

Route Availability Model for Inter-working Multi-hop Wireless Networks

Oladayo Salami, Antoine Bagula, and H. Anthony Chan

Abstract—In order to link a source-destination node pair in inter-working multi-hop wireless networks, links or routes must first be available. It is only after establishing the availability of links and routes between nodes that factors which affect connectivity e.g. interference can be considered. Connectivity in multi-hop wireless networks has been studied. However, the studies focused on network connectivity in ad-hoc networks. Since the next generation of wireless networks will be inter-working, an understanding of connectivity as it applies to such networks is needed. Specifically, this paper emphasizes that an analysis of route connectivity rather than network connectivity is needed for inter-working multi-hop wireless networks. With a focus on route connectivity, a route availability model for inter-working multi-hop wireless networks is presented.

Keywords—Inter-working, multi-hop, route availability, wireless network.

I. INTRODUCTION

CONNECTIVITY is a fundamental property of any network. Normally, in all networks, links are the basic elements that ensure connectivity. In wired networks, links are provided by communication cables and these links are stable and predictable to a large extent. On the other hand, in wireless networks, links are provided by the air interface (wireless channel).

Generally, in wireless networks, nodes have to be within an appreciable distance of each other before a communication link can be established between them. Any node that is not within the recommended range is said to be out of the network. In single hop wireless networks, it is sufficient for each node to be within the transmission range of at least one of the centralized base stations in order to communicate with another node. For multi-hop wireless networks, if source-destination pairs are not within each other's transmission range, packets reach their destination nodes after some hops on nodes in between the source and destination. One of the advantages of multi-hop communications is that it ensures efficient spatial reuse. In multi-hop wireless networks, the choice of the next hop depends on whether the node on this hop is able to link up to another node in the communication path en-route to the destination node. Most importantly, an available link must also

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be reliable for a good quality communication to be established between node pairs. One major characteristic of the wireless channel that affects the quality of communication is the variation in its strength over time and frequency. As a result of the variation, communication links in wireless networks tend to be unpredictable. Moreover, this variation affects the connectivity between two communicating nodes.

Another factor that affects connectivity between two communicating nodes is mobility. Since mobility may cause connected radio links to be disconnected, a critical issue is for nodes in the network to be able to communicate on links that can be sustained throughout the packet transmission duration. Therefore, the links between nodes have to be able to ensure lasting connectivity.

The developments of the theory of connectivity for wireless networks have been done in research works such as [2], [3], [8], [9], [10], [13], [21]. However, most of the theoretical and analytical investigations are focused on ad-hoc sensor networks. This paper studies the theory of connectivity in inter-working multi-hop wireless networks. Since the existence of a link or route between node pairs is essential for connectivity, the contribution of this paper is to present a model for the analysis of route availability between source-destination node pairs in inter-working multi-hop wireless networks.

Availability is the measure of the amount of nodes that are reachable by a node. It is also the probability that a link or route exist between any two nodes. Ultimately, it is determined by the probability that at least a node exists within a certain distance range from a particular node. In a wireless network, the availability of a link or route between node pairs depends on the distance between them, their transmission range and the network's node density. Link and route availability are probabilistic factors since the wireless network is stochastic in nature. The probability that a route is available between source-destination pair depends on the probability that intermediate nodes en-route to the destination have a link to another node closer to the destination node. Link availability is the probability that two nodes are within at most the maximum transmission range that is sufficient for a communication link to be established between them. Route availability is the probability that the adequate number of links that will form the communication path between a source-destination pair exists. In addition, this paper presents an analysis of the inter-dependency that exists between link and route availability in inter-working multi-hop wireless networks. Such analysis is needed to determine the availability of a route for packet transmission. To avoid ambiguity, a link refers to the connection between any node pair in the network, while a

route refers to the last mile connection path between a source and destination pair.

For this analysis, the fundamental models that are needed to represent the inter-working multi-hop wireless networks are:

1) *A model for the spatial distribution of nodes in the inter-working network.*

The network has been represented as Poisson Point process in two dimensions. The Poisson Point process is the most popular choice for modeling network nodes' spatial distribution [1] [5] [18] [19] [20]. Nodes are independently located and the average density of the nodes is uniform throughout the network.

2) *A model for the link distance between nodes.*

The model gives the probability that a node has a link to other nodes in the network. If the maximum transmission range of any node is R , then an independent communication link is available for any two nodes separated by a distance less than or equal to R . If β is the distance between two nodes, a link is available between them as long as $\beta \leq R$. Note that β refers to the distance between specific node pairs. The distance may be a single hop distance between and it may be the multi-hop distance between any source-destination pair.

The outline of this paper is as follows. In section 2, the node distribution, the inter-working network, and the node degree models are described. An analysis of the link models is given in Section 3. Section 4 presents the route availability model and section 5 concludes the paper. Numerical results of the link availability and route availability models are presented.

II. NETWORK MODEL

A. Node Distribution

Since nodes' locations are completely unknown a priori in wireless networks, they can be treated as completely random. The irregular location of nodes (in fig. 1), which is influenced by factors like mobility or unplanned placement of the nodes may be considered as a realization of a spatial point pattern (or process) [1].

A spatial point pattern in fig. 2 is a set of location, irregularly distributed within a designated region and presumed to have been generated by some form of stochastic mechanism. In most applications, the designation is essentially on planar \mathbb{R}^d (e.g. $d=2$ for two-dimensional) Euclidean space [7]. The lack of independence between the points is called complete spatial randomness (CSR) [6]. According to the theory of complete spatial randomness for a spatial point pattern, the number of points inside a planar region \mathbf{P} follows a Poisson distribution [7]. It follows that the probability of p points being inside region \mathbf{P} ($\Pr(p \text{ in } \mathbf{P})$) depends on the area of the region (A_p) and not on the shape or location of the plane. $\Pr(p \text{ in } \mathbf{P})$ is given by (1), where μ is the mean number of points (spatial density of points).

$$\Pr(p \text{ in } \mathbf{P}) = \frac{(\mu A_p)^p}{p!} e^{-\mu A_p}, p > 0. \quad (1)$$

This is a reasonable model for networks with random node placement such as the inter-working multi-hop wireless networks.

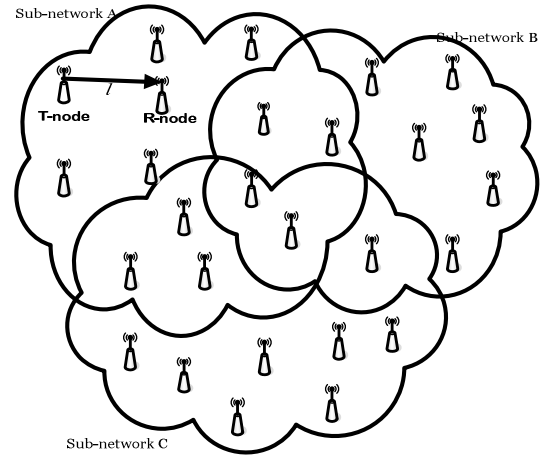


Fig. 1. Inter-working network model for multi-hop wireless networks with overlapping service area.

B. Inter-working Multi-hop Wireless Network

Fig. 1 represents network Ω , which is a set of inter-working multi-hop wireless networks (sub-networks A, B, and C). Each network is considered as a collection of random and independently positioned nodes. The nodes in the network are contained in a Euclidean space of 2-dimensions (\mathbb{R}^2). These sets of multi-hop wireless networks have some inter-domain co-ordination between them. The total number of nodes in Ω is denoted by N_Ω , while the number of nodes in sub-networks A, B, C are N_a , N_b and N_c respectively, where $N_a + N_b + N_c = N_\Omega$. The mean number of nodes (spatial density) of each sub-network is given by μ_a , μ_b , μ_c ($\mu = N/a$, N is the number of nodes in a sub-network, a is the sub-network's coverage area and μ is given in nodes /unit square). Theorem 1 states the merging property of a Poisson Point process.

Theorem 1: The superposition of N independent Poisson processes with spatial densities $\Phi_i \forall i \in N$ is a Poisson process with intensity $\Phi = \sum_i^N \Phi_i$ [11].

Using Theorem 1, the entire inter-working network can be considered as a merging Poisson process with mean number of nodes (spatial density): $\mu_{Net} = \mu_a + \mu_b + \mu_c$. Nodes in the network may communicate in a multi-hop manner and transmit at a data rate of Ψ bps.

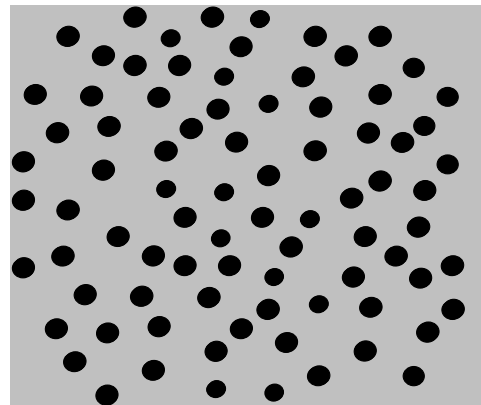


Fig. 2. Spatial point pattern.

In this paper, source-nodes are referred to as transmitter-nodes (t-nodes) while destination-nodes are referred to as receiver-nodes (r-nodes). $\{l : l = 1, 2, 3, \dots, n\} \in L$ represents the links between nodes, where L is the set of all links in the entire network. β represents the link distance (length of a communication link) between a T-node and an R-node.

C. Node Degree

The degree of a node in wireless multi-hop networks is defined as the number of neighbor nodes that it has [14]. A node is said to be a neighbor node to another node if the distance between the two nodes is less than or equal to their transmission range, which means that both nodes have a direct link to each other. Therefore, a node's degree is the number of nodes within its transmission range.

The degree of a node is denoted by $D(\cdot)$. In an instance where for a node, $D(\cdot)=0$, the node is termed a "lone node". The existence of a "lone node" in a multi-hop wireless network is an undesirable condition. Although a lone node maybe useless in terms of connectivity in a static multi-hop wireless network, yet in a mobile scenario, it becomes useful as it moves into the transmission range of another node or when another node moves into the node's transmission range. The desirable condition for connectivity in a multi-hop wireless network is for all nodes to have $D(\cdot) > 0$. The probability that $D(\cdot) > 0$ for any node is the same as the probability that a link is available for the node and it is given by equation 2. R is the transmission range of the node and $f(x)$ is the probability density function (PDF) of the distance between any two nodes in an inter-working multi-hop wireless network.

$$\Pr(D(\cdot) > 0) = \Pr(\text{link availability}) = \int_0^R f(x) dx \quad (2)$$

III. LINK MODELS

A. Link Distance Distribution Model

It is important to analyze the distribution of the link distances between nodes because the probability that a communication link is available for any node is related to the link distance distribution [17]. Also, in multi-hop wireless networks, the probability that a multi-hop communication path is available is related to the availability of the individual links that make up the path.

Let β denote the distance between a t-node and its nearest neighbor (a potential r-node). With Theorem 2 stated below, the probability that $\beta > R$ can be evaluated.

Theorem 2: For a Homogeneous Poisson Point Process in R^2 , the probability that there are no points within a distance x of an arbitrary point (p) is $e^{-\lambda\pi x^2}$, where the parameter λ is the mean number of points per unit area [6].

For any t-node within the network in fig. 1, the above theorem applies in the following ways:

1) The probability that there are no nodes within a distance $\beta \leq R$, (probability that a t-node node has no neighbor) is:

$$\Pr(D(\cdot) = 0) = \Pr(\beta > R) = e^{-\mu_{net}\pi R^2} \quad \forall R > 0 \quad (3)$$

2) Also, the probability that the distance between a t-node and its nearest neighbor node is less than the t- node's transmission range (the probability that a t-node has at least one neighbor) is:

$$\Pr(D(\cdot) > 0) = 1 - e^{-\mu_{net}\pi R^2} \quad \forall R > 0 \quad (4)$$

$$\Pr(\beta \leq R) = 1 - e^{-\mu_{net}\pi R^2} \quad \forall R > 0 \quad (5)$$

Equation 5 represents the cumulative distribution function (CDF) of the distance between any two randomly positioned nodes in the network in fig. 1. The CDF is represented by $F_\beta(R)$. Equation 5 also represents the probability that a link exists between a t-node and an r-node. Assuming that links become non-existent independently, this quantity can be taken as the probability that a link exists in a binomial trial. If the trial is repeated z times, then an estimate of the number of existing links for any node is given by $z \times F_\beta(R)$ [17].

B. Link Availability Model

As long as $\beta \leq R$, a link is available (exists) between any two arbitrary nodes. Therefore, the CDF of the link distance β can be taken as the probability that at least a link is available for transmission [12]. Thus, the availability of a link in a network is a function of R , β and μ_{Net} in the network. If P_{link} represents the availability of a 1-hop link for any node in the network, then, P_{link} can be expressed as:

$$\therefore P_{link} = \begin{cases} 1 - e^{-\mu_{net}\pi R^2} & \text{for } 0 < \beta \leq R \\ 0 & \text{for } \beta > R \end{cases} \quad (6)$$

Fig. 3 gives a plot of the availability of a link as the value R takes on increases. A network scenario in which $N=20$ nodes in a 10 square unit area has been considered. At $R=0.2$, only 22.2% of the total nodes are available for a 1- hop link to any node and 99.8% of nodes are available if $R=1$.

All (100%) of the links are available once $R > 1$, which means that every node has a link to all other nodes in the network. This phenomenon indicates that the network is fully connected.

For a network with N nodes in area A , as R increases, P_{link} increases. From Poisson distribution, equation 4 is analogous

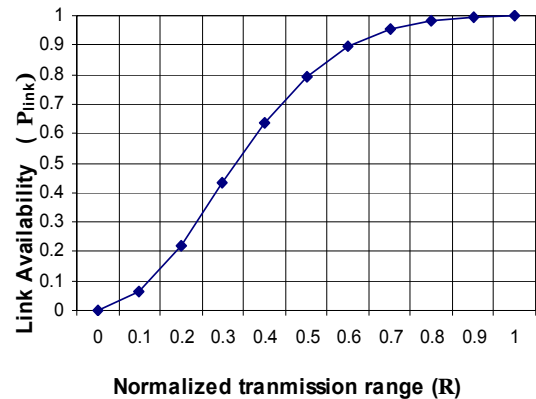


Fig. 3. Link Availability vs Normalized transmission range.

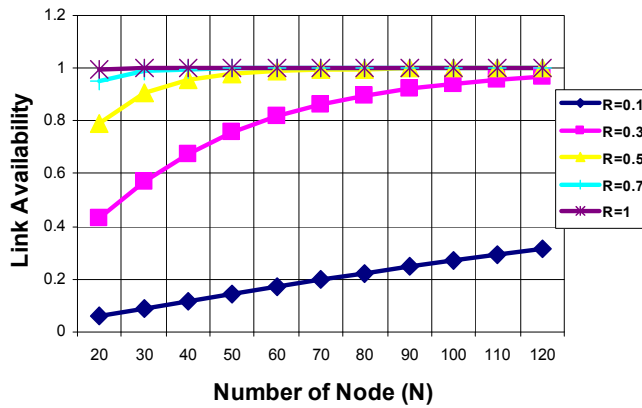


Fig. 4. P_{link} vs Number of Nodes (N) for different values of R.

to equation 7, which is the probability of > 0 nodes within a node's radio coverage area of πR^2 , for any value of R. The probability that the degree of a node is equal to n is expressed in 8.

$$\Pr(D(.) > 0) = \sum_{n=1}^{N-1} \frac{(\mu_{net} \pi R^2)^n}{n!} e^{-\mu_{net} \pi R^2} \quad \forall R > 0 \quad (7)$$

$$\Pr(D(.) = n) = \frac{(\mu_{net} \pi R^2)^n}{n!} e^{-\mu_{net} \pi R^2} \quad \forall R > 0 \quad (8)$$

The number of available 1-hop links for a t-node, given its transmission radius can be expressed as $P_{link}(N-1)$ for N nodes in the network. Note that a maximum of N-1 links are potentially available to all node in a network of N nodes.

From fig. 3, it can be observed that the CDF of β , ($F_{\beta}(R)$) given by equation 5 is a monotonically increasing function. Consequently, in a network with area A and N nodes, P_{link} increases as R increases.

Fig. 4 gives a plot of P_{link} at fixed transmission range (R) as the number of nodes in the network increases. Also, $A=10$ square units, and N was increased from 20 nodes to 120 nodes at fixed node transmission values of 0.1, 0.3, 0.5, 0.7 and 1. From fig. 4, generally for all the cases considered P_{link} increases as N increases; indicating that the probability of having an available link is higher in a dense network. For high values of R, the P_{link} is at very high values for large N in the network. If R is the same for all nodes, then the upper bound for P_{link} is:

$$P_{link-upper} = 1 - e^{-\mu_{net} \pi R^2} \quad \forall R > 0 \quad (9)$$

IV. ROUTE AVAILABILITY MODEL

If the distance (β) between a source and destination is greater than R, then from equation 6, $P_{link}=0$, therefore a multi-link (multi-hop) route has to be utilized for packet transmission. In this case, multiple hop routes in the direction of the destination node are used. [4] explains the different methods, which can be used to achieve this. To ensure end to end route availability, each intermediate node on the route must have at least two neighbour nodes. These two neighbours

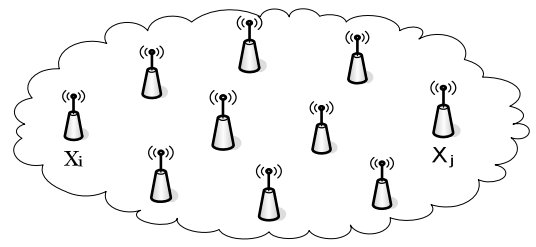


Fig. 5. A Subnetwork.

are for the purpose of packet reception from the preceding node and packet transmission to the subsequent node.

Let l represent the links (or hops) between any two nodes in the network, where $l \in L$ and L is the set of all links that exists in the network. If a transmitted packet from a node have to hop on a total of l links to arrive at the destination node, then, $l-1$ intermediate nodes will be required on this route. The number of hops depends on β , and the transmission range (R) of the t-node and the intermediate nodes. For analytical tractability, the transmission ranges of all nodes in the network are assumed to be equal. Thus, the minimum number of links (hops) that can connect any two nodes together is:

$$l = \left\lceil \frac{\beta}{R} \right\rceil \quad (10)$$

$\lceil x \rceil$ represents the greatest integer that is greater than x. However, a bound for l exists in every network. The bound occurs when β happens to be equal to the maximum distance (β_{max}) that can be between any two nodes in the network. In this case, the maximum number of hops $l_{max} = \beta_{max}/R$ cannot be exceeded.

Fig. 6 shows a plot of the number of hops versus the distance between a source-destination node pair. As in section III B, a 20 node network with an area of 10 square units has been considered. The value of β was increased from 0.2 units to 1.8 units, while the number of hops was observed for constant transmission range (R= 0.2, 0.3, 0.6 and 0.7). Fig. 6 confirms that the longer the distance between node pairs relative to their transmission range, the more the number of hops (links) that will be utilized to transmit a packet from

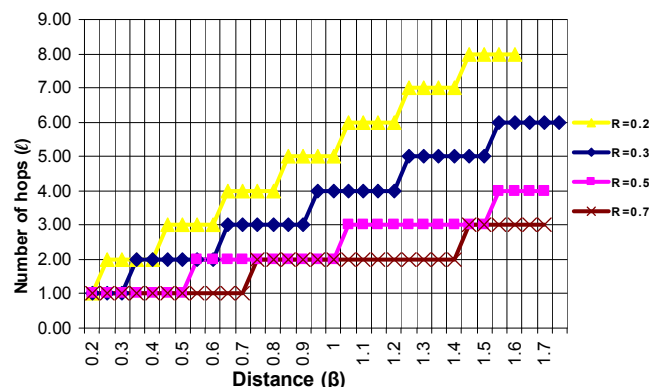


Fig. 6. Number of hops (l) vs distance between nodes.

source to destination. Equation 11 summarizes the minimum number of hops (ℓ_{\min}) for different values of β , and a general expression for evaluating ℓ_{\min} is obtained.

$$l_{\min} = \begin{cases} 1, \forall 0 < \beta \leq R \\ 2, \forall R < \beta \leq 2R \\ 3, \forall 2R < \beta \leq 3R \\ \dots \dots \dots \\ l, \forall (l-1)R < \beta \leq lR \end{cases} \quad (11)$$

$$\sum_{n=1}^{N-1} \frac{(\mu_{Net} \pi (lR)^2)^n}{n!} e^{-\mu_{Net} \pi (lR)^2} - \sum_{n=1}^{N-1} \frac{(\mu_{Net} \pi ((l-1)R)^2)^n}{n!} e^{-\mu_{Net} \pi ((l-1)R)^2} \quad (12)$$

Consider the sub-network in fig. 5 with N nodes. If a route is to be established between X_i and X_j , where X_j is the destination, there will be $N-2$ intermediate nodes between X_i and X_j . Depending on β and R , the maximum number of hops that can be used to transmit packets from X_i to X_j is $N-1$ and the minimum number of hop is 1. To establish a route with a definite number of hops (e.g. ℓ hops), R has to be at a certain maximum value as illustrated in fig.6. If R is lower than the maximum value, more hops will be utilized to set up such a route. In order to evaluate the probability that any source-destination node pair is linked by a certain number of hops, the following must be fulfilled:

The distance between the source-destination pair must fulfil the general expression for ℓ_{\min} in equation 10.

There should be at least a node between the distance $(\ell-1)R$ and ℓR and;

Every node along the route should have at least a neighbour node that is within the transmission range of another node 2hops away from it.

The condition in (2) is such that nodes must exist between distance $(\ell-1)R$ and ℓR and the probability of this happening is expressed in equation 12 (stated below this page). For condition (3), let A_{int} be the area of intersection of the transmission ranges of any two nodes along the route, which are 2-hops away from each other. From [16] A_{int} can be expressed as:

$$A_{\text{int}} = R^2 \left[2 \cos^{-1} \left(\frac{\beta}{2R} \right) - \sin \left(2 \cos^{-1} \left(\frac{\beta}{2R} \right) \right) \right] \quad (13)$$

The probability of at least 1 node in area A_{int} , where n_{int} is the number of nodes in area (A_{int}) is:

$$\Pr(n_{\text{int}} > 0) = 1 - e^{-\mu_{Net} A_{\text{int}}} \quad \forall A_{\text{int}} > 0 \quad (14)$$

For an ℓ -hop route, equation 14 is expressed as:

$$\Pr(n_{\text{int}} > 0) = (1 - e^{-\mu_{Net} A_{\text{int}}})^{\ell-1} \quad \forall A_{\text{int}} > 0 \quad (15)$$

Finally, the probability of an ℓ -hop route, ($P_{\ell\text{-hop}}$) between X_i and X_j is given by the multiplication of equation 12 and 15. However, as the network's node density increases, for constant

R , equation 15 tends towards 1. From equation 4, 7 and 12, an asymptotic probability for an ℓ -hop route can be evaluated with equation 16.

$$P_{\ell\text{-hop}} = \left(1 - e^{-\mu_{Net} \pi (lR)^2} \right) - \left(1 - e^{-\mu_{Net} \pi ((l-1)R)^2} \right) \quad (16)$$

$$= e^{-\mu_{Net} \pi ((l-1)R)^2} - e^{-\mu_{Net} \pi (lR)^2}$$

Using the network scenario in section III B, $P_{\ell\text{-hop}}$ versus ℓ is as shown in fig. 7. The sum of $P_{\ell\text{-hop}}=1$. From the data obtained for fig. 7, the probability that $\ell > \ell_{\max}$ tends to zero. In case of node or link failures, an alternative detour needs to be available at any point in order to ensure end-to-end packet transmission. This alternative route may require more than the minimum number of hops or the same number of hops as ℓ .

So now, what is the probability that a source-destination pair will be connected irrespective of the number of hops from source to destination?

Let P_{route} denote the probability that a route is available. Equation 17 gives the route availability for $\beta/R \leq \ell \leq \ell_{\max}$. P_r depends on the probability of establishing an ℓ -hop route between any pair of source-destination node in the inter-working multi-hop wireless network. It also depends on β (distance between the source-destination nodes) and R (transmission range of nodes).

$$P_{\text{route}} = \sum_{\ell=\frac{\beta}{R}}^{\ell_{\max}} P_{\ell\text{-hop}} \quad (17)$$

V. CONCLUSION

The research work focuses on connectivity in inter-working multi-hop wireless networks. The paper provided a study of link availability and presented a model for route availability for inter-working multi-hop wireless networks. In multi-hop wireless networks such as ad-hoc networks, a network connectivity analysis is needed. However, in inter-working multi-hop wireless networks, an analysis of the route connectivity is more desirable. Thus the emphasis of this paper was on route connectivity in inter-working multi-hop wireless networks.

For there to be connectivity between a source-destination node pair in an inter-working multi-hop wireless network, a

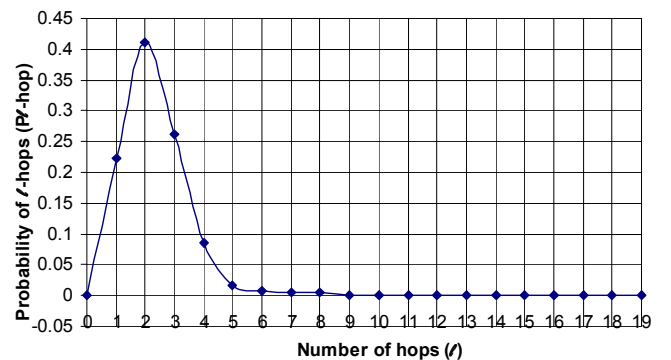


Fig. 7. Probability of ℓ -hops vs number of hops in a 20 node network.

