ARCHIVES OF ELECTRICAL ENGINEERING VOL. 64(3), pp. 351-359 (2015)

DOI 10.2478/aee-2015-0027

The modified procedures in coercivity scaling*

MARIUSZ NAJGEBAUER[1], KRZYSZTOF SOKALSKI[2], JAN SZCZYGŁOWSKI[1]

[1]*Częstochowa University of Technology*
Institute of Electric Power Engineering
ul. Armii Krajowej 17, 42-200 Częstochowa, Poland
e-mail: najgebauer@el.pcz.czest.pl, jszczyg@gamil.com

[2]*Częstochowa University of Technology*
Institute of Computer Science
ul. Armii Krajowej 17, 42-200 Częstochowa, Poland
e-mail: sokalski@el.pcz.czest.pl

(Received: 31.09.2014, revised: 10.12.2014)

Abstract: The paper presents a scaling approach to the analysis of coercivity. The Widom-based procedure of coercivity scaling has been tested for non-oriented electrical steel. Due to insufficient results, the scaling procedure was improved relating to the method proposed by Van den Bossche. The modified procedure of coercivity scaling gave better results, in comparison to the original approach. The influence of particular parameters and a range of measurement data used in the estimations on the final effect of the coercivity scaling were discussed.

Key words: coercivity, hyperbolic tangent transformation, scaling procedure

1. Introduction

Coercivity (or coercive field) is defined as the value of an external magnetic field, which is required to reduce the magnetization of the sample from saturation to zero. The range of coercivity determines the type of magnetic material (soft, semi-hard or hard). This phenomenon is also correlated with hysteresis losses and the distribution of impurities in magnetic materials [1].

Coercivity is a very important magnetic parameter in technical applications. Thereby its modeling is a field of theoretical and practical research. Coercivity can be expressed as the frequency dependence:

$$H_c(f) = H_0 + B \cdot (H_0 \cdot f)^{\frac{1}{n}}, \tag{1}$$

where H_0 is the static coercivity, B and n are the coefficients depending on the microstructure, intrinsic parameters and geometry of the materials [2-4]. In general, the frequency dependence

* This is extended version of a paper which was presented at the 23th Symposium on Electromagnetic Phenomena in Nonlinear Circuits, Pilsen, Czech Republic 02-04.07, 2014.

of coercivity (1) has the power law form, that may suggest its scaling properties [5, 6]. The scaling behavior of coercivity in thin magnetic films and multilayers has been investigated e.g. in papers [7-9], however scaling exponents and factors were obtained from individual fitting to experimental data for different excitation parameters. Thereby, the scaling exponents and factors did not have universal values. The assumptions of the data collapse and coercivity scaling with universal values of scaling parameters are investigated in the presented paper for non-oriented electrical steel.

2. Methods

2.1. The scaling analysis of coercivity

The proposed scaling analysis of coercivity made use of the Widom procedure, which was previously applied in the power loss modeling of soft magnetic materials [10-11]. The Widom procedure is based on the hypothesis that a functional dependency describing the scaling phenomenon is a generalized homogenous function, defined as follows:

$$\exists a,b,c:\forall\lambda>0 \quad f\left(\lambda^{a_1}x_1,\lambda^{a_2}x_{2,\dots,}\lambda^{a_n}x_n\right)=\lambda^p f\left(x_1,x_{2,\dots,}x_n\right). \tag{2}$$

In general, the scaling analysis has two fundamental areas of prediction: data collapse and scaling laws [12], which can be used in the modeling of the analyzed phenomena.

In our approach, the coercivity dependence on magnetizing frequency and peak induction in the form of $H_C\,(f, B_m)$ was assumed. This form of coercivity dependence was determined from our measurements. It was also proposed in [13]. After the scaling transformations, the following relationship of scaled coercivity was proposed:

$$H_C/B_m^{\delta}=H_0+H_1\cdot\left(f/B_m^{\varepsilon}\right)+H_2\cdot\left(f/B_m^{\varepsilon}\right)^2+\dots, \tag{3}$$

where H_0, H_1, H_2 are amplitudes, while δ and ε are so-called scaling exponents. The aforementioned scaling transformations were described in detail in [5, 6]. Taking into account the frequency dependence of the coercivity given by (1), the relationship (3) was reduced to:

$$H_C/B_m^{\delta}=H_0+H_1\cdot\left(f/B_m^{\varepsilon}\right). \tag{4}$$

The relationship (4) should allow one to obtain the data collapse of all coercivity measurements, for the appropriately estimated amplitudes and scaling exponents. The parameters were estimated from the measurement data using the least-square method. The measurements were carried out according to the international IEC 404 Standards, which requires the sine waveform of magnetic induction inside the samples [14, 15]. Values of coercivity were determined from hysteresis loops, measured at the frequency range from 5 to 400 Hz and for induction with amplitude varying from 0.1 to 1.6 T. The measurements and calculations have been made by the computer-aided Single Sheet Tester MAG-RJJ-2.0. The result of the coercivity scaling for non-oriented electrical steel is depicted in Figure 1. A quite large deviation of the scaled measurements from the theoretical curve of the data collapse given by (4) can be noticed. It may suggest that magnetic coercivity is not subjected to scaling analysis or that the scaling procedure should be modified.

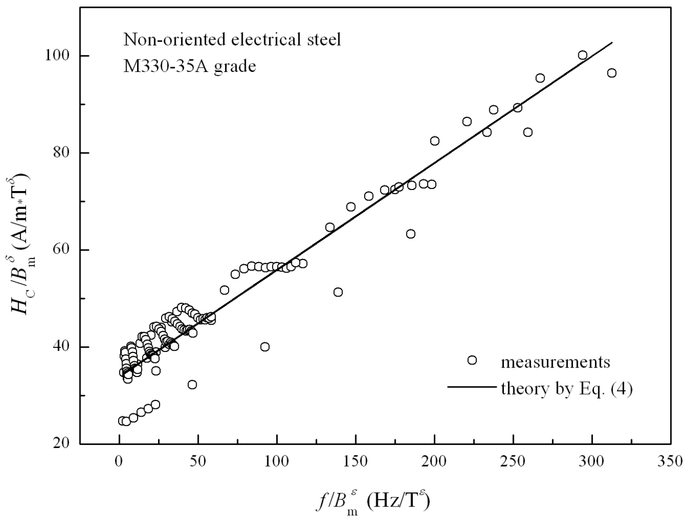

Fig. 1. The Widom-based scaling transformation of coercivity measurements for non-oriented electrical steel

2.2. The hyperbolic tangent modification of the scaling procedure

In order to improve the scaling procedure for coercivity, the method proposed in [16, 17] and used recently in the scaling of core losses under nonsinusoidal flux waveforms and DC bias conditions [18], has been applied. According to this method, the coercivity has been projected on the magnetization using the hyperbolic tangent transformation, as follows:

$$H_\mathrm{C} \to \tanh(d \cdot H_\mathrm{C} - r), \tag{5}$$

or in a more general form:

$$H_\mathrm{C} \to \sum_i a_i \cdot (\tanh(d \cdot H_\mathrm{C} - r_i))^{\,i}, \tag{6}$$

where a_i, d and r_i are free parameters to be estimated. The parameter r can be interpreted as a location of a single domain. Moreover, the small phenomenological tuning parameters x and z were introduced into the relationship (6) in order to enable a natural correction of the exponents [18]. Finally, the modified relationship for the scaled coercivity was proposed:

$$\frac{\sum\limits_i a_i \cdot (\tanh(d \cdot H_\mathrm{C} - r_\mathrm{i}))^{\,i\cdot(1\text{-}x)}}{B_\mathrm{m}^\delta} = H_0 + \sum_j H_j \cdot \left(\frac{f}{B_\mathrm{m}^\varepsilon}\right)^{j\cdot(1\text{-}z)}, \tag{7}$$

where j denotes the number of terms of the Maclaurin series used in the computations.

2.3. Estimation algorithms

The modified relationship of the scaled coercivity (7) is composed of many parameters that need to be determined. These parameters were estimated from the coercivity measurement data using the least-square method, which has been implemented in Microsoft Excel as well as in Fortran environments. The estimation algorithm in Excel was based on the Solver tool with the Generalized Reduced Gradient method of nonlinear optimization, whereas in the case of Fortran the quasi-Newton method of minimization was applied. The parameter estimation was implemented in two computing environments in order validate estimation results, *i.e.* to check whether local or global minimum of the analyzed relationship for coercivity scaling has been achieved. In both algorithms the starting values of the estimated parameters, besides the tuning parameters x, z, were randomly generated. The starting values of the x, z parameters were fixed at 0, because for these values the relationship (7) takes the classical form of the Maclaurin series. It allows us to investigate a range of a natural correction of the exponents as well as to determine how these parameters affect the results of coercivity scaling. It should be noted that the starting values of the x, z different form 0 were also considered in the parameter estimation – however it resulted in worse results of coercivity scaling.

3. Results and discussion

A structure of the relationship (7), proposed for the scaled coercivity, makes it possible to generate less or more complex formulas, consisting of a diverse number of parameters. In order to find the optimum solution for coercivity scaling as well as to examine the influence of the particular parameters on the effect of coercivity scaling, different formulas – starting from more complex to simpler ones – have been tested. The initial formula consisted of five amplitudes (H_0, H_1 … H_4) and five free parameters a_i, r_i as well as tuning parameters z, x. In the next steps, the formulas with decreasing number of amplitudes or parameters were analyzed. The quality of coercivity scaling for the tested formulas was evaluated using the following criteria:

1) average error δ_{Av} – the computed average of all percentage errors of the coercivity scaling (*i.e.* the discrepancy between the measured and theoretical values of scaled coercivity),
2) estimation coefficient ΔE – the sum of relative errors of coercivity scaling, which indicates how biased the estimation of parameters is,
3) coefficient of determination R2 – a kind of statistical measure, which indicates how well the data points fit a model (as R_2 is closer to 1, the model fits is better).

In our computations over 20 formulas for coercivity scaling were tested, which resulted in a very similar visualization of the coercivity scaling, i.e. the distribution of the scaled data points around the data collapse curve. The best scaling results were obtained for three formulas with the following structure:

$$\frac{\sum_i a_i \cdot (\tanh(d \cdot H_C - r))^i}{B_m^{\delta}} = H_0 + H_1 \cdot \left(\frac{f}{B_m^{\varepsilon}}\right) + H_2 \cdot \left(\frac{f}{B_m^{\varepsilon}}\right)^2, \tag{8}$$

for $i = 1$, $i = 2$ and $i = 3$, respectively. The relationship (8) has a limited range of application $B_m > 0$, because for $B_m = 0$ it has a singular point, which may gives non-physical results.

The structure of the relationship (8) directly indicates that: 1) increasing the number of amplitudes H_i doesn't improve the effect of coercivity scaling, whereas further reduction of amplitudes results in the linearization of the relationship and thereby in the breaking down of coercivity scaling, 2) increasing the number of free parameters r_i doesn't affect the scaling quality, and 3) tuning parameters z and x have a negligible influence on coercivity scaling, due to their low values (close to zero).

The values of quality criteria δ_{Av}, Δ_E and R_2, calculated for the considered structures of the relationship (8), are compared in Table 1.

Table 1. Quality criteria of coercivity scaling given by the relationship (8)

Structure of relationship (8)	δ_{Av} (%)	Δ_E (–)	R^2 (–)
$i = 1$	0.381	0.00031	0.9925
$i = 2$	0.312	0.00019	0.9962
$i = 3$	0.407	0.00090	0.9829

The best sets of quality criteria of coercivity scaling were obtained for $i = 2$, but differences between their values were very small (especially for $i = 2$ and $i = 1$). Therefore, for the further considerations the relationship (8) for $i = 1$ was chosen:

$$\frac{a \cdot \tanh(d \cdot H_C - r)}{B_m^{\delta}} = H_0 + H_1 \cdot \left(\frac{f}{B_m^{\varepsilon}}\right) + H_2 \cdot \left(\frac{f}{B_m^{\varepsilon}}\right)^2, \tag{9}$$

because the impact of the proposed simplification on the results of coercivity scaling is negligible. Moreover due to simpler structure of (9), it is easier to find analytically the inverse function with respect to H_C, comparing to the case for $i = 2$ or 3. Thereby, the prospective use of the simplified relationship (9) in coercivity modelling should be simpler.

The scaling parameters of the relationship (9), estimated using different algorithms, are compared in Table 2.

Table 2. The scaling parameters estimated by different algorithms

Scaling parameters	Algorithms of parameter estimation	
	excel based	fortran based
δ (–)	$-1.688 \cdot 10^{-3}$	$-1.693 \cdot 10^{-3}$
ε (–)	–1.051	–1.049
a (A/m)	1.703	1.710
d (m/A)	$-1.843 \cdot 10^{-4}$	$-1.833 \cdot 10^{-4}$
r (–)	–0.843	–0.837
H_0(A/m·T$^{\delta}$)	1.168	1.172
H_1(A · T$^{\varepsilon-\delta}$/m ·Hz)	$-4.698 \cdot 10^{-5}$	$-4.722 \cdot 10^{-5}$
H_2(A · T$^{2\varepsilon-\delta}$/m ·Hz^2)	$6.478 \cdot 10^{-8}$	$6.465 \cdot 10^{-8}$

Minor discrepancies between their values confirm that the parameters have been correctly estimated as well as a global minimum of the analyzed relationship has been achieved. A visualization of coercivity scaling given by the relationship (9) is depicted in Figure 2.

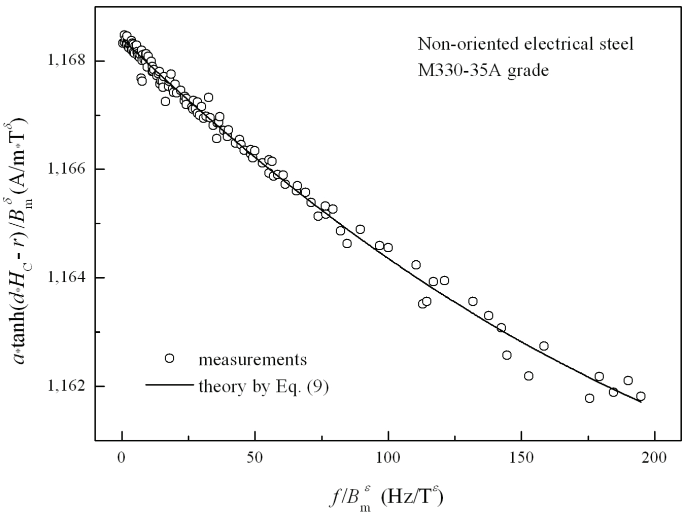

Fig. 2. The modified scaling transformation of coercivity measurements for non-oriented electrical steel

Much better agreement between the scaled coercivity data and the collapse curve can be observed in Figure 2, compared to the previous study presented in Figure 1. It indicates that the proposed modification of the scaling procedure was successful and that significant improvement in coercivity scaling was obtained, as well. However, it should be noted that some scaled data points show noticeable deviations from the data collapse curve for higher values of the scaled frequency f_{scal}. It might result from the following reasons: 1) two or more so-called classes of universality with different sets of parameters exist in the analyzed frequency range, thus the estimated parameters are valid only in a limited range of the frequency, or 2) the considered data points have been measured with higher errors compared to the other ones.

In order to verify the first presumption, the scaling parameters were estimated from separate sets of measurement data, related to the scaled frequency ranges lower and higher to $f_{scal} = 75$ Hz/T$^{\varepsilon}$, respectively. The scaling results obtained for the range of higher frequencies are depicted in Figure 3.

The distribution structure of the scaled data points and their deviations from the collapse curve are very similar to the case of the full frequency range. Moreover, the scaling parameters have comparable values for all frequency ranges considered in the estimation algorithms, which is presented in Table 3. It proves that all the measurement data belongs to the same class of universality. Thus, the analyzed cause is not responsible for the observed deviation of the scaled data points.

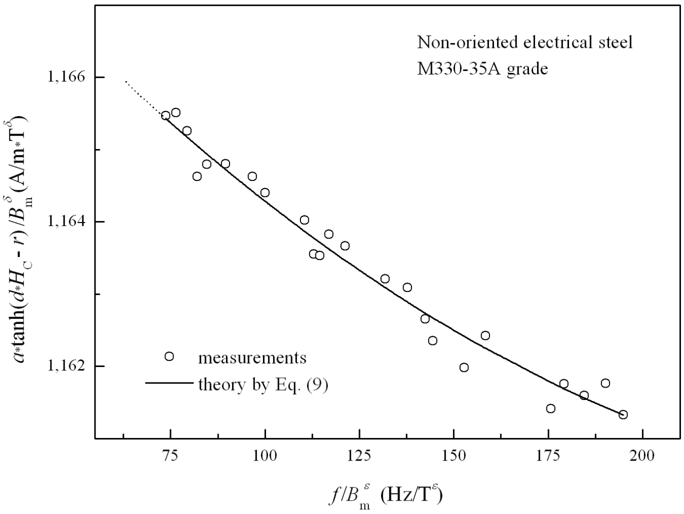

Fig. 3. Distribution structure of scaled data points for higher frequencies

Table 3. The scaling parameters for different sets of measurement data

Scaling parameters	Frequency range		
	5-400 (Hz)	5-100 (Hz)	100-400 (Hz)
δ (–)	$-1.69\cdot10^{-3}$	$-1.70\cdot10^{-3}$	$-1.67\cdot10^{-3}$
ε (–)	–1.05	–1.05	–1.05
A (A/m)	1.70	1.70	1.70
D (m/A)	$-1.84\cdot10^{-4}$	$-1.83\cdot10^{-4}$	$-2.07\cdot10^{-4}$
R (–)	–0.84	–0.84	–0.84
H_0 (A/m·T$^{\delta}$)	1.17	1.17	1.17
H_1 (A · T$^{(\varepsilon-\delta)}$/m·Hz)	$-4.70\cdot10^{-5}$	$-4.63\cdot10^{-5}$	$-5.14\cdot10^{-5}$
H_2 (A · T$^{(2\varepsilon-\delta)}$/m·Hz^2)	$6.48\cdot10^{-8}$	$7.58\cdot10^{-8}$	$8.32\cdot10^{-8}$

In the next step, the data points with noticeable deviations were identified. It should be stated, that these data points were measured in specific conditions, i.e. at low frequency and high peak induction or at high frequency and low peak induction. Therefore, the considered data points were probably measured with higher errors, related to the limitations of the measuring system. These data points were eliminated from the estimation of the scaling parameters. This process had no influence on the values of the scaling parameters. The results of the coercivity scaling obtained for the reduced range of measurement data are depicted in Figure 4, in which further improvement in the coercivity scaling is noticed. These results point towards measurement errors as a source of observed discrepancies.

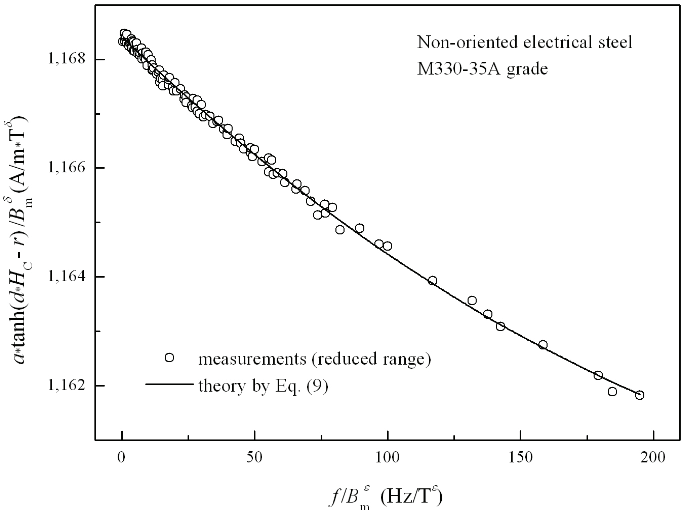

Fig. 4. The modified scaling transformation of coercivity measurements for non-oriented electrical steel in the case of reduced measurement data

4. Conclusions

In the paper, the modified procedure for coercivity scaling using the hyperbolic tangent transformation has been proposed. This procedure was examined for non-oriented electrical steel (M330-35A grade) and compared to the previous results, obtained for the Widom-based scaling. The use of the modified scaling procedure resulted in the decreasing of deviations of the scaled measurement data from the theoretical curve of data collapse.

The proposed formula of coercivity scaling has a general form, which is composed of many free parameters to be estimated. In order to find the optimum solution of coercivity scaling, over 20 formulas with different numbers of scaling parameters were tested. It allowed us to examine the influence of the particular parameters on the coercivity scaling results, as well. It was revealed that only some parameters have the crucial influence on the scaling results, whereas the other ones may be neglected.

It should be noted that all tested formulas of coercivity scaling resulted in a very similar distribution of the scaled data points around the data collapse curve. This fact as well as the final results of the coercivity scaling allowed us to conclude, that magnetic coercivity is subjected to the scaling analysis in the assumed excitation conditions. Moreover, the data collapse of the coercivity depicted in Figure 4 indicates the universal nature of the estimated scaling parameters.

Future research will be focused on testing the proposed procedure of coercivity scaling for soft magnetic materials with different internal structures such as amorphous, micro- and nanocrystalline ones. Moreover, the proposed procedure for coercivity scaling will be tested for sample exposed to non-sine waveform of magnetic induction. Prospective applications of the modified scaling analysis in coercivity modeling will also be examined.

References

[1] Vértesy G., Magni A., *Frequency dependence of coercive properties*. Journal of Magnetism and Magnetic Materials 256: 7-12 (2003).
[2] Zhukov A., Vázquez M., Velázquez J. et al., *Frequency dependence of coercivity in rapidly quenched amorphous materials.* Materials Science and Engineering A, 226-228: 753-756 (1997).
[3] Grössinger R., Mehboob N., Suess D. et al., *An eddy-current model describing the frequency dependence of the coercivity of polycrystalline Galfenol.* IEEE Transactions on Magnetics 48: 3076-3079 (2012).
[4] Groessinger R., Mehboob N., Kriegisch M. et al., *Frequency dependence of the coercivity of soft magnetic materials.* IEEE Transactions on Magnetics 48: 1473-1476 (2012).
[5] Najgebauer M., *The concept of scaling analysis in description of soft magnets' properties.* Proceedings of The 9th International Conference Mechatronic Systems and Materials, MSM'2013, Vilnius, Technica, pp. 180-181 (2013).
[6] Najgebauer M., *The concept of scaling analysis in describing the properties of soft magnets.* Solid State Phenomena 220-221: 646-651 (2015).
[7] Steinke N.-J., Moore T.A., Mansell R. et al., *Nonuniversal dynamic magnetization reversal in the Barkhausen-dominated and mesofrequency regimes.* Physical Review B, 86: 184-434 (2012).
[8] Handoko D., Lee S.-H., Lee K.M. et al., *Comparison of hysteresis loop area scaling behavior of Co/Pt multilayers: Discrete and continuous field sweeping*. Journal of Magnetism and Magnetic Materials 351: 82-86 (2014).
[9] Sokalski K., Szczygłowski J., Najgebauer M., Wilczyński W., *Losses scaling in soft magnetic materials.* COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering 26: 640-649 (2007).
[10] Sokalski K., Szczygłowski J., *Formula for energy loss in soft magnetic materials and scaling.* Acta Physica Polonica A, 15: 920-924 (2007).
[11] Najgebauer M., *Scaling theory and its chosen applications in electromagnetism.* Przegląd Elektrotechiczny 12: 213-216 (2008).
[12] Stanley H.E., *Scaling, universality and renormalization: three pillars of modern critical phenomena.* Reviews of Modern Physics 71: S358-S366 (1999).
[13] Horvat J., Babić E., Zadro K., Marohnić Ž., *Frequency and peak magnetization dependence of the coercive field in Fe-Ni-B-Si amorphous alloys.* Journal of Magnetism and Magnetic Materials 110: 215-220 (1992).
[14] IEC Standard, Publication 404-2, Third Edition 1996, *Magnetic materials, part 2: Methods of measurement of the magnetic properties of electrical steel sheet and strip by means of an Epstein frame.* International Electrotechnical Commission (1996).
[15] IEC Standard, Publication 404-2, *Third Edition 1997, Magnetic materials, part 3: Methods of measurement of specific total losses of magnetic sheet and strip by means of a single sheet tester*, International Electrotechnical Commission (1997).
[16] Van den Bossche A., Valchev V., Georgiev G., *Measurement and loss model of ferrites in non-sinusoidal waves*. IEEE Power Electronics Specialists Conference, PESC04, IEEE, pp. 4814-4818 (2004).
[17] Van den Bossche A., Valchev V., van de Sype D., Van den Bossche L., *Ferrite losses of cores with square wave voltage and DC bias.* Journal of Applied Physics 99: 08M908-08M908-3 (2006).
[18] Ruszczyk A., Sokalski K., Szczygłowski J., *Scaling in modeling of core losses in soft magnetic materials exposed to nonsinusoidal flux waveforms and DC bias conditions.* http://arxiv.org/pdf/1309.0022.pdf (accessed 20 August 2013).