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A Novel Multi-Exponential Function-based Companding Technique for Uniform Signal Compression over Channels with Limited Dynamic Range

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***Abstract*—Companding, as a variant of audio level compression, can help reduce the dynamic range of an audio signal. In analog (digital) systems, this can increase the signal-to-noise ratio (signal to quantization noise ratio) achieved during transmission. The μ-law algorithm that is primarily used in the digital telecommunication systems of North America and Japan, adapts a companding scheme that can expand small signals and compress large signals especially at the presence of high peak signals. In this paper, we present a novel multi-exponential companding function that can achieve more uniform compression on both large and small signals so that the relative signal strength over the time is preserved. That is, although larger signals may get considerably compressed, unlike μ-law algorithm, it is guaranteed that these signals after companding will definitely not be smaller than expanded signals that were originally small. Performance of the proposed algorithm is compared with μ-law using real audio signal, and results show that the proposed companding algorithm can achieve much smaller quantization errors with a modest increase in computation time.**

***Keywords*—Companding, multi-exponential function, Mu-law, quantization, uniform signal compression.**

I. Introduction

COMPANDING is a common technique for reducing the data rate of audio signals by making the quantization levels *unequal*. If the quantization levels are equally spaced, 12 bits must be used to obtain telephone quality speech. However, only 8 bits are required if the quantization levels are made *unequal*, matching the characteristics of human hearing [12]. This can be carried out by an operation called companding [9], [3]. In this way, the signal levels need to be very close together for smaller signals, whereas a larger spacing is often needed for larger signals. However, since the companded signal is not uniformly distributed, this leads to increased quantization errors [7].

One direct solution to reduce quantization error is to increase the number of quantization intervals. This would require the number of code words to be increased in proportion to the number of quantization intervals, resulting in increased system capacity and thus a higher cost [11].

Currently, two nearly identical standards are used for companding signals: μ-law in North America and Japan and A-law in Europe [11]. Although μ-law, similar to A-law, improves voice quality at lower signal levels compared to uniform quantization as in uniform PCM, this technique focuses on enlarging small signals and does not change high signal peaks, which leads to a higher average power level of output signals [1], [2], [10]. As shown in Fig. 1, the first peak (between sample intervals 1 and 2 in Fig. 1) is not compressed when μ-law technique is applied. To combat this problem, several nonlinear companding transform techniques have been proposed in the literature [1], [4], [5], [6], [10]. However, comparing the original signals, the compressed signals have a larger average power level and still exhibit nonuniform distributions [2], [5], as exemplified in a case also shown in Fig. 1. Although this technique is successful in compressing a large peak (sample interval between 1 and 2 in Fig. 1) and also expanding the smaller peaks, the average amplitude level is enlarged at the same time. This problem was addressed in [2] by employing an exponential companding technique. This technique is shown to be capable of adjusting the amplitudes of both large and small input signals while leaving the average power unchanged, but it can be effective only if the original signals are Gaussian-distributed in nature, which may not be the case in real applications.

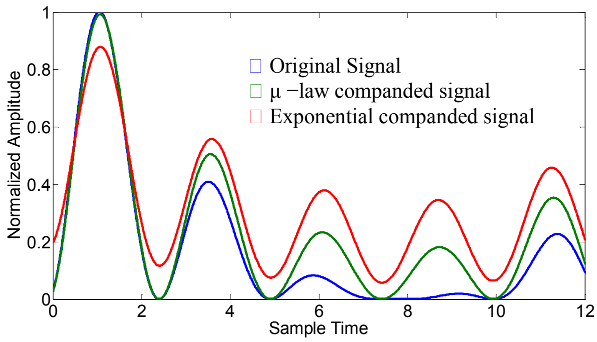

Fig. 1. Signal amplitude vs time for (i) original signal, (ii) μ-law companded signal, and (ii) exponential companded signal.

In this paper, we present a non-signal structure based technique, herein referred to as "multi-exponential companding", which can achieve more uniform compressions on both large and small signals so that the relative signal strength over the time is still preserved. That is, although

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larger signals may get considerably compressed, unlike μ-law algorithm, it is guaranteed that these signals after companding will definitely not be smaller than originally small but expanded signals. The more uniform the source signal is, the less quantization error and thus a higher signal to noise ratio is achieved.

This paper is organized as follows. In sections 2, μ-law algorithm is briefly discussed. Section 3 describes the proposed companding algorithm, followed by quantization error analysis in Section 4. Section 5 presents the simulation results, and finally, conclusion is drawn in Section 6.

II. μ-LAW

μ-law is a logarithmic conversion algorithm used in North America and Japan that is defined by CCITT G.711. It compresses 16−bit linear PCM data down to eight bits of logarithmic data. The compressing and expanding functions of this algorithm are defined by the following equations, respectively.

$$F(x) = \operatorname{sgn}(x)\frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \qquad -1 \le x \le 1 \tag{1}$$

where μ is the compression parameter (μ in the U.S. and Japan) and x is the normalized integer to be compressed.

$$F^{-1}(y) = \operatorname{sgn}(y)(1/\mu)\left[(1+\mu)^{|y|} - 1\right] \qquad -1 \le y \le 1 \tag{2}$$

III. PROPOSED COMPANDING ALGORITHM

To compand the signal, more than one function is utilized: an exponential function for compressing stage and a logarithmic function for expanding. In each exponential function, to make the output signal more uniformly distributed, a set of companding parameters are properly adjusted. This algorithm is described below.

A. Compression Stage

To compress the original signal, the whole signal range is divided into several subintervals and each set of these subs is mapped to a new interval. In this work, we consider 4 subintervals and assign a different exponential function as a compressor function for each subinterval. To do so, the following steps are necessary:

1) *Normalizing signal to [0, 1]*: for simplicity, the amplitude of original signal is normalized to [0, 1].

2) *Choosing the range of each subinterval*: in this work, the whole range of original signal is divided into 4 subintervals, $[0, r_1)$, $[r_1, r_2)$, $[r_2, r_3)$, and $[r_3, 1]$.

3) *Counting the number of signal data that fall into each subinterval.* In this step, it is necessary to determine the number of data points in each subinterval. Let n_1, n_2, n_3 and n_4 be the number of data points falling into subintervals $[0, r_1)$, $[r_1, r_2)$, $[r_2, r_3)$ and $[r_3, 1]$, respectively. The percentage of data, p_i, that fall in the subinterval i in compressed space can be found by $p_i = \frac{n_i}{N}$, where N is the total number of signal data points. That is,

$$[0, r_1) \to p_1$$
$$[r_1, r_2) \to p_2$$
$$[r_2, r_3) \to p_3$$
$$[r_3, 1] \to p_4$$

4) *Calculation of compressing function*,
The following functions are employed for compressing function,

$$F(x) = \begin{cases} 1 - b_1 \exp(a_1|x|) & 0 \le |x| < r_1 \\ b_2 \exp(a_2|x|) & r_1 \le |x| < r_2 \\ b_3 \exp(a_3|x|) & r_2 \le |x| < r_3 \\ b_4 \exp(a_4|x|) & r_3 \le |x| \le 1 \end{cases} \tag{3}$$

As shown in Fig. 2, in order to map signal data from the original space to the compressed space, by applying the above function, we shall have,

$$a_1 = \frac{1}{r_1}\ln(1-p_1), \qquad b_1 \text{ is set to } 1,$$
$$a_2 = \frac{1}{r_2 - r_1}\ln\left(\frac{r_2}{r_1}\right), \qquad b_2 = r_2\exp(-a_2 r_2)$$
$$a_3 = \frac{1}{r_3 - r_2}\ln\left(\frac{r_3}{r_2}\right), \qquad b_3 = r_3\exp(-a_3 r_3)$$
$$a_4 = \frac{1}{1 - r_3}\ln\left(\frac{1}{r_3}\right), \qquad b_4 = \exp(-a_3)$$

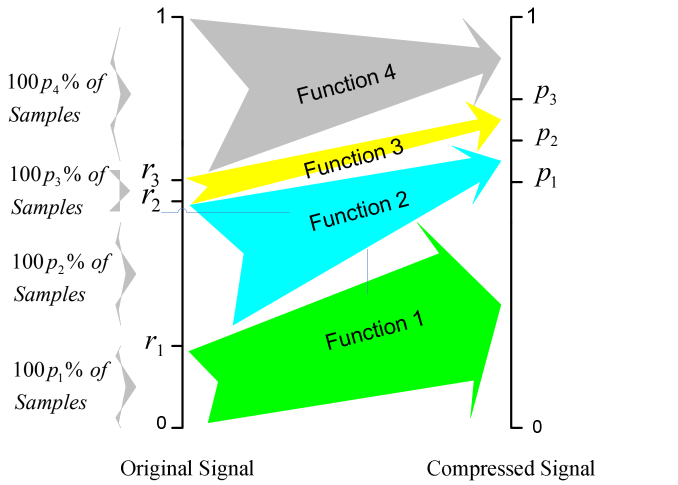

Fig. 2. A schematic of the technique.

B. Expansion Stage

The de-companding function is the inverse function of that can be found as,

$$F^{-1}(y)=\begin{cases}\frac{1}{a_1}\ln(1-|y|) & 0\le |y|<p_1\\ \frac{1}{a_2}\ln(\frac{|y|}{b_2}) & p_1\le |y|<p_2\\ \frac{1}{a_3}\ln(\frac{|y|}{b_3}) & p_2\le |y|<p_3\\ \frac{1}{a_4}\ln(\frac{|y|}{b_4}) & p_3\le |y|\le 1\end{cases} \tag{4}$$

IV. Quantization Error Analysis

As described in previous section, the original signal is mapped to a compressed space in such a way that the number of data in each subinterval is proportional to the length of that subinterval. By doing so, the compressed signal exhibits a uniform distribution and it can be quantized using a uniform quantizer. For uniform quantization of a uniform distributed signal, since the quantization error is also uniform over each quantization interval, we have,

$$\sigma_q^2=\frac{1}{\Delta}\int_{-\Delta/2}^{+\Delta/2} q^2dq=\frac{\Delta^2}{12}=\frac{1}{12}(\frac{2x_{\max}}{M})^2=\frac{(x_{\max})^2}{3M^2} \tag{5}$$

where σ_q^2 is the mean square quantization error, Δ is the length of quantization interval, $x_{\max}$ is the maximum signal amplitude level, and M is the number of quantization levels.

In general, it can be shown that the mean square quantization error is obtained by [7], [8],

$$\sigma_q^2=\frac{\alpha^2\sigma_x^2}{3M^2} \tag{6}$$

$$\alpha=\frac{x_{\max}}{|x|C'(x)}$$

where σ_x^2 is the variance of the signal, and $C'(x)$ is the derivative of the companding function with respect to x.

For the μ-law companding, α is close to the companding parameter μ which is typical equal to 255. Assuming that the number of quantization level, M is 256, Eq. (6) becomes,

$$\sigma_q^2=\frac{\sigma_x^2}{3} \tag{7}$$

Since $\sigma_x^2\ge\left(\frac{x_{\max}}{M}\right)^2$, from Eqs. (5), (7), one can arrive at

$$\left(\sigma_q^2\right)_U\le\left(\sigma_q^2\right)_\mu \tag{8}$$

V. Simulation Results

To verify the performance of the proposed technique in terms of quantization error, which is given in Eq. (5) and computation efficiency given in CPU time, we used both synthetic and real data as source signals. In the case where synthetic data are used for verification, we generated 4000 random numbers with a Gaussian distribution; for real data verification, we constructed an audio signal using MATLAB audio recording command (Fig. 3). The amplitude of the signals are normalized into [0, 1], and the whole dynamic range is divided into 128 (7 bits) and 256 (8 bits) quantization levels, respectively.

From Tables 1 and 2, it can be seen that in terms of quantization error, in good agreement with the theoretical analyses shown in Section 4, the proposed companding algorithm shows less quantization errors than that obtained from μ-law at a modest increase of computation cost. It is also

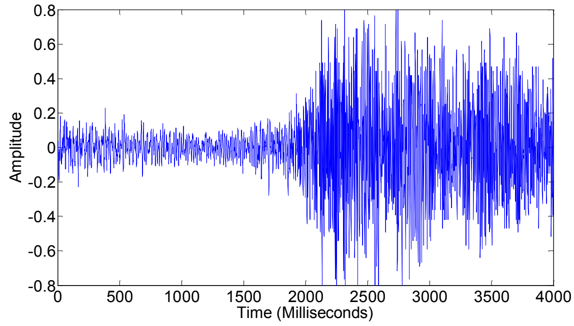

Fig. 3. A real audio signal for data processing.

TABLE I
Results for Synthetic Data

Criteria		No Companding	μ-law	Multi-Exponential	Theoretical Results
Quantization	8 Bits	0.00018	0.00010	0.00006	0.00005
Error	7 Bits	0.00034	0.00019	0.00010	0.00008
Relative CPU Time		1	1.06	1.92	---

TABLE II
Results for Real Data

Criteria		No Companding	μ-law	Multi-Exponential	Theoretical Results
Quantization	8 Bits	0.00098	0.00075	0.00042	0.00035
Error	7 Bits	0.00190	0.00140	0.00110	0.00088
Relative CPU Time		1	1.07	2	---

seen that the proposed algorithm would require smaller number of code words for the same quantization error level than when μ-law is applied.

VI. Conclusions

In this work, quantization error in companding techniques due to non-uniformly distributed companded signal was investigated. To address this problem, a new algorithm that uses multi-exponential companding was proposed, and it was verified using real data. The results showed that the proposed technique has a few distinct advantages: it has lower quantization error than popular μ-law, or it needs smaller number of code words for the same quantization error than that of the case when μ-law is applied. These advantages are achieved at a cost of a slight increase in computation requirement, which is less of a concern, given modern fast computing platforms.

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