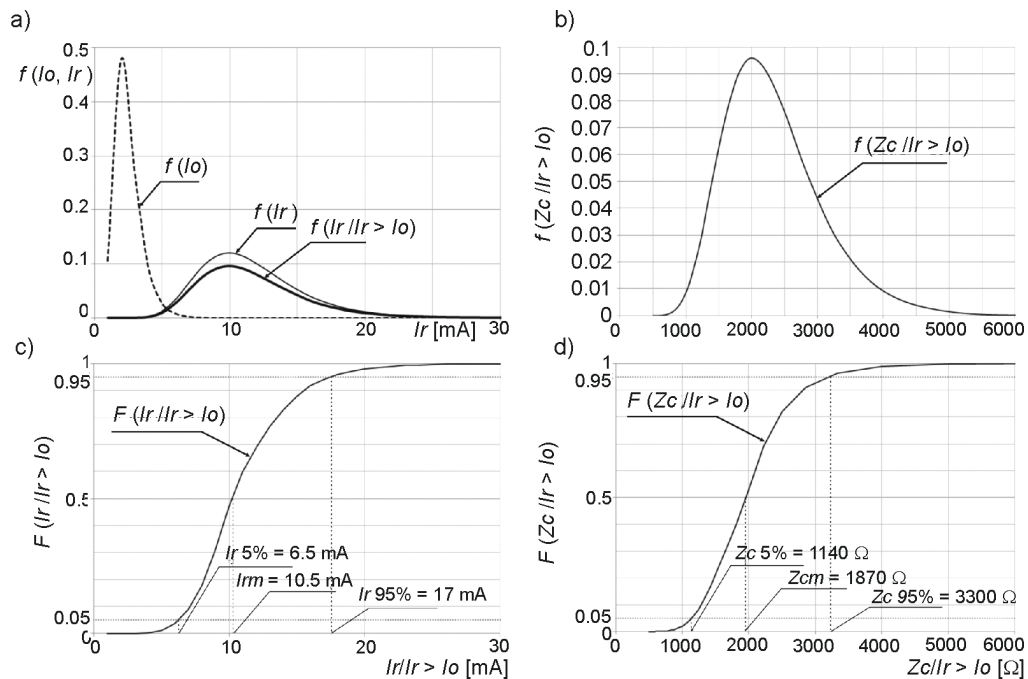


Fig. 2. For the shock conditions described in Table 1 and with occurring sensory symptoms: a) probability density function of touch current values $f(I_r)$, current causing sensory symptoms $f(I_o)$, as well as victim's touch current $f(I_r / I_r > I_o)$, b) cumulative distribution function of victim's touch current values $F(I_r / I_r > I_o)$, c) probability density function of impedance values $Z_c / I_r > I_o$ of the victim's body $f(Z_c / I_r > I_o)$, d) cumulative distribution function of the victim's body impedance values $F(Z_c / I_r > I_o)$

- in the case of ventricular fibrillation – values of touch current $f(I_r)$, current causing ventricular fibrillation $f(I_f)$, victim's touch current $f(I_r / I_r > I_f)$ and $F(I_r / I_r > I_f)$, as well as impedance values of the victim's body $f(Z_c / I_r > I_f)$ and $F(Z_c / I_r > I_f)$ are presented in Figure 4.

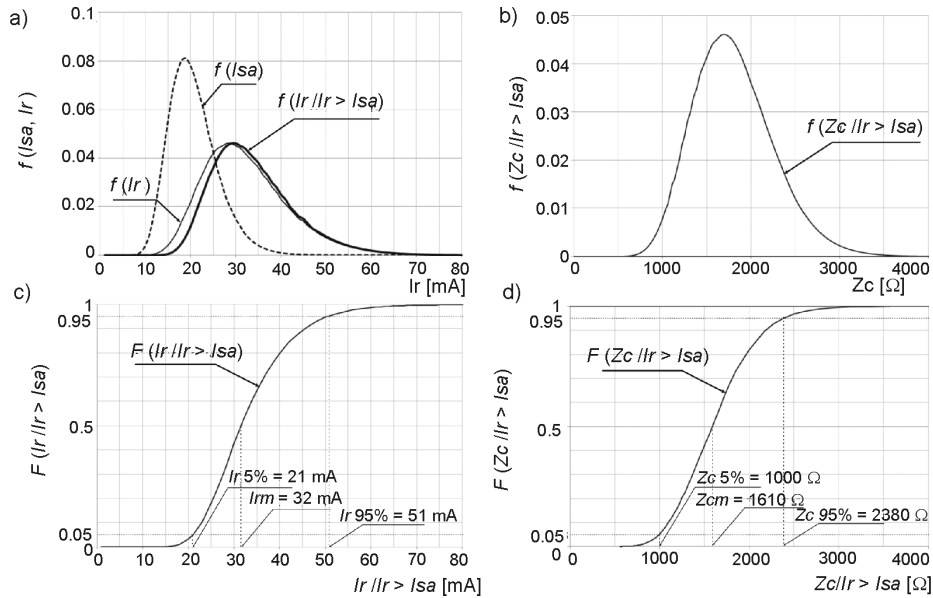


Fig. 3. For the shock conditions described in Table 1 and with the let-go threshold current exceeded: a) probability density function of touch current values $f(I_r)$, let-go threshold current values $f(I_{sa})$, as well as victim's touch current $f(I_r / I_r > I_{sa})$, b) cumulative distribution function of victim's touch current values $F(I_r / I_r > I_{sa})$, c) probability density function of impedance values $Z_c / I_r > I_{sa}$ of the victim's body $f(Z_c / I_r > I_{sa})$, d) cumulative distribution function of the victim's body impedance values $F(Z_c / I_r > I_{sa})$

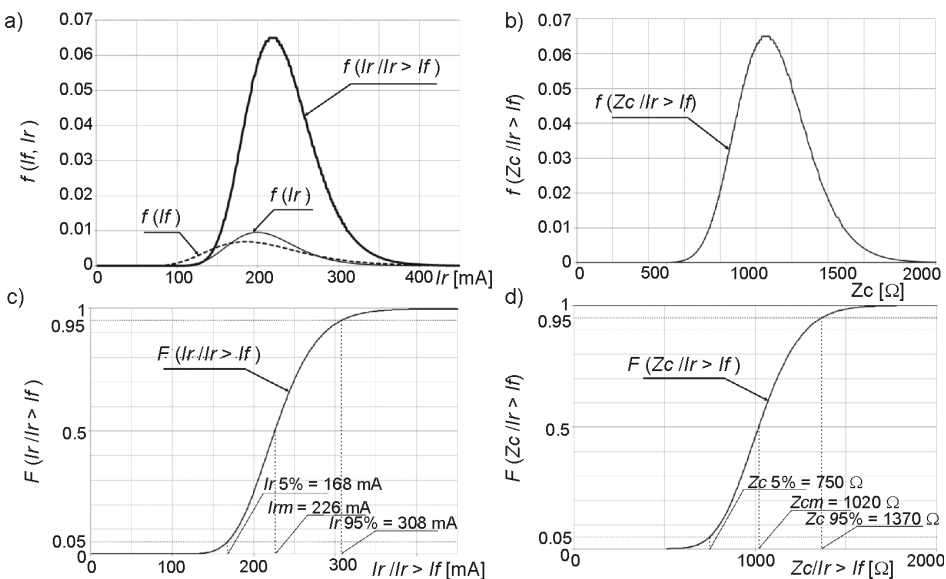


Fig. 4. For the shock conditions described in Table 1 and with ensuing ventricular fibrillation: probability density function of touch current values $f(I_r)$, current causing ventricular fibrillation $f(I_f)$, as well as victim's touch current $f(I_r / I_r > I_f)$, b) cumulative distribution function of victim's touch current values $F(I_r / I_r > I_f)$, c) probability density function of impedance values $Z_c / I_r > I_f$ of the victim's body $f(Z_c / I_r > I_f)$, d) cumulative distribution function of the victim's body impedance values $F(Z_c / I_r > I_f)$

5. Final conclusion

With the known probability density functions of human body impedance, touch currents, and currents causing specific shock effects, it is possible to estimate probabilistic characteristics of body impedance and of touch currents for a shock victim in whom specific shock effects were observed under defined electric shock circumstances.

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