ARCHIVES OF ELECTRICAL ENGINEERING VOL. 65(1), pp. 5-18 (2016)

DOI 10.1515/aee-2016-0001

Optimum design of non-uniform symmetrical linear antenna arrays using a novel modified invasive weeds optimization

EL HADI KENANE [1,2], FARID DJAHLI[2]

[1] *Institute of Electrical Engineering and Electronics*
University of M'sila
M'sila, 28000, Algeria
e-mail: kenaneh@yahoo.fr

[2] *LIS Laboratory, Institute of Electronics*
University of Setif 1
19000 Setif, Algeria
e-mail: fdjahli@yahoo.fr

(Received: 26.12.2014, revised: 17.08.2015)

Abstract: This paper presents a new modified method for the synthesis of non-uniform linear antenna arrays. Based on the recently developed invasive weeds optimization technique (IWO), the modified invasive weeds optimization method (MIWO) uses the mutation process for the calculation of standard deviation (SD). Since the good choice of SD is particularly important in such algorithm, MIWO uses new values of this parameter to optimize the spacing between the array elements, which can improve the overall efficiency of the classical IWO method in terms of side lobe level (SLL) suppression and nulls control. Numerical examples are presented and compared to the existing array designs found in the literature, such as ant colony optimization (ACO), particle swarm optimization (PSO), and comprehensive learning PSO (CLPSO). Results show that MIWO method can be a good alternative in the design of non-uniform linear antenna array.

Key words: invasive weeds optimization, non uniform linear array, optimization, synthesis

1. Introduction

The recent advances in wireless communications require networks with enhanced characteristics such as capacity, quality and coverage. Whatever its complexity, a single element antenna is unable to fit with current constraints, but some of these limitations could be overcome by the design of antenna arrays. For the base station in mobile telecommunications, the most commonly used topology is the uniform linear array [1]. Moreover, in the last decade, many researches have been done in the field of non-uniform linear array. The aim is to fit with new requirements in radiation characteristics by designing the spacing between elements. The principal requirements to be satisfied are minimum side lobe level (SLL) [2-3], nulls towards interfered directions [4], patterns shape [5], and interference reduction capability [6].

Since the classical synthesis methods often fall in local optima, many stochastic methods have been employed for the design of non-uniform linear antenna array. Among the most recent and powerful algorithms which have been successfully used, one has noticed: neural networks [7], genetic algorithm (GA) [8-10], modified least mean square algorithm (MLMS) [11], ant colony optimization (ACO) [12-13], particle swarm optimization (PSO) [14-16], comprehensive learning PSO (CLPSO) [17], and cat swarm optimization [18].

In [19], Mehrabian and Lucas proposed a new algorithm based on the ecological behavior of colonizing weeds, known as invasive weeds optimization (IWO). Among all stochastic methods, the IWO has emerged as a powerful optimization method and has been successfully applied to solve many electromagnetic problems.

In the last few years, IWO has been used for the design of linear arrays [20-21], circular arrays [22], time modulated linear arrays [23], and adaptive beam-forming [24]. Mainly, the IWO method has been used for real continuous problems, but a binary version has also been applied [25]. Recently, a developed hybrid IWO with iterative Fourier technique (IWO-IFT) has been also proposed for the synthesis of a large planar thinned array [26]. Some other hybrid versions of IWO are also applied to the design of antenna arrays in [27-30]. In the same way, a discrete IWO (DIWO) has been developed for cooperative multiple task assignment of unmanned aerial vehicles [31].

In this paper, a new modified version of IWO, referred as MIWO, is presented for the design of non-uniform linear array with optimized performance with respect to SLL and nulls control. All the steps in our proposed MIWO are the same as the standard IWO except the reproduction process. The procedure of reproduction is based on a new dispersal technique as follows. For each iteration index, a random number is generated and compared to a fixed number called probability of mutation (Pm). If the generated number is greater than Pm, the algorithm will replace the value of the calculated standard deviation (SD) of the actual iteration by its initial value (SD of the first iteration). This mutation allows a larger dispersion of seeds around the parent plant in the IWO process, thus, avoiding local minima. To illustrate the performance of the proposed algorithm, a comparison is presented with the state-of-the-art optimization algorithms such ACO, PSO, CLPSO, and standard IWO. Accordingly, existing arrays designs found in the literature involving 10, 26, 28, and 32 elements arrays are calculated and compared. This comparison reflects the superiority of our proposed algorithm in terms of convergence speed and accuracy for most designs.

2. Antenna array model

Assuming that a $2N$-element linear array is symmetric with respect to its center, placed along x axis as shown in Figure 1. Mathematically, the Array Factor (AF) is given by

$$AF(\theta) = 2\sum_{n=1}^{N} I_n \cos\left(k(n-0.5)d_n(\cos\theta - \cos\theta_0)\right), \tag{1}$$

where $k = 2\pi/\lambda$, λ is the wavelength, θ is the measured angle from the axis of the array and θ_0 is the scanned angle. I_n represents the current excitation (amplitude and phase excitation) of the n^{th} element on each side of the array and d_n is the separation between the $(n + 1)^{th}$ and n^{th} element in the array.

Let us assume that the linear array under investigation is a broadside ($\theta_0 = 90°$) and all elements have the same current excitation ($I_n = 1$). The expression of AF will be simplified to

$$AF(\theta) = 2\sum_{n=1}^{N} \cos\left(k(n-0.5)d_n \cos\theta\right). \tag{2}$$

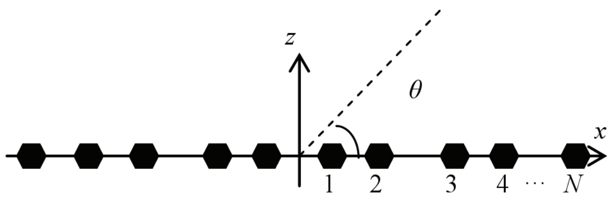

Fig. 1. The geometry of $2N$ element symmetric linear array of isotropic sources positioned along x axis

In our study, by varying the separation d_n between the elements, the characteristics of the array factor will be controlled during the design process. When the separation inter-element in an array is reduced, the mutual coupling affects, strongly, the different radiation characteristics. To overcome the effect of mutual coupling, the distance of inter-elements must be greater than 0.25λ.

The last equation of AF is used to find the optimal locations x_n of the array elements that minimize the SLL, while maintaining the width of the main beam equal or less than those of the equidistant array.

For that, the cost function of these requirements is given by [17]

$$CF1(X) = \max_{\upsilon \in S}\left\{AF_{dB}^{X}(\upsilon)\right\} + \Xi \cdot \max\left\{0, \left|BW_c - BW_d\right| - 1\right\}, \tag{3}$$

where x is the vector of the locations of the elements, S is the space spanned by the angle υ excluding the main beam. BW_c and BW_d denote the calculated and desired beamwidth, respectively. Ξ is a very large number.

The relation between the separation d_n and the location x_n is given by

$$x_n = \frac{d_1}{2} + \sum_{i=2}^{n} d_i. \tag{4}$$

In [16], a null control towards specified directions will be added. The cost function will be

$$CF2(X)=\sum_{m=1}^{M}\frac{1}{\Delta\upsilon_m}\int_{\upsilon_{lm}}^{\upsilon_{um}}\left|AF^{\upsilon}(X)\right|^2 d\upsilon+\sum_{k=1}^{K}\left|AF^{X}(\upsilon_k)\right|^2, \tag{5}$$

where K is the number of nulls on each side, υ_k is the direction of k^{th} null. $\Delta\upsilon_m$ is the m^{th} region of SLL suppression. M denotes the number of regions of SLL suppression and $\Delta\upsilon_m = \upsilon_{um} - \upsilon_{lm}$.

3. MIWO algorithm

IWO is a recent method, compared to GA and other traditional stochastic methods, based on the colonizing behavior of the invasive weeds in the nature. Plants invade cropping field, occupy the free space around this field and grow to get new weeds, and so on. Starting from random positions of a non-uniform linear array, the optimum positions of the elements will be found by suppressing side lobes in both regions outside the main beam, with symmetric nulls in some directions. In the proposed MIWO, a mutation process in the calculation of SD is added; this allows a larger dispersion of seeds around the parent plant. Hence, new weeds will then grow randomly. Their number is related to the fitness value of their parents. The algorithm of MIWO method is organized as follows.

3.1. Initialization

First, a finite number of plants are randomly spread over search space (N dimensions). This initial population is denoted as $POP= [P_1, P_2\ldots,P_{pop_{ini}}]$ where pop_{ini} is the number of generated plants. Each plant is considered as a proposed solution in the search space and termed as $P_i= [D_1, D_2,\ldots,D_N]$.

3.2. Evaluation

The fitness reflects an evaluation of how good the plant is. The optimal plant (vector solution) is the one which minimizes the cost function defined by the designer.

3.3. Reproduction

Depending on its fitness value, a plant can produce a number of seeds from 0 to a fixed number. The best plant in the colony will produce a maximum number of seeds, while the worst plant cannot produce any seed. Between these extremely plants fitness, the number of produced seeds is given by

$$Ns(P_i)=\text{integer}[Nbr], \tag{6}$$

where

$$Nbr = ms+\left(\frac{Ms-ms}{BC-WC}\right)(C(P_i)-WC), \tag{7}$$

where, Ms and ms are the maximum and minimum numbers of generated seeds respectively. BC and WC are the best and worst cost in the actual population respectively. $C(P_i)$ is the cost function of i^{th} plant in the population and integer $[Nbr]$ is a mathematical function that gives the greatest integer of Nbr.

Next, the produced seeds will be spread around its parent with a random distance using uniform distribution with the mean equal to 0 and a standard deviation SD that will decrease iteratively following the equation below

$$SD_{iter} = \frac{(iter_{\max} - iter)}{(iter_{\max})^{mod}}(SD_{ini} - SD_{fnl}) + SD_{fnl}, \tag{8}$$

where, $itr_{\max}$ is the maximum number of iterations and itr is the actual iteration index. SD_{ini} and SD_{fnl} present the initial and final values of standard deviation respectively. mod is the modulation index, usually equal to 3.

3.4. Mutation

The most works in modified IWO investigate on the choice of standard deviation value, as the most important parameter in IWO. A good parameter choice is essential for a fast convergence of the process. Many works have been done to achieve its optimal value. For instance, in [31], the spatial dispersal was modified to random selection of solutions from neighboring hypercube in the discrete solutions around the plant with normal distribution. In [32], each weed in the population has a standard deviation value on the basis of its cost function in the actual population; whereas a periodical variation can be added to the standard deviation.

In the present study, the idea is inspired from the mutation process of the GA [9]. First, a probability P_m is fixed. If P_m is lower than a randomly generated value in the range [0, 1], then the standard deviation value of the current iteration SD_{iter} will be replaced by its initial value SD_{ini}, otherwise, the standard deviation SD_{iter} will be calculated as indicated in Equation (8). The steps of the proposed algorithm are as follows:

Step 1: A random number is generated.
Step 2: A fixed probability of mutation P_m is chosen.
Step 3: If the random number is greater than P_m then $SD_{iter} = SD_{ini}$;
Else SD_{iter} is calculated from Equation (8).
Step 4: Repeat this algorithm for each iteration.

The initial value of the probability of mutation P_0 is fixed to 0.8 and P_m will decrease iteratively as follows

$$P_{m_{iter}} = 1 - P_0^{\left(1 - \frac{iter}{nbr_{iter}}\right)}, \tag{9}$$

where P_{miter} is the probability of mutation at iteration $iter$ and nbr_{iter} is the maximum number of iterations. At this point, by decreasing P_m, iteratively, the aim is to reduce the standard deviation when the cost function is near to its minimum value.

3.5. Limitation

The seeds will grow and become new weeds. These weeds will be added to the colony till a maximum number of plants $pop_{\max}$ is reached. In this case, a competitive exclusion will begin to keep only the best $pop_{\max}{}^{\text{th}}$ plants in the colony, and discard the rest.

3.6. Stop criteria

Generally, a maximum number of iterations is taken as a stop criterion. All these points are summarized in the flowchart presented in Figure 2.

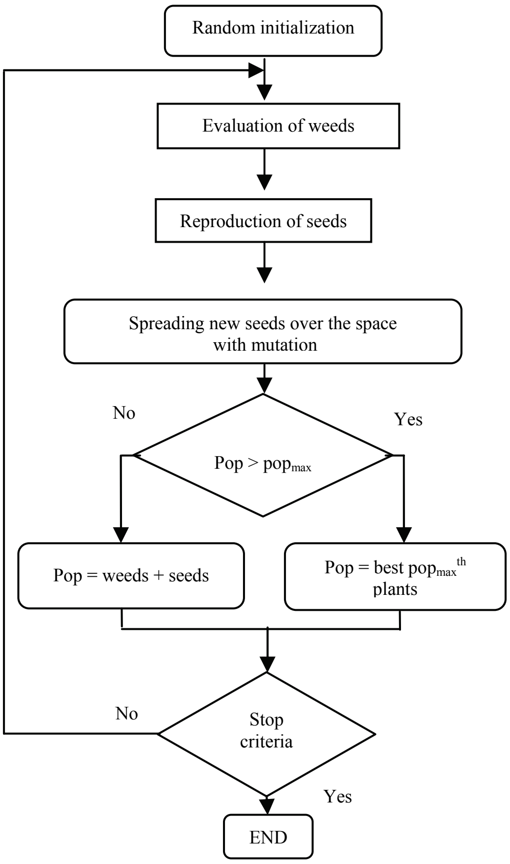

Fig. 2. Flowchart structure for the modified IWO algorithm

4. Design examples

To assess the performance of MIWO for the synthesis of non-uniform linear arrays, four design examples have been chosen to be evaluated. Each example contains a different number of elements (2*N*) and different constraints in the beam width. A performance comparison between the proposed MIWO and the state of the art optimization techniques such ACO, PSO, and CLPSO methods for the three first design examples and with standard IWO for the last one has been carried out. All designs assume a symmetric array with a uniform excitation (amplitude = 1 and phase = 0).

All algorithms have been run 20 times. The best results are compared. Results are presented in terms of the spacing between the elements in function of wavelength λ.

Since MIWO controls the spacing between elements, the corresponding positions have been calculated using Equation (4).

In all examples, the scanning angle is fixed to 90°, the size of the initial population, pop_{ini}, is fixed to $4N$ and the maximum number of population $pop_{\max}$ is fixed to twice of pop_{ini}. Depending on the case, standard deviations, SD_{ini} and SD_{fnl} are respectively fixed to 0.05 and 1e-7 or 0.01 and 1e-5. The iteration number is fixed to 1000 or 2000, and in the cost function Ξ = 10^6.

The first example concerns the design of a $2N$ = 10 elements array. The constraint is a minimizing SLL while keeping the beam width closed to that of equidistant linear array. MIWO results are compared to PSO [16], ACO [13], and CLPSO [17] results as well as equidistant linear array ones. Table 1 presents the positions of the 10 elements array whereas the corresponding normalized patterns are shown in Figure 3. Beam width is characterized by the half power beam width (HPBW) and the first null beam width (FNBW). Table 5 shows the results extracted from Figure 3, a SLL minimum of −19.07 dB is obtained by MIWO method with a FNBW and a HPBW closed to that of equidistant array.

In the second example, ($2N$ = 28 elements array design), the optimized radiation pattern should present three nulls directed to 120, 122.5, and 125° with a beam width less or equal to that of the uniform array. Again, MIWO results are compared with those using PSO, ACO, and CLPSO. Positions of the 28 elements array are reported in Table 2, corresponding radiation patterns are shown in Figure 4 and their principal characteristics are tabulated in Table 6. As it can be seen, the best design is obtain by MIWO with null depths around −80 to −90 dB.

In the third example, (design of a $2N$ = 32 elements with a desired null towards 99°), Table 3 shows the element positions while Figure 5 and Table 7 present radiation patterns and pattern properties, respectively. The obtained null depth was −61.5 dB for MIWO, −59 dB for CLPSO and only −17.81 dB for a uniform array, MIWO design is better than ACO and CLPSO ones. In this case, best results are obtained with PSO with a null depth of −63.45 dB. Our MIWO results present a lower PSLL and closer beam width (FNBW and HPBW) compared to those of equidistant array.

It can be seen from Figure 5 that the MIWO algorithm gives a good enhancement in suppression of the first peak by more than 7dB as compared to the uniform array.

In order to compare our modified method with standard IWO [21], a last design example of a $2N$ = 26 elements array is presented. Two nulls are placed towards 12° and 60°, where a minimum SLL is required. Optimized positions of the array elements are reported in Table 4, Figure 6 presents the radiation patterns and Table 8 shows the corresponding pattern properties. The obtained results, using MIWO, show better characteristics in terms of null depth, minimum SLL with beam width constraint.

Another criterion is the minimum cost function versus iterations, which indicates how fast the algorithm converges. Convergence curves for the best runs, for the algorithms CLPSO and MIWO are depicted in Figure 7. It can be seen that both curves converge to the same level. However, MIWO converge faster than CLPSO. Figure 8 shows the convergence for the best runs by both algorithms MIWO and standard IWO. MIWO shows a fast convergence compared to standard IWO. In addition, the level of the cost function achieved by MIWO is better than the one achieved by standard IWO. It is seen from Figures 7 and 8 that a fast degradation of the cost function is obtained in both cases leading to an optimal value around the 200th and 600th iteration respectively.

Caused by the mutation process, the proposed MIWO appears to be more stable where the speed of convergence and the level of cost function are slightly changed. For the same examples, evolutions of standard deviation values versus the number of iterations are presented in Figures 9 and 10. The mutation probability was decreased iteratively as illustrated in the evplution of standard deviation values, this means that most of the mutations are found in the first half range of iterations.

Table 1. Optimized positions of the 10 element linear array obtained using many approaches in term of λ.

Uniform	0.2500	0.7500	1.2500	1.7500	2.2500
ACO	0.2500	0.5500	1.0500	1.5500	2.1500
PSO	0.2515	0.5550	1.0650	1.5000	2.1100
CLPSO	0.2515	0.7110	1.2080	1.8350	2.5585
MIWO	**0.2286**	**0.7343**	**1.2475**	**1.8954**	**2.6431**

Table 2. Optimized positions of the 28 element linear array obtained using many approaches in term of λ/2

Uniform	0.500	1.500	2.500	3.500	4.500	5.500	6.500
	7.500	8.500	9.50	10.50	11.50	12.50	13.500
ACO	0,900	1.100	2.500	3.100	4.500	4.900	6.300
	7.500	8.300	9.50	10.50	11.50	12.50	13.900
PSO	0.530	1.560	2.440	3.500	4.540	5.480	6.530
	7.540	8.380	9.50	10.63	11.42	12.32	13.680
CLPSO	0.470	1.322	2.26	3.178	4.142	5.369	6.212
	7.135	8.313	9.794	11.19	12.79	14.36	15.960
MIWO	**0.500**	**1.499**	**2.491**	**3.393**	**4.393**	**5.393**	**6.313**
	7.032	**8.032**	**9.032**	**9.875**	**10.56**	**11.47**	**12.471**

Table 3. Optimized positions of the 32 element linear array obtained using many approaches in term of λ/2

Uniform	0.500	1.500	2.500	3.500	4.500	5.500	6.500	7.500
	8.500	9.500	10.500	11.500	12.500	13.500	14.500	15.500
ACO	0.300	1.500	2.100	3.500	4.500	5.100	5.900	7.500
	8.300	9.100	9.500	10.700	12.100	14.100	15.500	16.900
PSO	0.530	1.370	2.350	3.110	3.970	4.660	5.330	6.110
	6.860	7.800	8.760	9.900	11.100	12.480	14.100	15.510
CLPSO	0.450	1.475	2.202	3.024	3.800	4.743	5.873	6.914
	7.833	8.835	9.982	11.322	12.922	14.522	16.122	17.722
MIWO	**0.724**	**0.979**	**2.316**	**2.843**	**3.927**	**4.715**	**5.712**	**6.655**
	7.596	**8.845**	**9.932**	**10.960**	**12.634**	**14.334**	**16.034**	**17.734**

Table 4. Optimized positions of the 26 element linear array obtained using many approaches in term of λ/2

Uniform	0.500 1.500 2.500 3.500 4.500 5.500 6.500 7.500 8.500 9.500 10.500 11.500 12.500
IWO	0.4483 1.488 2.275 3.279 4.254 5.326 6.161 7.168 8.046 9.091 9.928 11.240 12.680
MIWO	**0.500 1.330 2.232 3.030 3.974 4.909 5.909 6.834 7.834 8.834 9.834 10.742 11.742**

Table 5. Pattern properties of the 10 element linear array (extracted from Fig. 3)

Pattern	Uniform	PSO	ACO	CLPSO	MIWO
SLL (dB)	−12.96	−17.40	−18.27	−19.04	**−19.07**
FNBW(°)	23.08	27.25	27.04	22.93	**22.14**
HPBW(°)	10.19	11.48	11.30	9.53	**9.22**

Table 6. Pattern properties of the 28 element linear array (extracted from Fig. 4)

Pattern		Uniform	PSO	ACO	CLPSO	MIWO
FNBW(in °)		8.16	8.18	8.40	8.32	**8.60**
HPBW(in °)		3.62	3.62	3.64	3.40	**3.84**
SLL in Nulls window (in dB)		−26.39	−40.37	−46.62	−37.86	**−44.62**
PSLL (in dB)		−13.20	−13.23	−15.40	−21.60	**−12.01**
Null depth (in dB)	**120°**	−70.00	−45.22	−55.00	−70.00	**−80.00**
	122.5°	−26.44	−43.45	−55.00	−78.04	**−92.30**
	125°	−47.40	−47.40	−45.00	−60.00	**−80.00**

Table 7. Pattern properties of the 32 element linear array (extracted from Fig. 5)

Pattern	Uniform	PSO	ACO	CLPSO	MIWO
FNBW(in °)	7.16	8.46	7.52	7.68	**7.88**
HPBW(in °)	3.16	3.52	3.16	3.10	**3.12**
Null depth at 99° (in dB)	−17.81	−63.45	−50.00	−59.00	**−61.50**
PSLL (in dB)	−13.23	−18.72	−17.09	−22.73	**−23.86**
Null direction (in °)	100.82	99.02	99.09	99.02	**98.99**

Table 8. Pattern properties of the 26 element linear array (extracted from Fig. 6)

Pattern		Uniform	Classical IWO	MIWO
FNBW		8.83	9.25	**9.69**
HPBW		3.90	4.03	**4.22**
Null depth	**12°**	−30.50	−61.5	**−83.00**
	60°	−25.30	−56.60	**−65.00**
Null direction	**12°**	–	12.49	**12.00**
	60°	–	59.95	**60.00**
PSLL		−13.21	−14.40	**−14.56**

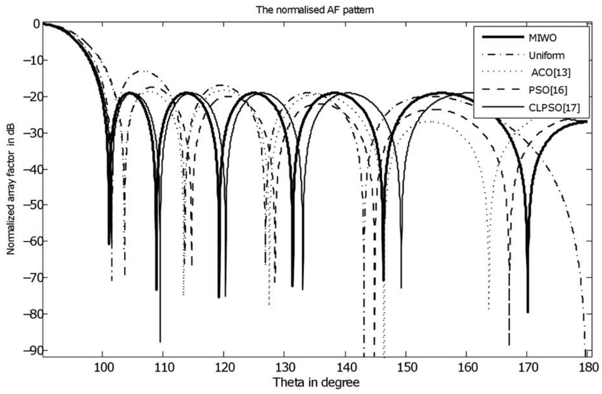

Fig. 3. Normalized pattern of 10 element linear array obtained by MIWO (thick solid line), standard PSO (hair solid line), CLPSO (dotted line), Ant colony (dashed line), and uniform array (dash dot line)

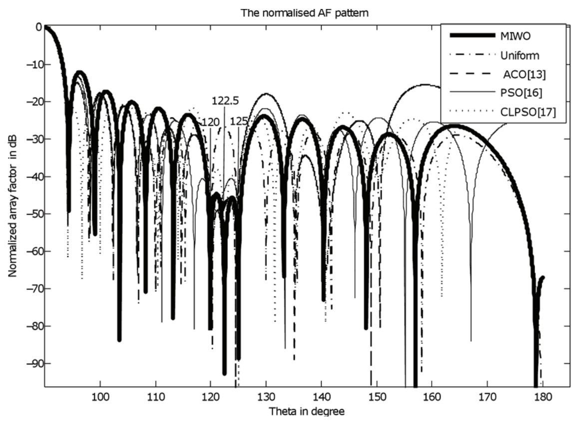

Fig. 4. Normalized pattern of 28 element linear array obtained by MIWO (thick solid line), standard PSO (hair solid line), CLPSO (dotted line), Ant colony (dashed line), and uniform array (dash dot line)

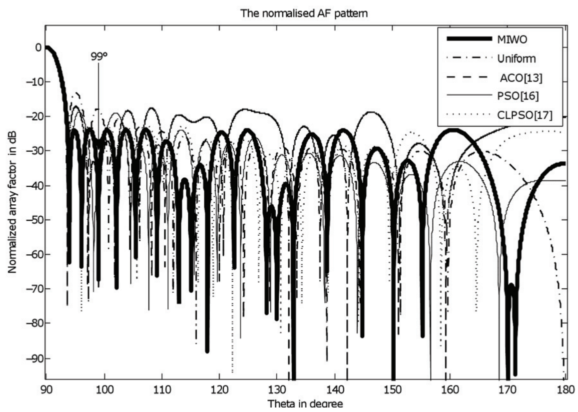

Fig. 5. Normalized pattern of 32 element linear array obtained by MIWO (thick solid line), standard PSO (hair solid line), CLPSO (dotted line), Ant colony (dashed line), and uniform array (dash dot line)

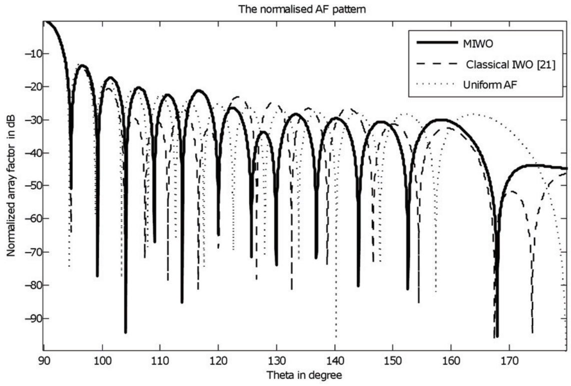

Fig. 6. Normalized pattern of $2N = 26$ element linear array obtained by MIWO (solid line), classical IWO (dashed line), and uniform array (dotted line)

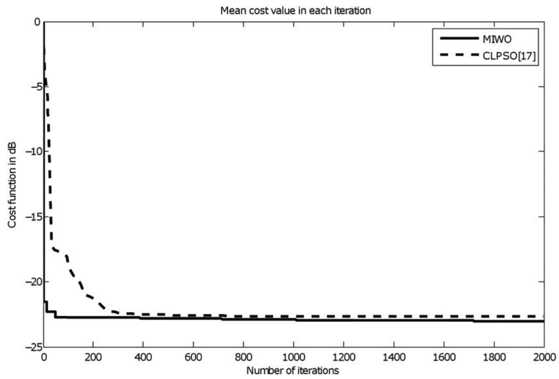

Fig. 7. Evolution of the minimum cost function versus iterations, for the 32 element linear array, Compared with cost with CLPSO in [17]

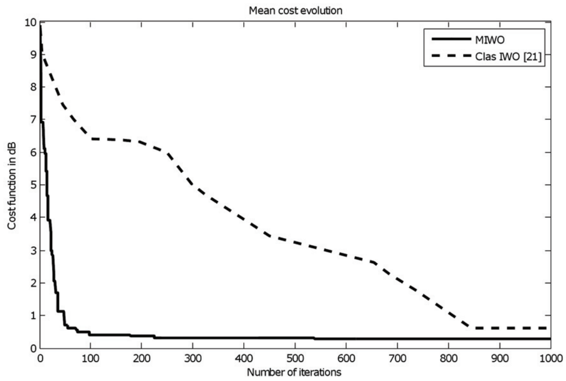

Fig. 8. Evolution of the minimum cost function versus iterations, for the 26 element linear array, compared to cost function with standard IWO [21]

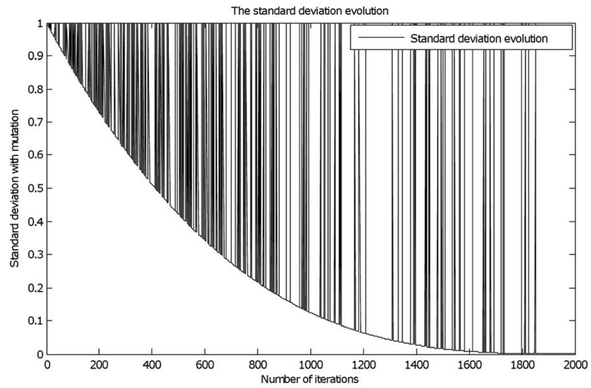

Fig. 9. Evolution of the standard deviation values versus number of iterations for the 32 element linear array (207 mutations)

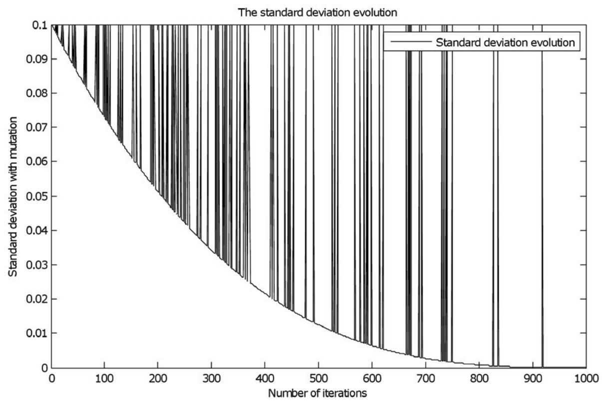

Fig. 10. Evolution of the standard deviation values versus number of iterations for the 26 element linear array (117 mutations)

5. Conclusions

In this paper, a new algorithm denoted as MIWO was used for the synthesis of non-uniform linear antenna arrays. MIWO has been compared to other stochastic methods used to suppress side lobe level and to control nulls positions in the radiation pattern of this kind of antenna arrays. Examples of 10, 26, 28, and 32 element linear arrays have been investigated with different types of constraints and in all cases, MIWO design was better in terms of pattern properties as well as convergence speed and robustness. Research is going on to further suppress side lobes and control nulls towards interferers which improve the performance of array antenna systems. Many additional extensions in array synthesis could be also improved using MIWO, such as mutual coupling, array topology, and other antenna types.

References

[1] Godara L.C., *Applications of Antenna Arrays to Mobile Communications, Part I: Performance Improvement, Feasibility, and System Considerations.* Proceeding of the IEEE 85(7): 1031-1060 (1997).
[2] Rupcic S., Mandric V., Zagar D., *Reduction of Side lobes by Non uniform Elements Spacing of a Spherical Antenna Array.* Radio Engineering 20(1): 299-306 (2011).
[3] Abdul Rani K.N., Abdul Malek M.F., Siew-Chin N., *Nature-inspired Cuckoo Search Algorithm for Side Lobe Suppression in a Symmetric Linear Antenna Array.* Radio Engineering 21(3): 865-874 (2012).
[4] Gomez N.G., Rodriguez J.J., Melde K.L., McNeill K.M., *Design of Low-Side lobe Linear Arrays With High Aperture Efficiency and Interference Nulls.* IEEE Antennas and Wireless Propagation Letters 8(1): 607-610 (2009).
[5] Merad L., Bendimerad F.T., Meriah S.D., *Controlled Random Search Optimization for Linear Antenna Arrays*. Radio Engineering 15(3): 10-14 (2006).
[6] Panduro M.A., *Design of non-uniform linear phased arrays using Genetic Algorithm to Provide Maximum Interference Reduction Capability in a Wireless Communication System.* Journal of the Chinese Institute of Engineers 29(7): 1195-1201 (2006).
[7] Elkamchouchi H.M., Saleh G.A., *Solution of the non-uniformly spaced array problem using Neural Networks.* 17th National Radio Science Conference, Minufia University, Egypt, B21, NRSC'2000: 1-6 (2000).
[8] Yan K.K., Lu Y., *Side lobe Reduction in Array pattern Synthesis using Genetic Algorithm.* IEEE Transactions on Antennas and Propagation 45(7): 1117-1122 (1997).
[9] Haupt R.L., *Phase-Only Adaptive Nulling with a Genetic Algorithm.* IEEE Transaction on Antennas and Propagation 45(6): 1009-1015 (1997).
[10] Hussein A.H., Abdullah H.H., Salem A.M. et al., *Optimum Design of Linear Antenna Arrays Using a Hybrid MoM/GA Algorithm.* IEEE Antennas And Wireless Propagation Letters 10(1): 1232-1235 (2011).
[11] Zhang J.Li, M., Shi H., Zheng S., Zhang A., *Design and Implementation of an Adaptive Space-Time Antenna Array for GPS Receivers.* Archives of Electrical Engineering 69(3-4): 83-87 (2015).
[12] Dorigo M., Maniezzo V., Colorni A., *Ant System: Optimization by a Colony of Cooperating Agents.* IEEE Transactions On Systems, Man, And Cybernetics-Part B Cybernetics 26(1): 29-41 (1996).
[13] Iglesias Eva R., Teruel O.Q., *Linear array synthesis using an Ant colony optimization based algorithm.* IEEE Antennas and propagation Magazine 49(2): 70-79 (2007).
[14] Mandal D., Bhattacharjee A.K., Ghoshal S.P., *Linear Antenna Array Synthesis Using Improved Particle Swarm Optimization.* Second International Conference on Emerging Applications of Information Technology IEEE, pp. 365-368 (2011).
[15] Korany R.M., Eladawy M.I., Bansal R. et al., *Analysis of Uniform Circular Arrays for Adaptive Beamforming Applications Using Particle Swarm Optimization Algorithm.* International Journal of RF and Microwave Computer-Aided Engineering 18(1): 42-52 (2007).
[16] Khodier M., Christodoulou C.G., *Linear Array geometry synthesis with minimum side lobe level and null control using Particle Swarm Optimization.* IEEE transaction on antennas and propagation 53(8): 2674-2679 (2005).
[17] Goudos S.K., Moysiadou V., Samaras T. Et al., *Application of a Comprehensive Learning particle Swarm Optimizer to unequally spaced linear array synthesis with side lobe level suppression and null control.* IEEE antennas and wireless propagation letters 9(1): 125-129 (2010).
[18] Pappula L., Ghosh D., *Linear Antenna Array Synthesis Using Cat Swarm optimization.* International journal of electronics and communications, AEU Journal 68(6): 540-549 (2014).
[19] Mehrabian A.R., Lucas C., *A Novel Numerical Optimization Algorithm Inspired from Weed Colonization.* Ecology Information 1(4): 355-366 (2006).
[20] Karimkashi S., Kishk A., *Invasive Weed Optimization and its Features in Electromagnetics.* IEEE Transactions on Antenna and Propagation 58(4): 1269-1278 (2010).
[21] Pal S. et al., *Linear Antenna Array Synthesis with Invasive Weed Optimization Algorithm.* International Conference of Soft Computing and Pattern Recognition, pp.161-166 (2009).

[22] Basak A., Pal S., Das S., Abraham A., *Circular Antenna Array Synthesis with a Differential Invasive Weed Optimization Algorithm.* Tenth International Conference on Hybrid Intelligent Systems, IEEE Conference publication, USA, pp. 153-158 (2010).

[23] Basak A., Pal S., Das S. at al., *A Modified Invasive Weed Optimization Algorithm for Time-Modulated Linear Antenna Array synthesis.* IEEE Congress on Evolutionary Computation, pp. 1-8 (2010).

[24] Roshanei M., Lucas C., Mehrabian A.R., *Adaptive beamforming using a novel numerical optimization algorithm.* IET Microwaves, Antennas & Propagation 3(5): 765-773 (2009).

[25] Veenhuis C., *Binary Invasive Weed Optimization.* Second World Congress on Nature and Biologically Inspired Computing, IEEE, NaBIC'2010, pp. 449-454 (2010).

[26] Wang X., Jiao Y., Liu Y., Tan Y., *Synthesis Of Large Planar Thinned Arrays Using Iwo-Ift Algorithm.* Progress in Electromagnetics Research 136(1): 29-42 (2013).

[27] Zhang X., Niu Y., Cui G., Wang Y., *A modified invasive weed optimization with crossover operation*. 8th World Congress on Intelligent Control and Automation (WCICA), China, pp. 11-14 (2010).

[28] Bai Y.Y., Xiao S., Liu C., Wang B.Z., *A Hybrid IWO/PSO Algorithm for Pattern Synthesis of Conformal Phased Arrays.* IEEE Transactions on antennas and Propagation 61(4): 2328-2332 (2013).

[29] Dastranj A., Abiri H., Mallahzadeh A., *Design of a Broadband Cosecant Squared Pattern Reflector Antenna Using IWO Algorithm.* IEEE Transactions on Antennas and Propagation 61(7): 3895-3900 (2013).

[30] Zaharis Z.D., Skeberis C., Xenos T.D. et al., *Design of a Novel Antenna Array Beam former Using Neural Networks Trained by Modified Adaptive Dispersion Invasive Weed Optimization Based Data.* IEEE Transactions on Broadcasting 59(3): 455-460 (2013).

[31] Ghalenoei M.R., Hajimirsadeghi H., Lucas C., *Discrete Invasive Weed Optimization Algorithm Application to Cooperative Multiple Task Assignment of UAVs.* 48th IEEE conference on decision and control, China, pp. 1665-1670 (2009).

[32] Roy G.G., Das S., Chakraborty P., Suganthan P.N., *Design of Non-uniform Circular Antenna Arrays Using a Modified Invasive Weed Optimization Algorithm.* IEEE Transactions on Antennas and Propagation 59(1): 110-118 (2011).