Chemical and Process Engineering 2015, 36 (2), 135-149
DOI: 10.1515/cpe-2015-0011

LOCAL TURBULENT ENERGY DISSIPATION RATE IN A VESSEL AGITATED BY A RUSHTON TURBINE

Radek Šulc*, Vít Pešava, Pavel Ditl

Czech Technical University in Prague, Faculty of Mechanical Engineering, Department of Process Engineering, Technická 4, 166 07 Prague, Czech Republic

The scaling of turbulence characteristics such as turbulent fluctuation velocity, turbulent kinetic energy and turbulent energy dissipation rate was investigated in a mechanically agitated vessel 300 mm in inner diameter stirred by a Rushton turbine at high Reynolds numbers in the range 50 000 < *Re* < 100 000. The hydrodynamics and flow field was measured using 2-D TR PIV. The convective velocity formulas proposed by Antonia et al. (1980) and Van Doorn (1981) were tested. The turbulent energy dissipation rate estimated independently in both radial and axial directions using the one-dimensional approach was not found to be the same in each direction. Using the proposed correction, the values in both directions were found to be close to each other. The relation $\varepsilon/(N^3 \cdot D^2) \propto$ const. was not conclusively confirmed.

Keywords: mixing, Rushton turbine, local turbulent energy dissipation rate, energy spectrum function, fluctuation velocity

1. INTRODUCTION

The theory of mixing operations such as liquid – liquid, gas – liquid and powder dispersion, and also flocculation and scale of segregation in chemical reactions occurring in a turbulent regime is based on Kolmogorov theory of turbulence whose basic parameter is the local dissipation rate of turbulent kinetic energy (Paul et al. (2004), Bałdyga, Bourne (1999)).

The following experimental techniques such as Wire Hot Anemometry (WHA), Laser Doppler Anemometry (LDA) or Particle Image Velocimetry (PIV) have been used for estimation of some turbulence characteristics in stirred tanks. In this work a 2D-PIV method was used. Baldi and Yianneskis (2004) measured directly turbulence energy dissipation rate in the impeller stream of a vessel of diameter T = 100 mm stirred by a Rushton turbine of diameter $D = T/3$ by particle image velocimetry (PIV) for 15 000 < *Re* < 40 000. The values of the ratio $\varepsilon/N^3\ d^2$) in the impeller stream were in the range of 5-10 (i.e. = 7.5 ± 2.5 = 7.5 ± 33.3 % of the average value). The turbulent energy dissipation was calculated by means of turbulent velocity gradients. Ducci and Yianneskis (2005) carried out measurements in a vessel of diameter T = 294 mm stirred by a six-blade Rushton turbine of diameter $D = T/3$ for 20 000 < *Re* < 40 000. The dissipation rate normalised with N^3D^2, where N is the impeller rotational speed and D is the impeller diameter, respectively, was found to be approximately constant. The average integral length scale determined by the LDA data in the vicinity of ε_{max} was around 0.05D, about 50% of that normally assumed in dimensional relation estimates for ε. The turbulent energy dissipation was calculated by means of turbulent velocity gradients.

*Corresponding author, e-mail: Radek.Sulc@fs.cvut.cz

The aim of this work is to study scaling of turbulence characteristics such as turbulent fluctuation velocity, turbulent kinetic energy and turbulent energy dissipation rate in a mechanically agitated vessel in a fully turbulent region at high Reynolds numbers in the range of 50 000 < *Re* < 100 000. The hydrodynamics and flow field were measured in an agitated vessel using Time Resolved Particle Image Velocimetry (TR PIV).

2. THEORETICAL BACKGROUND

2.1. Mean and fluctuation velocity

Using PIV, the data set of i^{th} instantaneous velocity component $U_i\ (t_j)$ for $j = 1, 2, \ldots N_R$ at observation times t_j with an equidistant time step Δt_S (i.e. $\Delta t_S = t_{j+1} - t_j$) was obtained in a given location. Assuming the ergodic hypothesis, the time-averaged mean velocity $\overline{U_i}$ was determined as the average value of velocity data set $U_i\ (t_j)$:

$$\overline{U_i} = \frac{1}{N_R} \cdot \sum_j U(t_j) \ \text{ for } j = 1, 2, \ldots, N_R \tag{1}$$

where $\overline{U_i}$ is i^{th} mean velocity component, $U_i\ (t_j)$ is i^{th} instantaneous velocity component at observation time t_j, and N_R is the number of data items in the velocity data set. Consequently, the i^{th} fluctuation velocity component $u_i\ (t_j)$ at observation time t_j is obtained by decomposition of the instantaneous velocity:

$$u_i(t_j) = U_i(t_j) - \overline{U_i} \ \text{ for } j = 1, 2, \ldots, N_R, \tag{2}$$

where $u_i\ (t_j)$ is the i^{th} fluctuation velocity component at observation time t_i, $\overline{U_i}$ is i^{th} mean velocity component, and $U_i\ (t_i)$ is i^{th} instantaneous velocity component at observation time t_i.

The root mean squared fluctuation velocity is determined as follows:

$$\overline{u}_i = \left(\frac{1}{N_R} \cdot \sum_j u_i^2(t_j) \right)^{1/2} \ \text{ for } j = 1, 2, \ldots, N_R, \tag{3}$$

where $\overline{u}_i$ is the root mean squared fluctuation velocity, and $u_i\ (t_i)$ is the fluctuation velocity at observation time t_i.

2.2. Turbulent kinetic energy

The time-averaged total turbulent kinetic energy $\overline{q}$ is calculated from rms fluctuation velocities as follows:

$$\overline{q} = \sum_{i=1}^{3} \overline{q}_i = \frac{1}{2} \cdot \sum_{i=1}^{3} \overline{u}_i^2 \tag{4}$$

2.3. Turbulent energy dissipation rate

The distribution of the turbulent kinetic energy in eddies of different size can be described by energy spectrum function (to be precise: energy spectrum density function) $E(k)$. Hence the following relation can be written:

$$\overline{q} = \int_0^\infty E(k)dk \tag{5}$$

where $\overline{q}$ is total turbulent kinetic energy, $E(k)$ is three-dimensional energy spectrum density function, and k is wave number.

Kolmogorov (1941) derived in the inertial subrange the following form of spectrum function known as Kolmogorov –5/3 power law:

$$E\ (k) = A_{iner} \cdot \varepsilon^{2/3} \cdot k^{-5/3} \tag{6}$$

The local turbulent kinetic energy dissipation rate ε can be estimated easily by a spectral fitting method from the energy spectrum function in this subrange. This evaluation method enables to estimate local energy dissipation rates without any knowledge of the local velocity gradients whose estimation is very difficult and inaccurate. However, the problem is how to obtain the three-dimensional energy spectrum function. In the literature it can be found that assuming the local isotropy the three-dimensional energy spectrum $E(k)$ can be replaced by the one-dimensional spectrum (Ståhl Wernersson and Trägårdh, 2000):

$$E_{1D}(k_1) = A_{iner1D} \cdot \varepsilon^{2/3} \cdot k_1^{-5/3} \tag{7}$$

For this case, Grant et al. (1962) evaluated the constant A_{iner1D} as 0.47 ± 0.02 and Sreenivasan (1995) presented the value of 0.53 as the universal empirical constant.

The one-dimensional energy spectrum function $E_{1D}(f_1)$ was obtained from the time course of fluctuation velocity $u_1(t)$ using the Fourier transform. Then the one-dimensional energy spectrum function $E_{1D}(f_1)$ is transformed from the frequency domain (f) to the wavenumber domain (k) by the following Eqs. (8) and (9):

$$E_{1D}(k_1) = \frac{f_1}{k_1} \cdot E_1(f_1) = \frac{U_{conv}}{2\pi} \cdot E_{1D}(f_1) \tag{8}$$

$$k_1 = \frac{2 \cdot \pi \cdot f_1}{U_{conv}} \tag{9}$$

where U_{conv} is so-called convective velocity. The convective velocity method is a way how to transform turbulent time scales into length scales (Ståhl Wernersson and Trägårdh, 2000) and enables to correct a method developed for one-dimensional and low intensity flow to be used for application to a three – dimensional or highly turbulent flow field (Kresta and Wood, 1993).

The following definitions of convective velocity were used:

- Antonia et al. (1980):

$$U_{conv}^2 = \overline{U}_i^2 + \overline{u}_i^2 \tag{10}$$

- Van Doorn (1981):

$$U_{conv}^2 = \overline{U}_1^2 + \overline{u}_1^2 + 2 \cdot \overline{u}_2^2 + 2 \cdot \overline{u}_3^2 \tag{11}$$

where $\overline{U}_i$ and $\overline{u}_i$ are i^{th} mean and rms fluctuation velocity components respectively.

The third component of fluctuation velocity in Van Doorn definition was estimated using the pseudo-isotropic assumption as an average of the first and second squared velocity components, then:

$$U_{conv}^2 = \overline{U}_1^2 + \overline{u}_1^2 + 2 \cdot \overline{u}_2^2 + 2 \cdot ((1/2) \cdot (\overline{u}_1^2 + \overline{u}_2^2)) \tag{12}$$

2.4. Data block averaging

The technique of data block averaging was used for smoothing of the spectrum. The raw data were split into blocks of equal duration. The entire dataset of record time length T_R containing a number of data N_R was divided into a number of blocks n_B with equal block time T_B containing block number of data N_B so that:

$$T = \Delta t_S \cdot N_R = n_B \cdot T_B \tag{13a}$$

$$T_B = \Delta t_S \cdot N_B \tag{13b}$$

$$N_R = n_B \cdot N_B \tag{13c}$$

where Δt_S is sampling time, i.e. time step between two consecutive values of measured velocity.

The separate energy spectrum density is calculated for each individual block and the final energy spectra density is determined as the average of separate energy spectra density $E_j(f)$:

$$E(f) = \frac{1}{n_B} \cdot \sum_j E_j(f) \quad \text{for } j = 1, 2, \ldots, n_B. \tag{14}$$

3. EXPERIMENTAL

The hydrodynamics and the flow field were measured in an agitated vessel using Time Resolved Particle Image Velocimetry (TR PIV). Experiments were carried out in a fully baffled cylindrical flat bottom vessel 300 mm in inner diameter (Kotek et al., 2012). The tank was agitated by a Rushton turbine 100 mm in diameter, i.e. the dimensionless impeller diameter D/T was 1/3. The dimensionless impeller clearance C/D taken from the lower impeller edge was 0.75. The tank was filled by degassed distilled water and the liquid height was 300 mm, i.e. the dimensionless liquid height H/T was 1. The dimensionless baffle width B/T was 1/10. To prevent air suction, the vessel was covered by a lid. Velocity fields were measured for three impeller rotation speeds: 300 rpm, 450 rpm and 600 rpm, at which fully developed turbulent flow was reached. Distilled water at a temperature of 23°C (density $\rho = 997.4$ kg.m^{-3}, dynamic viscosity $\mu = 0.9321$ mPa·s) was used as the agitated liquid.

The time resolved LITRON LDY 304 2D-PIV system (Dantec Dynamics (Denmark)) consists of a Neodyme-YLF laser (light wave length 532 nm, impuls energy 2×30 MJ), a SpeedSence 611 high speed PIV-regime camera (resolution 1280×1024 pixels) with a Sigma MacroDg objective equipped with an optical filter with wave length 570 nm. Rhodamine B the fluorescent particles of mean diameter 11.95 ± 0.25 µm were used as seeding particles. The fluorescent particles lit by 532 nm light emit 570 nm light. In this way, non-seeding particles such as impurities and bubbles are separated and are not recorded. The operating frame rate was 1 kHz (1000 vector fields per second), i.e. the sampling time Δt_s was 1 ms.

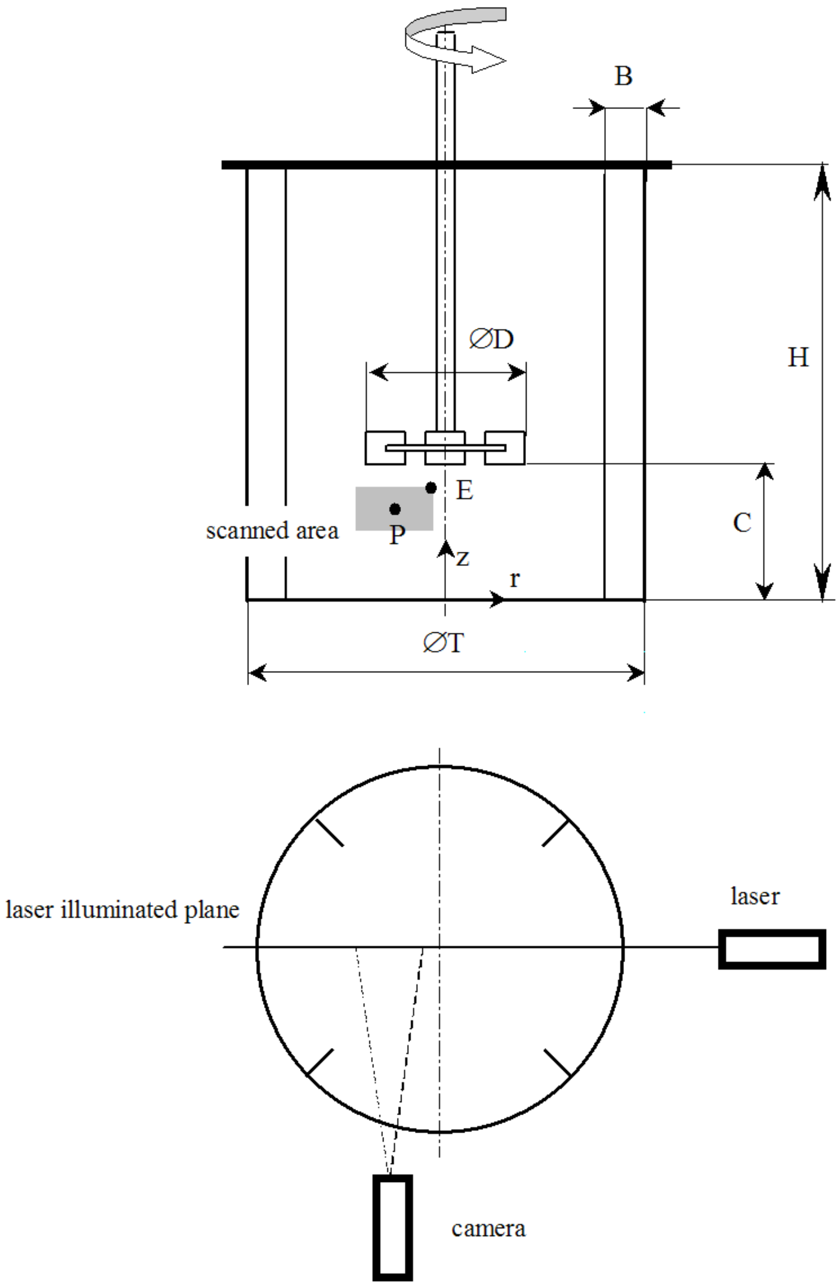

Fig. 1. Scheme of experimental apparatus and investigated area

The measured vertical plane was located in the center of the vessel and in the middle of the baffles. The plane was illuminated by a laser sheet 0.7 mm in thickness. The investigated area was 43×27 mm. The position of right top apex $E[r_E; z_E]$ was [10; 55], i.e. $2r/T = 20/300$ and $z/T = 55/300$, i.e. the right top edge was located 20 mm below the impeller paddle edge and 10 mm from the impeller axis. A scheme of the experimental apparatus and the investigated area is depicted in Fig. 1. The camera was positioned orthogonally to the laser sheet. The experiments were conducted in cooperation with Dr M. Kotek (Technical University Liberec) and Dr B. Kysela (Institute of Hydrodynamics, Czech Academy of Sciences). The data obtained in the position $P[r_P; z_P]$ = [33.584; 41.478], i.e. $2r/T$ = 0.224 and z/T = 0.138, are presented in this paper. Since this point is relatively far from the impeller, we expect the state of local isotropy.

For all experiments, 5 000 images were taken at a sampling interval of 0.001 s, i.e. the total record time length was 5 s. Unfortunately, a recording time of only 3.865 s was available for the 300 rpm measurement, due to damage to the storage disk.

4. RESULTS

The evaluated mean and fluctuation velocities are presented in a dimensionless form in Table 1 and graphically in Figs. 2 and 3. According to the theory of mixing, the dimensionless velocities normalised by the product of impeller speed N and impeller diameter D should be independent of the impeller rotational speed.

Table 1. Dimensionless velocities and dimensionless turbulent kinetic energy

Operational conditions		Radial direction			Axial direction			Isotropy
N [rpm]	Re [-]	$\frac{\overline{U}_r}{ND}$ [-]	$\frac{\overline{u}_r}{ND}$ [-]	$\frac{\overline{q}_r}{(ND)^2}$ [-]	$\frac{\overline{U}_{ax}}{ND}$ [-]	$\frac{\overline{u}_{ax}}{ND}$ [-]	$\frac{\overline{q}_{ax}}{(ND)^2}$ [-]	$\frac{\overline{u}_{ax}}{\overline{u}_r}$ [-]
300	53 503	-0.0716	0.2587	0.0335	0.4145	0.1381	0.0095	0.5340
450	80 254	0.0064	0.3023	0.0457	0.4234	0.1540	0.0119	0.5093
600	107 006	-0.0057	0.2223	0.0247	0.4423	0.1388	0.0096	0.6243

The effect of impeller rotational speed on dimensionless velocities was tested by hypothesis testing (Bowerman and O´Connell (1997)). A statistical method for hypothesis testing can estimate whether the differences between the predicted parameter values (e.g. predicted by a proposed theory) and the parameter values evaluated from the measured data are negligible. If this is the case, we assumed the dependence of the tested parameter on the impeller rotational speed, described by the simple power law parameter = $B \cdot N^{\beta}$, and the difference between predicted exponent β_{pred} and evaluated exponent β_{calc} was tested. The hypothesis test characteristics are given as $t = (\beta_{calc} - \beta_{pred})/ s_{\beta}$ where s_{β} is the standard error of parameter β_{calc}. If the calculated $|t|$ value is less than the critical value of the t-distribution for (m-2) degrees of freedom and significance level α, the difference between β_{calc} and β_{pred} is statistically negligible (statisticians state: "the hypothesis cannot be rejected").

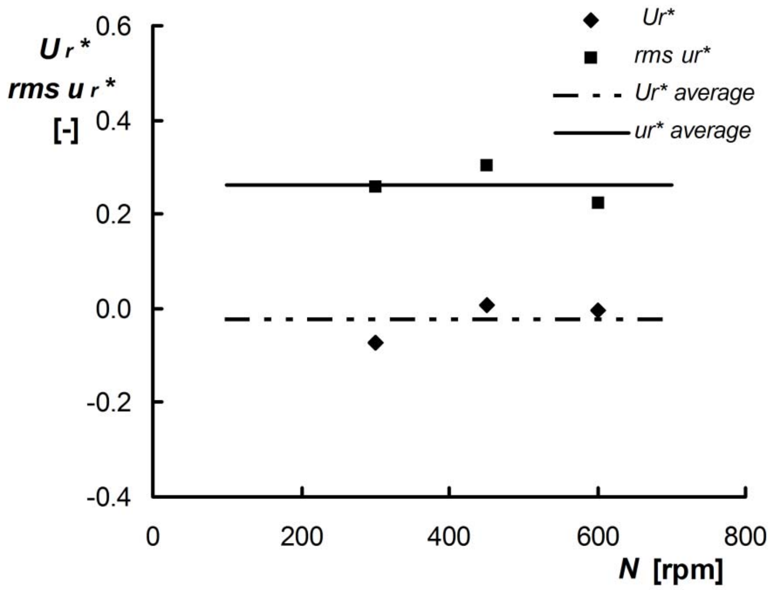

Fig. 2. Dimensionless velocity – radial direction

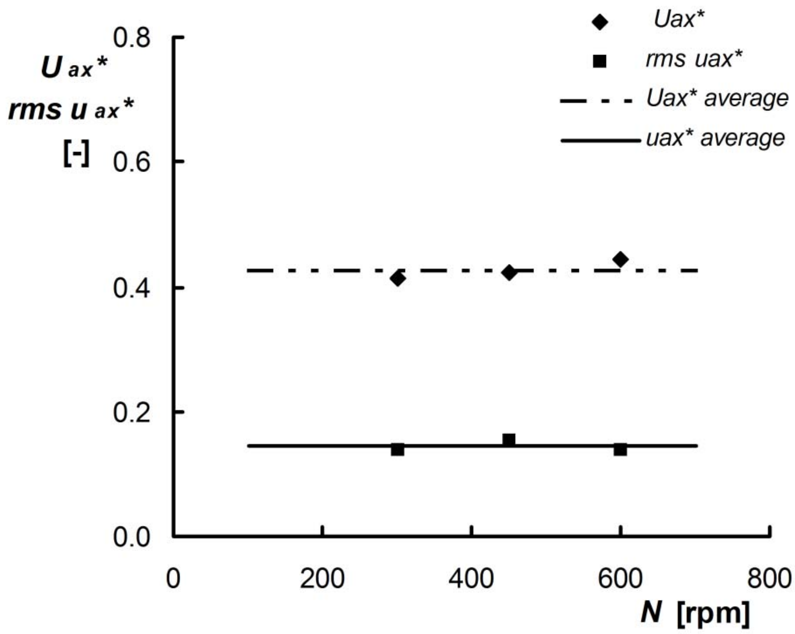

Fig. 3. Dimensionless velocity – axial direction

In our case, the independence of dimensionless velocities and dimensionless turbulent kinetic energies from the impeller speed was tested as a hypothesis, i.e. parameter = $B\cdot(N)^0$ = const., i.e. $\beta_{pred} = 0$. The t-distribution coefficient $t_{(m\text{-}2),\ \alpha}$ for three impeller rotational speeds and significance level $\alpha = 0.05$ is 12.706. The hypothesis test result and parameter β evaluated from the experimental data are presented in Table 2.

It was found, that all dimensionless velocities except radial mean velocity can be statistically taken as constant and independent of the impeller rotational speed. The average values were calculated and are presented in Table 2. The dimensionless radial mean velocity cannot be tested due to the alternation of positive and negative values.

Table 2. Dimensionless velocities and turbulent kinetic energy – effect of impeller speed

Parameter	m [-]	t-distribution $t_{(m\text{-}2),\ \alpha=0.05}$	Relation parameter = $B\cdot(N)^{\beta}$ β_{calc}	t-characteristics $\lvert t\rvert$ Hypothesis parameter = $B\cdot(N)^0$ $\beta_{pred} = 0$	Average value
$\overline{U}_r/(ND)$	3	12.706	n/a	n/a	-0.0236 ± 0.0479
$\overline{u}_r/(ND)$	3	12.706	-0.179	0.4 (acceptable)	0.2611 ± 0.0412
$\overline{q}_r/(ND)^2$	3	12.706	-0.358	0.4 (acceptable)	0.0346 ± 0.0111
$\overline{U}_{ax}/(ND)$	3	12.706	0.091	3.3 (acceptable)	0.4267 ± 0.0156
$\overline{u}_{ax}/(ND)$	3	12.706	0.024	0.1 (acceptable)	0.1436 ± 0.0103
$\overline{q}_{ax}/(ND)^2$	3	12.706	0.048	0.1 (acceptable)	0.0103 ± 0.0015
$\overline{u}_{ax}/\overline{u}_r$	3	12.706	0.203	0.9 (acceptable)	0.5559 ± 0.0684

Note: The t-distribution for (m-2) degrees of freedom and significance level $\alpha = 0.05$.

The radial mean velocity was found to be close to zero. This finding corresponds to a chosen position in an upward flow to the impeller. This region contains predominantly ascending flow along the vessel axis towards the impeller (Fořt et al, 1982). Owing to the fact that the selected position is relatively far from the impeller and outside of the impeller discharge flow we expected a state of local isotropy. The state of local isotropy was defined on the length-scale level corresponding to integral length scale as an equality of the fluctuation velocity components. This expectation was not confirmed. The axial rms fluctuation velocity was approx. one half of radial component.

The calculated values of dimensionless turbulent kinetic energy in radial and axial directions are presented in Table 1. The values of dimensionless turbulent kinetic energy were also found to be independent of the impeller rotational speed. The test results and averaged values are presented in Table 2.

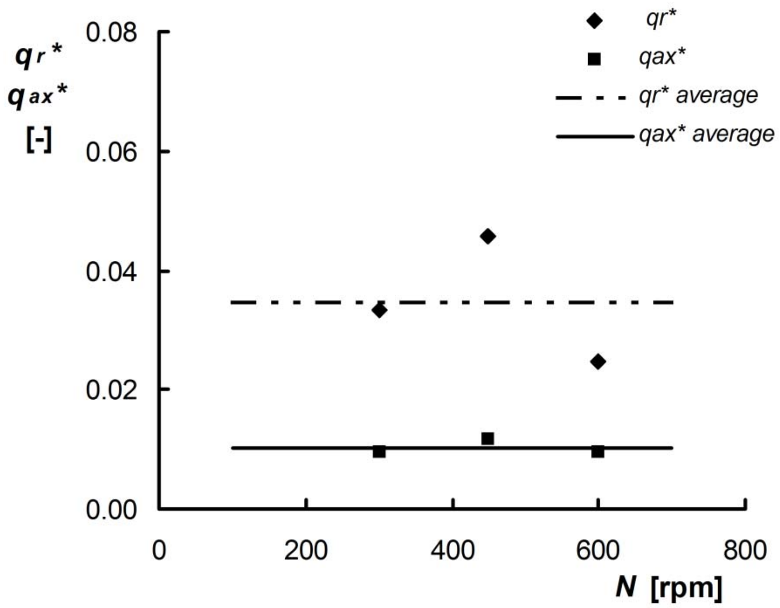

Fig. 4. Dimensionless velocity – axial direction

The estimation of local turbulent energy dissipation rate from the energy spectrum function is traditionally based on the assumption that the three-dimensional energy spectrum function can be replaced at a local isotropy by the one-dimensional energy spectrum function and the so-called convective velocity coupling the time and space derivatives (Wu et al., 1989) has been used for spectrum conversion from frequency domain to wave number domain. Following on from that, the correct estimation of dissipation rate depends on the degree of local isotropy and a proper estimation of convective velocity. We tested convective velocity formulas proposed by Antonia et al. (1980) and Van Doorn (1981).

The energy spectrum functions were obtained using Fast Fourier Transform (FFT). Owing to the nature of FFT algorithm, the number of samples must be an integer power of 2. For smoothing of the spectrum, the technique of data block averaging was used. Calculations were made for a block number of data N_B equal to 1024. The obtained averaged energy spectrum $E_1(f)$ was transformed into wave number domain and statistically treated by regression line:

$$E_{1D}(k) = B_k \cdot k^{-5/3} \tag{15}$$

Then, combining Eqs. (7) and (15) the local dissipation rate can be estimated as follows:

$$\varepsilon = \left(\frac{B_k}{A_{iner}} \right)^{3/2} \tag{16}$$

where the constant A_{iner1D} as 0.47 ± 0.02 evaluated by Grant et al. (1962) was used.

The convective velocity according to Antonia et al. (1980) was calculated as follows:

- for radial direction:

$$U^2_{conv\ r} = \overline{U}^2_r + \overline{u}^2_r \tag{17a}$$

- for axial direction:

$$U^2_{conv\ ax} = \overline{U}^2_{ax} + \overline{u}^2_{ax} \tag{17b}$$

The convective velocity according to Van Doorn (1981) was calculated as follows:

- for radial direction:

$$U^2_{conv-r} = \overline{U}^2_r + \overline{u}^2_r + 2 \cdot \overline{u}^2_{ax} + 2 \cdot ((1/2) \cdot (\overline{u}^2_r + \overline{u}^2_{ax})) \tag{18a}$$

- for axial direction:

$$U^2_{conv-ax} = \overline{U}^2_{ax} + \overline{u}^2_{ax} + 2 \cdot \overline{u}^2_r + 2 \cdot ((1/2) \cdot (\overline{u}^2_{ax} + \overline{u}^2_r)) \tag{18b}$$

The velocity components in a given direction were presumed as the dominant velocity components for van Doorn definition of the convective velocity in a given direction. The averaged dimensionless velocities presented in Table 2 were used to calculate convective velocity.

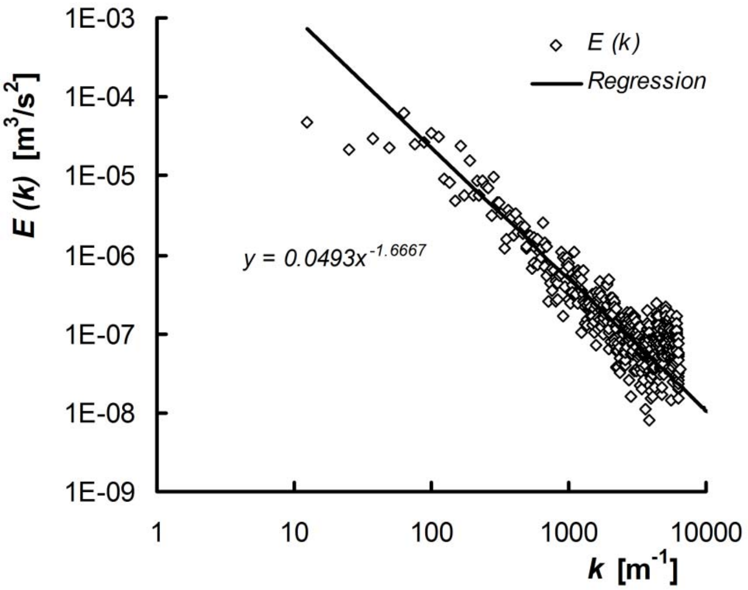

Fig. 5. Energy spectrum function – axial direction, N = 450 rpm, Van Doorn´s convective velocity

The convective velocity and energy spectrum function were calculated independently in both axial and radial axes and then dissipation rate was estimated for each direction independently. The obtained results are presented in Tables 3 and 4. For comparison purposes, a specific impeller power input (P/m) was also calculated for the impeller power number of 5.1. For illustration purposes, we treated (-5/3) law trend line for axial direction and Van Doorn´s convective velocity at the impeller speed of 450 rpm (Fig. 5). The effect of impeller speed on the spectrum function is demonstrated in Fig. 6 for radial direction and Van Doorn´s convective velocity.

The used convective velocity formula plays an important role for the dissipation rate estimation. The convective velocities calculated using Van Doorn´s formula are 1.7 - and 1.45 times higher for radial and axial directions respectively compared with Antonia´s formula. Unlike this the dissipation rates are different in opposite directions, i.e. the dissipation rates estimated using Antonia´s formula are approx.

1.7 - and 1.5 times higher for radial and axial directions respectively compared with the dissipation rates estimated by Van Doorn´s formula.

Table 3. Local turbulent kinetic energy dissipation rate – convective velocity according to Antonia et al. (1980)

Operational conditions		Radial direction		Axial direction			Ratio
N [rpm]	P/m [W/kg]	$U_{conv\ r}$ [m/s]	$\varepsilon(r)$ [m²/s³]	$U_{conv\ ax}$ [m/s]	$\varepsilon(ax)$ [m²/s³]	$\varepsilon(ax)_{corr}$ [m²/s³]	$\varepsilon(ax)_{corr}/\varepsilon(r)$ [-]
300	0.301	0.131	0.039	0.225	0.012	0.038	0.974
450	1.015	0.197	0.143	0.338	0.049	0.160	1.119
600	2.405	0.262	0.491	0.450	0.152	0.492	1.002

Table 4. Local turbulent kinetic energy dissipation rate – convective velocity according to Van Doorn (1981)

Operational conditions		Radial direction		Axial direction			Ratio
N [rpm]	P/m [W/kg]	$U_{conv\ r}$ [m/s]	$\varepsilon(r)$ [m²/s³]	$U_{conv\ ax}$ [m/s]	$\varepsilon(ax)$ [m²/s³]	$\varepsilon(ax)_{corr}$ [m²/s³]	$\varepsilon(ax)_{corr}/\varepsilon(r)$ [-]
300	0.301	0.223	0.023	0.327	0.008	0.026	1.130
450	1.015	0.334	0.084	0.491	0.034	0.11	1.310
600	2.405	0.446	0.289	0.654	0.104	0.338	1.170

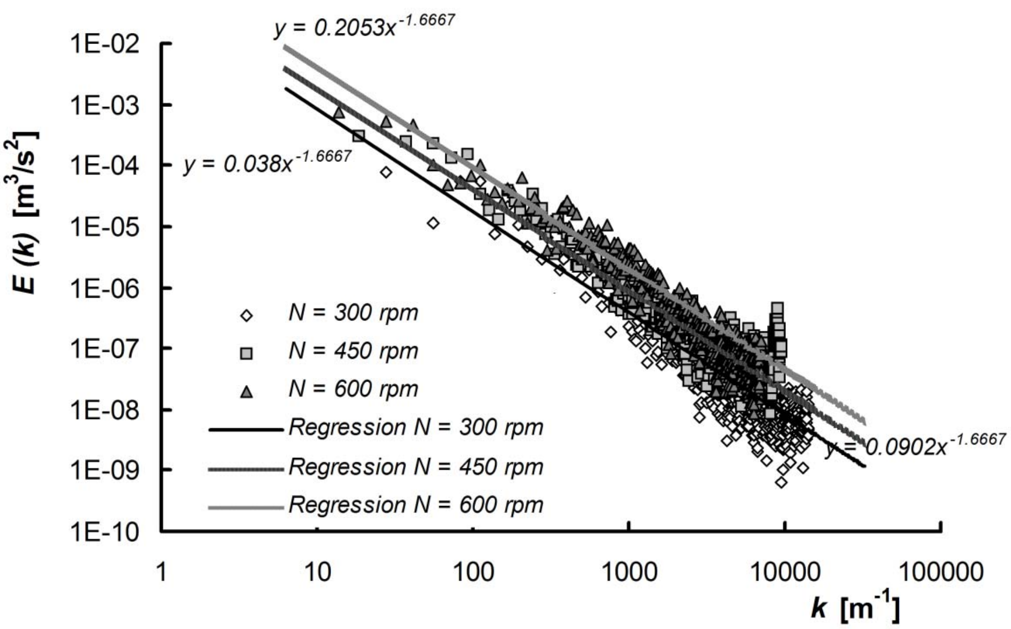

Fig. 6. Energy spectrum function – radial direction, effect of impeller speed, Van Doorn´s convective velocity

For illustration purposes, the effect of convective velocity formula on energy spectrum function is presented in Fig. 7 for radial direction and impeller speed of 600 rpm. Although the difference between constants of proportionality is negligible in log-log diagram, the differences of dissipation rates are more significant.

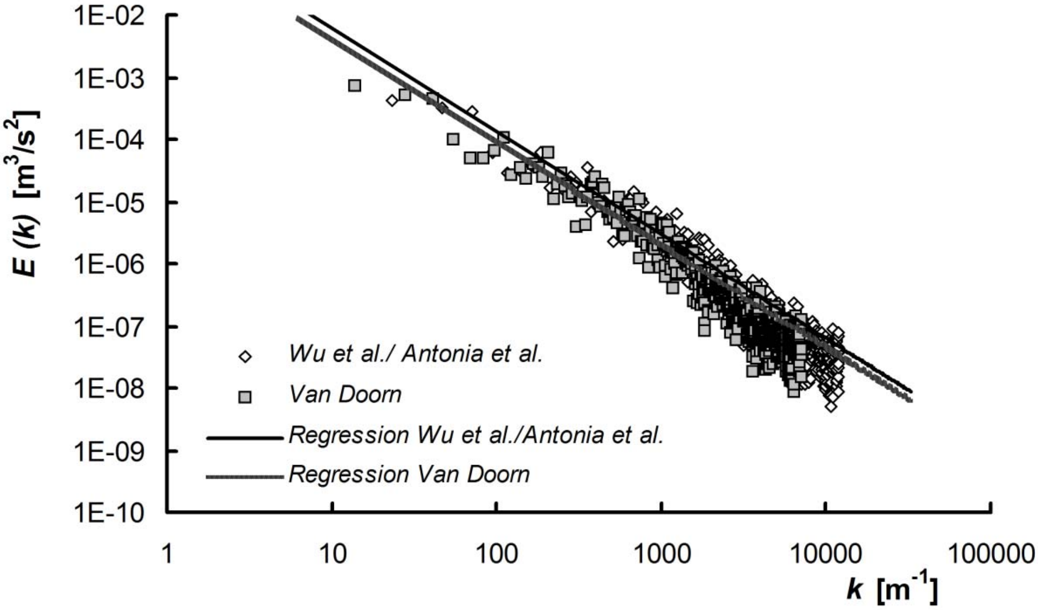

Fig. 7. Effect of convective velocity formula on energy spectrum function - radial direction, N = 600 rpm

It was observed that the dissipation rates estimated based on axial fluctuation velocity components are approx. 2.5-3.3 times lower compared with the dissipation rates estimated based on radial fluctuation velocity component both for Antonia´s formula and for Van Doorn´s formula of convection velocity. We found that this dissipation rate ratio corresponds to the squared ratio of axial velocity and radial velocity components.

We proposed and tested the following empirical formula to correct the dissipation rate estimated from axial direction:

$$\varepsilon(ax)_{corrected} = \left(\frac{\overline{u}_{ax}}{\overline{u}_r}\right)^{-2} \cdot \varepsilon(ax)_{uncorrected} \tag{19}$$

The corrected values of dissipation rate estimated from axial direction are presented in Tables 3 and 4. For correction purposes the average value $\overline{u}_{ax}/\overline{u}_r$ = 0.5559 was used. Using this correction the values in both directions were found to be close to each other as expected having applied the one-dimensional spectrum fitting method for dissipation rate estimation.

For the local turbulent kinetic energy dissipation rate estimated by Van Doorn´s convective velocity in radial direction the integral and Kolmogorov scales were calculated and are presented in Table 5. The integral length scale Λ was estimated using the dimensional assumption as follows:

$$\varepsilon = C_\varepsilon \frac{\overline{u}^3}{\Lambda} \tag{20}$$

where empirical constant of proportionality C_ε equal to 1 was used. The integral time scale was estimated in the following way:

$$\tau_\Lambda = \frac{\Lambda}{\overline{u}} \tag{21}$$

The Kolmogorov scales were determined as follows:

- length scale

$$\eta_K = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \tag{22}$$

- velocity scale

$$v_K = (\nu.\varepsilon)^{1/4} \tag{23}$$

- time scale

$$\tau_K = \left(\frac{\nu}{\varepsilon}\right)^{1/2} \tag{24}$$

Table 5. Turbulence characteristics - Van Doorn – radial direction

N [rpm]	ε [m^2/s^3]	$\varepsilon/(N^3 \cdot D^2)$ [-]	η_K [μm]	v_K [m/s]	τ_K [s]	Λ [mm]	τ_Λ [s]
300	0.023	0.0184	77	0.0121	0.0064	97	0.742
450	0.084	0.0199	56	0.0167	0.0033	89	0.456
600	0.289	0.0289	41	0.0228	0.0018	62	0.236

Finally, the dimensionless local turbulent energy dissipation rate defined as ratio $\varepsilon/(N^3 \cdot D^2)$ was calculated and presented in Table 5 and graphically in Fig. 8. According to the test of hypothesis the relation $\varepsilon/(N^3 \cdot D^2) \propto$ const. can be accepted. However, the visual assessment of the Fig. 8 signalises a trend line. Similarly, the relation $\Lambda \propto d$ was not conclusively confirmed.

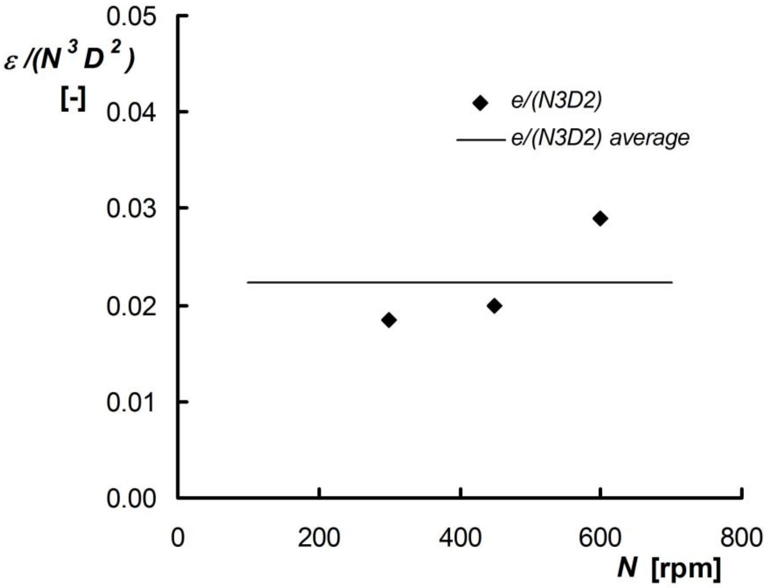

Fig. 8. Dimensionless local turbulent dissipation rate – radial direction, effect of impeller speed, Van Doorn convective velocity

5. CONCLUSIONS

- The scaling of turbulence characteristics such as turbulent fluctuation velocity, turbulent kinetic energy and turbulent energy dissipation rate was investigated in a mechanically agitated vessel 300 in inner diameter stirred by a Rushton turbine in a fully turbulent region at high Reynolds numbers in the range of 50 000 < Re < 100 000. The hydrodynamics and flow field were measured in an agitated vessel using 2-D Time Resolved Particle Image Velocimetry (TR PIV). The data obtained in the chosen point in upward flow to the impeller are presented.

- In accordance with the theory of mixing, the dimensionless mean and fluctuation velocities and dimensionless turbulent kinetic energies in the measured directions were found to be constant and independent of the impeller rotational speed.
- The turbulent energy dissipation rate was estimated by fitting of energy spectrum function. The energy spectrum was obtained via FFT of time course of fluctuation velocity using block averaging technique for smoothing of the spectrum. We tested convective velocity formulas proposed by Antonia et al. (1980) and Van Doorn (1981) for spectrum conversion from frequency domain to wave number domain.
- The convective velocity formula plays an important role for dissipation rate estimation. The dissipation rates estimated using Antonia´s formula were approx. 1.7 - and 1.5 times higher for radial and axial direction respectively compared with the dissipation rates estimated by Van Doorn´s formula.
- The state of local isotropy was defined on the length-scale level corresponding to integral length scale by an equality of fluctuation velocity components. Although the selected observation point was relatively far from the impeller in upward flow to the impeller, the state of local isotropy was not found. Therefore, the turbulent energy dissipation rate estimated using one-dimensional approach independently in both directions was not found to be the same in both directions. We found that the ratio of dissipation rates calculated in radial and axial directions corresponds to the squared ratio of axial velocity and radial velocity components. We proposed and tested an empirical formula for correction of dissipation rate estimated from the axial direction. Using this correction, the values in both directions were found to be close to each other as expected having applied the one-dimensional spectrum fitting method for dissipation rate estimation.
- Finally, the dimensionless local turbulent energy dissipation rate defined as ratio $\varepsilon/(N^3 \cdot D^2)$ was calculated. According to the test of hypothesis, the relation $\varepsilon/(N^3 \cdot D^2) \propto$ const. can be accepted. However, the visual assessment of ratios $\varepsilon/(N^3 \cdot D^2)$ signalizes a trend line. Similarly, the relation $\Lambda \propto d$ was not conclusively confirmed.

This research has been supported by Grant Agency of the Czech Republic project No. 101/12/2274 "Local rate of turbulent energy dissipation in agitated reactors & bioreactors" and by CTU in Prague SGS14/061/OHK2/1T/12.

SYMBOLS

A_{iner}	constant of proportionality; Eq. (6)
B	baffle width, m
B_k	constant of proportionality; Eq. (15)
C	impeller clearance (to the bottom impeller edge), m
C_ε	constant of proportionality; Eq. (20)
D	impeller diameter, m
$E(k)$	three-dimensional energy spectrum function in wavenumber domain, m^3/s
$E_{1D}(f)$	one-dimensional energy spectrum function in frequency domain, m^2/s
$E_{1D}(k)$	one-dimensional energy spectrum function in wavenumber domain, m^3/s
f	frequency, s^{-1}
$\boldsymbol{H}$	liquid height, m

k	wavenumber; $k = 2.\pi.f/U_{conv}$, m^{-1}
m	number of experimental points
m	liquid weight, kg
n_B	number of blocks
N	impeller rotational speed, s^{-1}
N_B	block number of data
N_R	number of data items in velocity data set
P	impeller power input, W
$\overline{q}$	time-averaged total turbulent kinetic energy, m^2/s^2
$\overline{q}_i$	i^{th} time-averaged turbulent kinetic energy component, m^2/s^2
r	radial coordinate, m
t	time, s
t_i	observation time, s
$t_{(m-2),\ \alpha=0.05}$ - t	distribution for (m-2) degrees of freedom and significance level α
T	tank diameter, m
T_B	block time, s
T_R	record time length, s
u_i	i^{th} fluctuation velocity component, m/s
$\overline{u}_i$	i^{th} root mean squared fluctuation velocity component, m/s
U_i	i^{th} instantaneous velocity component, m/s
$\overline{U}_i$	i^{th} mean velocity component, m/s
$U_{conv\ i}$	convective velocity calculated for i^{th} direction, m/s
v_K	Kolmogorov velocity scale, m/s
x_i	flow coordinates, i = 1, 2, 3, m
z	axial coordinate, m

Greek symbols

β	exponent
ε	local dissipation rate of turbulent kinetic energy , m^2/s^3
η_K	Kolmogorov length scale, m
μ	dynamic viscosity, Pa.s
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3
τ_K	Kolmogorov time scale, s
τ_{Ai}	integral time scale, s
Δt_S	sampling time, s
Λ	integral length scale, m

Subscripts

ax	axial direction
r	radial direction

REFERENCES

Antonia R.A., Satyaprakash B.R., Hussain A.K.M.F., 1980. Measurements of dissipation rate and some other characteristics of turbulent plane and circular jets. *Phys. Fluids*, 23, 695-700. DOI: 10.1063/1.863055.

Baldi S., Yianneskis M., 2004. On the quantification of energy dissipation in the impeller stream of a stirred vessel from fluctuating velocity gradient measurements. *Chem. Eng. Sci.*, 59, 2659–2671. DOI: 10.1016/j.ces.2004.03.021.

Bałdyga, J., Bourne, J.R., 1999. *Turbulent mixing and chemical reactions.* Wiley, England.

Bowerman B.L., O´Connell R.T., 1997. *Applied statistics: Improving business processes*. Richard D. Irwin, USA.

Ducci A., Yianneskis M., 2005. Direct determination of energy dissipation in stirred vessels with two-point LDA. *AIChE J.*, 51, 2133–2149. DOI 10.1002/aic.10468.

Grant H.L., Stewart R.W., Moilliet A., 1962. Turbulence spectra from a tidal channel. *J. Fluid Mechanics*, 12, 241-263. DOI: 10.1017/S002211206200018X.

Fořt I., Obeid A., Březina V., 1982. Flow of liquid in a cylindrical vessel with a turbine impeller and radial baffles. *Coll. Czechoslov. Chem. Comm.*, 47, 226-239. DOI: 10.1135/cccc19820226.

Kolmogorov A.N., 1941. The local structure of turbulence in incompressible viscous fluid for very large Reynolds' numbers. *Doklady Akad. Nauk SSSR*, 30, 301-305. DOI: 10.1070/PU1968v010n06ABEH003710.

Kotek M., Pešava V., Kopecký V., Jašíková D., Kysela B., 2012. PIV measurement in a vessel of D = 0.3 m agitated by Rushton turbine. *Research report for project No.* 101/12/2274, Liberec.

Kresta S.M., Wood P.E., 1993. The flow field produced by pitched blade turbine: characterization of the turbulence and estimation of the dissipation rate. *Chem. Eng. Sci.,* 48, 1761-1774. DOI: 10.1016/0009-2509(93)80346-R.

Paul E.L., Atiemo-Obeng V.A., Kresta S.M. (Eds.), 2004. *Handbook of industrial mixing. Science and Practice*. Wiley, Hoboken, NJ.

Sreenivasan K.R., 1995. On the universality of the Kolmogorov constant. *Phys Fluids*, 7, 2778-2784. DOI: 10.1063/1.868656.

Ståhl Wernersson E., Trägårdh C., 2000. Measurements and analysis of high-intensity turbulent characteristics in a turbine-agitated tank. *Exp. Fluids*, 28, 532–545. DOI: 10.1007/s003480050414.

Van Doorn M., 1981. On Taylor´s hypothesis in turbulent shear flows. *Internal note 811123*, Univ. of Missouri-Rolla.

Wu H., Patterson G.K., Van Doorn M., 1989. Distribution of turbulence energy dissipation rates in a Rushton turbine stirred mixer. *Exp. Fluids*, 8, 153-160. DOI:10.1007/BF00195789.

Received 22 November 2014
Received in revised form 18 February 2015
Accepted 24 April 2015