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EVENT DRIVEN CONTROL OF VIBRATORY CONVEYORS OPERATING ON THE FRAHM'S ELIMINATOR BASIS

STEROWANIE ZDARZENIOWE PRZENOŚNIKIEM WIBRACYJNYM DZIAŁAJĄCYM W OPARCIU O ELIMINATOR FRAHMA

The new, original control method of the vibratory conveyor operating on the Frahm's dynamic eliminator basis, is presented in the paper. The proposed method is based on the application of the control of the feed-forward controller, together with the events detection based on the generalised likelihood ratio (GLR) algorithm. Such approach leads to the controller intervention only when it is justified by the current process situation, (e.g. in case of an essential change of the feed mass) to enable the stable machine operations and to limit transient states. The results are presented in a form of numerical simulations.

Keywords: Frahm eliminator, vibratory conveyors, control, Generalized Likelihood Ratio

W artykule przedstawiono oryginalny sposób sterowania przenośnikiem wibracyjnym, działającym w oparciu o dynamiczną eliminację drgań na zasadzie eliminatora Frahma. Zaproponowana metoda polega na zastosowaniu sterowania typu feed-forward wraz z detekcją zdarzeń opartą na algorytmie sum kumulacyjnych (GLR). Podejście takie prowadzi do interwencji regulatora tylko wtedy gdy jest to uzasadnione bieżącą sytuacją procesową, np. w przypadku istotnej zmiany masy nadawy, co zapewnia stabilną pracę maszyny i ogranicza stany przejściowe. Wyniki przedstawiono w postaci symulacji numerycznych.

1. Introduction

Vibratory conveyors are utilised – usually at distances not longer than 20 m – in metallurgical industry for continuous transport of hot materials (furnace slag, small steel elements, etc.), caustic substances or substances emitting gases hazardous for the environment. In addition, vibratory conveyors enable recovery of heat from the transported materials (used later e.g. for warming furnace blowers, drying, humidifying, etc.). They also allow transporting in closed conduits [1,2].

A negative side of vibratory conveyors is transferring large dynamic loads on foundations. To lower dynamic reactions, the application of the Frahm's dynamic eliminator [4] in the structure of vibratory conveyors was proposed, already in the sixties of the previous century [3]. In conveyors operating on the Frahm's eliminator base, m_r is the mass of the conveyor trough, while m_b is the mass of the frame excited for vibrations by the excitation force: $P_0 \sin \omega t$ (Fig. 1). The conveyor trough is connected with the frame by leaf springs of a coefficient of elasticity k_τ satisfying the equation:

$$k_\tau = m_r \omega^2 \tag{1}$$

forming the Frahm's eliminator. A real development of this type of conveyors started at the beginning of the 21st century. Thus far, conveyors or feeders were applied in industry for transporting feed materials of small masses [5].

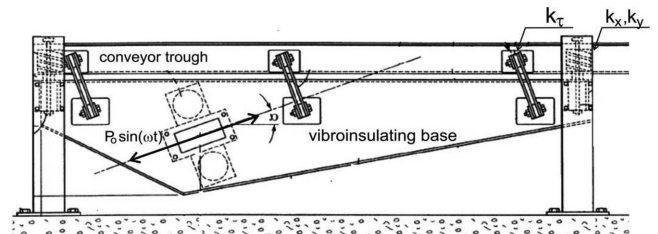


Fig. 1. Conveyor in the FMC Technologies [5]

A broad interest of this type of construction among producers inspired a lot of publications related to the problem [6]. The authors analysed the conveyor response to the assumed excitation frequency (resulting as vibroinsulating Frahm's damper). The author of paper [7] drew a conclusion that the feed mass influences the force transmitted to the foundation and its value has to be small if the conveyor is to operate properly. This thesis was challenged in study [8].

This type of solution is not, so far, applied in metallurgical industry, due to a general believe that with an increase of the feed mass a significant increase of forces transmitted to foundations occurs. However, it was shown – in paper [8] – that at the proper selection of parameters of the conveyor operating on the Frahm's eliminator bases, in spite of its loading with a significant feed mass, very small values of forces transmitted to foundations can be obtained, lower than in vibratory conveyors placed on vibroinsulating frames.

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Since – in this type of conveyors – a feed transferred on the trough in a certain sense cooperates with it, the dependence (1) is not valid in such system. It was proofed in paper [8], that there is the possibility of such changing of the excitation frequency ω in dependence on the mass of the feed (2), to have the conveyor operating properly and transmitting minimal forces to the foundation, regardless of a large load.

$$\omega = \sqrt{\frac{k_f}{m_r + m_n \sin^2 \beta}} \quad (2)$$

where:

m_r – mass of the conveyor trough,

k_f – total stiffness coefficient of leaf springs in working direction f ,

m_n – mass of the feed,

β – angle of deviation of the direction of trough vibrations from the level.

The system enabling the control of such machines by means of the feed-forward controller – equipped with the events detection module based on the algorithm of cumulative sums (GLR) – is presented in Chapter 3 of the hereby paper.

2. Mathematical model of the conveyor loaded with a feed

In order to estimate the influence of the feed on the efficiency of the reductions of forces transmitted to the foundations the simulation model was developed (Fig. 2).

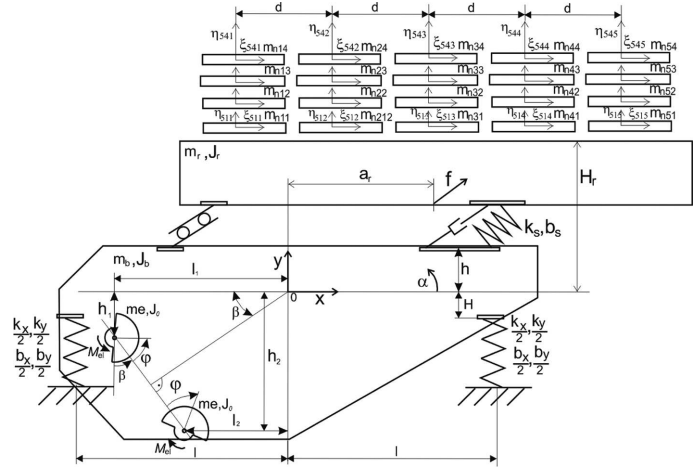


Fig. 2. Model of the vibratory conveyor loaded with a feed

The analysed system consists of two inertial vibrators, in which induction motors (described by static characteristics) excite for vibrations the frame suspended on the system of spiral springs. The conveyor trough together with the feed [9] is suspended on the frame by means of the leaf springs system allowing for the relative motion of the trough and frame along the axis s only.

The mathematic model of such system consists of the matrix equation (3) describing the machine motion, equation (10) describing the electromagnetic moment of drive motors, equations (9) used for the determination of motions of successive feed layers and dependencies (7) and (8) describing normal and tangent interactions in between feed layers as well as between a feed and the machine frame.

$$[M] \cdot [\ddot{q}] = [Q] \quad (3)$$

$$[M] = \begin{bmatrix} m_b + 2m + m_r & 0 & m(h_1 + h_2) - m_r h_r & 2me \cos \beta \cdot \cos \phi & m_r \cos \beta \\ 0 & m_b + 2m + m_r & -m(a_1 + a_2) + m_r a_r & 2me \sin \beta \cdot \cos \phi & m_r \sin \beta \\ m(h_1 + h_2) - m_r h_r & -m(a_1 + a_2) + m_r a_r & m(h_2^2 + l_2^2 + h_1^2 + l_1^2) + m_r(h_r^2 + a_r^2) + J_b + J_r & me(h_1 \cos(\beta + \phi) - a_1 \sin(\beta + \phi) + h_2 \cos(\phi - \beta) + l_2 \sin(\phi - \beta)) & m_r(a_1 \sin \beta - h_1 \cos \beta) \\ 2me \cos \beta \cdot \cos \phi & 2me \sin \beta \cdot \cos \phi & me(h_1 \cos(\beta + \phi) - a_1 \sin(\beta + \phi) + h_2 \cos(\phi - \beta) + l_2 \sin(\phi - \beta)) & 2me^2 + 2J_0 & 0 \\ m_r \cos \beta & m_r \sin \beta & m_r(a_1 \sin \beta - h_1 \cos \beta) & 0 & m_r \end{bmatrix} \quad (4)$$

$$[\ddot{q}] = [\ddot{x} \ \ddot{y} \ \ddot{\alpha} \ \ddot{\phi} \ \ddot{f}]^T \quad (5)$$

$$[Q] = \begin{bmatrix} -2me\dot{\phi}^2 \sin \phi \cdot \cos \beta - k_x(x + H\alpha) - b_x(\dot{x} + H\dot{\alpha}) - T_{1(01)} - T_{1(02)} - T_{1(03)} - T_{1(04)} - T_{1(05)} \\ me\dot{\phi}^2(\sin(\phi - \beta) + \cos(\beta + \phi)) - \frac{1}{2}k_y(y + l_1\alpha) - \frac{1}{2}k_y(y - l_2\alpha) - \frac{1}{2}b_y(\dot{y} + l_1\dot{\alpha}) - \frac{1}{2}b_y(\dot{y} - l_2\dot{\alpha}) - F_{1(01)} - F_{1(02)} - F_{1(03)} - F_{1(04)} - F_{1(05)} \\ -me\dot{\phi}^2(h_1 \sin(\beta + \phi) + l_1 \cos(\beta + \phi)) + m_2e_2\dot{\phi}^2(-h_2 \sin(\phi - \beta) + l_2 \cos(\phi - \beta)) - k_x H^2 \alpha - k_x H \dot{x} - b_x H^2 \dot{\alpha} - \\ \frac{1}{2}k_y(y + l\alpha)l + \frac{1}{2}k_y(y - l\alpha)l - \frac{1}{2}b_y(\dot{y} + l\dot{\alpha})l + \frac{1}{2}b_y(\dot{y} - l\dot{\alpha})l + (T_{1(01)} + T_{1(02)} + T_{1(03)} + T_{1(04)} + T_{1(05)})H_r + \\ F_{1(01)}(2d - a_r) + F_{1(02)}(d - a_r) - F_{1(03)}a_r - F_{1(04)}(d + a_r) - F_{1(05)}(2d + a_r) \\ 2M_{el} - mge (\sin(\beta + \phi) + \cos(\phi - \beta)) \\ -k_f f - b_f \dot{f} - (T_{1(01)} + T_{1(02)} + T_{1(03)} + T_{1(04)} + T_{1(05)}) \cos \beta - (F_{1(01)} + F_{1(02)} + F_{1(03)} + F_{1(04)} + F_{1(05)}) \sin \beta \end{bmatrix} \quad (6)$$

where:

f – dependent co-ordinate,

$F_{j,(j-1,k)}$ – normal component of the j -th layer pressure on the $j-1$ in the k -th column,
 $T_{j,(j-1,k)}$ – tangent component of the j -th layer pressure on the $j-1$ in the k -th column,
 j – indicator of the material layer, $j = 0$ is related to the machine frame,
 k – indicator of the material layer column.

If the successive feed layers j -th and $j-1$ are not touching (in the given column), then the contact force in the normal $F_{j,(j-1,k)}$ and tangent $T_{j,(j-1,k)}$ direction – equals zero. Otherwise the contact force occurs in the normal direction between layers j -th, k and $j-1, k$ of the feed (or in case of the first layer – between the layer and trough) and its model is as follows [10]:

$$\left\{ 1 - \frac{1-R^2}{2} \left[1 - \operatorname{sgn}(\eta_{j-1,k} - \eta_{j,k}) \cdot \operatorname{sgn}(\dot{\eta}_{j-1,k} - \dot{\eta}_{j,k}) \right] \right\} F_{j,(j-1,k)} = (\eta_{j-1,k} - \eta_{j,k})^p \cdot k \quad (7)$$

and – originated from friction – the force in the tangent direction:

$$T_{j,(j-1,k)} = -\mu F_{j,(j-1,k)} \operatorname{sgn}(\dot{\xi}_{j,k} - \dot{\xi}_{j-1,k}) \quad (8)$$

Equations of motion in directions ξ and η of individual layers of a feed, with taking into account the influence of the conveyor on lower feed layers, are of the form:

$$m_{n,j,k} \ddot{\xi} = T_{j,(j-1,k)} - T_{j+1,(j,k)} \quad (9)$$

$$m_{n,j,k} \ddot{\eta} = -m_{n,j,k} g + F_{j,(j-1,k)} - F_{j+1,(j,k)}$$

The moment generated by drive motors is:

$$M_{el} \frac{2M_{ut}(\omega_{ss} - \dot{\phi}_{i1}) \cdot (\omega_{ss} - \omega_{ut})}{(\omega_{ss} - \omega_{ut})^2 + (\omega_{ss} - \dot{\phi}_i)^2} \quad (10)$$

where:

M_{ut} – stalling torque of drive motors,
 ω_{ss} – synchronous frequency of drive motors,
 ω_{ut} – frequency of a stall of drive motors.

Simulations were performed at the following parameters:

$l = 4$ [m]	$m = 15$ [kg]
$l_1 = 1.54$ [m] (variable)	$J_0 = 0$
$l_2 = 1.05$ [m] (variable)	$e =$ variable, depending on the coefficient of throw K
$h_1 = 0.31$ [m] (variable)	$m_r = 3700$ [kg]
$h_2 = 1.18$ [m] (variable)	$m_b = 2600$ [kg]
$H = 0.0$ m	$\mu = 0.4$
$h = 0.0$ m	$J_k = 500$ [kgm ²]
$H_r = 0.0$ m	$J_r = 700$ [kgm ²]
$a_r = 0.0$ m	$M_{ut} = 50$ [Nm]
$\beta = 45^\circ$ (variable)	$\omega_{ss} = 106$ [rad/s] (variable)
$k_x = k_y = 2328000$ [N/m]	$\omega_{ut} = 15.9 \cdot 2\pi$ [rad/s] (variable)
$k_f = 42171481$ [N/m]	$R = 0.13$
$b_f = \frac{\psi_s k_s}{2\pi\phi}$ [Ns/m]	$\sum m_n = 800$ [kg] (variable)
$b_x = b_y = \frac{\psi_{x,y} k_{x,y}}{2\pi\phi}$ [Ns/m]	$\psi_{x,y} = 0.14$
$\psi_f = 0.09$	

The time-series example of the force transmitted to the foundation is presented in Fig. 8, while Fig. 3 presents examples of vertical displacements of the successive feed layers for

the total feed mass $m_n = 800$ kg at its dividing into 8 layers ($n = 8$).

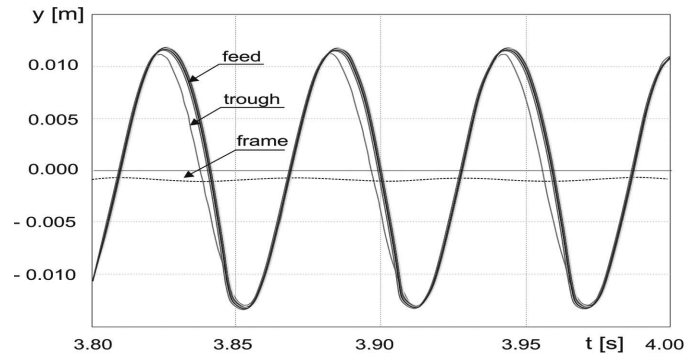


Fig. 3. Vertical displacements of the trough and eight feed layers for the feed mass of 800 kg

3. Control

A vibratory conveyor of the Frahm's eliminator type, equipped with the system of the excitation frequency control, allowing to transfer larger feed masses, would significantly broaden the applicability range of this type of devices. Especially, as it was mentioned in the introduction, such conveyors could be applied in metallurgy, e.g. for transporting hot slag.

However, the classic feedback loop control would be in such case difficult for the application since it would cause the machine destabilisation due to a highly oscillatory character of its motion. On account of this, the implementation of the simple and the most common in industry PID controller can not be applied. In return, an application of a feed-forward control seems reasonable. In this control system the mass of a feed is treated as a measurable disturbance, directly influencing the control value, it means the excitation frequency of vibrators (Fig. 4).

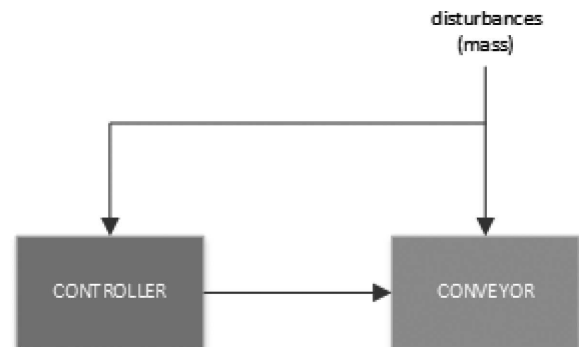


Fig. 4. Scheme of the proposed control system

The availability of the measured mass of a feed constitutes here a problem. In industrial applications the mass of a feed being on the conveyor trough is rather seldom measured. More

TABLE 1

Constant components of the force and optimal frequencies for various masses of the feed

m [kg]	50	100	150	200	250	300	350
F_f [N]	-1.379e03	-2.759e03	-4.139e03	-5.519e03	-6.900e03	-8.279e03	-9.659e03
ω_{ss} [rad/s]	105.3848	103.8348	102.4421	101.0722	99.5634	98.2617	97.0215

realistic for obtaining, seems the measurement of the constant component of the force transmitted by the whole conveyor (with a feed) to the foundation.

In order to find out the dependence between the feed mass m and the constant component of force¹⁾ F_f on the basis of the mathematical model of the conveyor (described in the previous chapter), the force values corresponding to the mass of the feed from 50 to 350 kg, were numerically calculated, with a step of 50 kg. These values are listed in Table 1. On their base the linear regression model of these two values was calculated. As the result the linear model of the first order regression was obtained:

$$F_f(m) = -27.6m + 0.4857 \quad (11)$$

It is possible to calculate – from this model – the mass value of the feed on the bases of the force measurements (see Fig. 5).

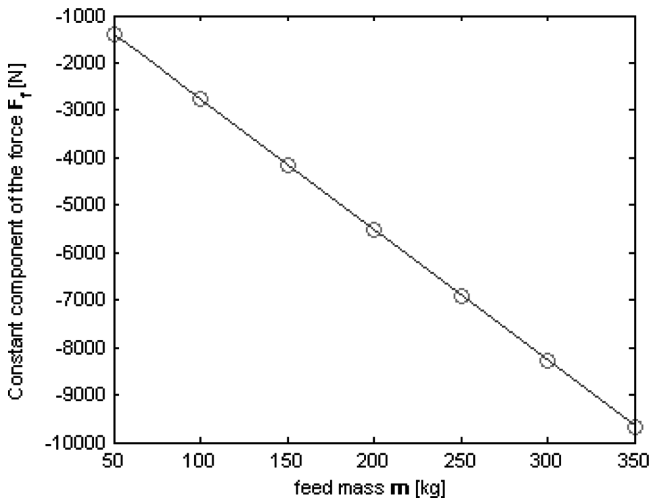


Fig. 5. Linear dependence between the mass of the feed and the constant component of the force transmitted to foundations

The optimal control value, it means the excitation frequency of vibrators ω_{ss} , is the one which ensures the minimal amplitude of vibrations of the force transmitted to foundations. This value depends, among others, on the mass of the feed. In order to find out the optimal frequency values the mathematical model of the conveyor (described in the previous chapter) was applied. By means of simulations of this model and the optimisation techniques, based on the golden cut and parabolic interpolation [11], the set of optimal frequencies corresponding to the mass of the feed from 50 to 350 kg with the step every 50 kg was obtained (presented in Table 1). For these data the numerical model of the linear regression was

calculated. As the result the linear model of the first order regression was obtained:

$$\omega_{ss}(m) = -0.028m + 106.67 \quad (12)$$

presented in Fig. 6.

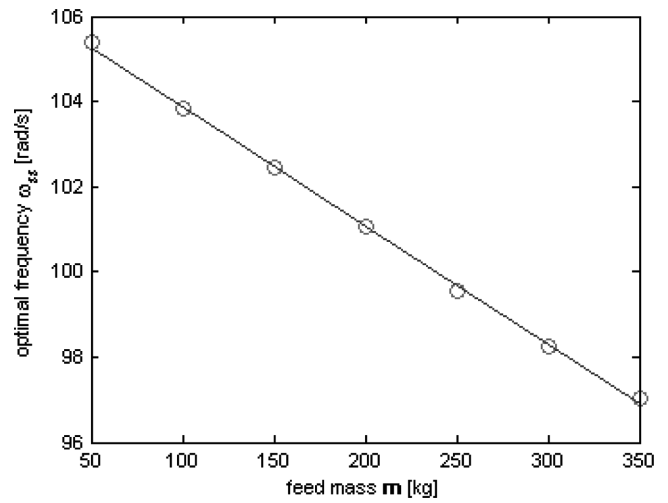


Fig. 6. Linear dependence of the optimal frequency on the feed mass

Equation (12) is simultaneously the control law, which theoretically can be directly implemented in the controller. However, in practice, such implementation without the analysis of the current process situation can destabilise the process. The presented model of the conveyor is very sensitive to control changes (excitation frequency of vibrators). Each change introduces the system in the transient state characterised by large overshoot. Disturbances in measurements of the constant force component cause disturbances in the feed mass estimation from the regression model (11). These, in turn, causes fast-changing fluctuations of the excitation frequency, which brings the system in the permanent transient state.

Due to the above, the controller intervention must be preceded by analysing whether it is justified in this very moment. Such approach means resigning from the continuous control and substituting it by the event driven control. The event, which initiates a new control value – on the bases of the regression model (12) – could constitute an essential change of the feed mass on the conveyor trough. Thus, the detection of the mass change would be reduced, in this case, to the detection of the change of the constant component of the force transmitted by the conveyor to foundations.

¹⁾ Due to an oscillatory character of the force time-series, its average value was taken in calculations in the window of the length corresponding to 2 seconds of the machine operation in a steady state.

4. Detection of the mass change

The algorithm of the GLR (*Generalized Likelihood Ratio*) based on the idea of the so-called *Cumulative Sum* (CUSUM) was applied for the detection of a step change of the mass. This idea is based on constructing the so-called *sufficient statistics* (it means certain transformations of measurements), which will draw out a possibly full information on residues²⁾ contained in the original signal.

Next, on the basis of this information, the statistical inference is performed. In other words, on the basis of residues the decision rule is calculated to select one out of two statistical hypothesis concerning the investigated signal parameter θ :

$$\mathbf{H}_0 : \theta = \theta_0$$

$$\mathbf{H}_1 : \theta = \theta_1$$

The H_0 hypothesis means that none significant change of the parameter θ occurred and it is equal θ_0 . The H_1 hypothesis means that the significant change of the parameter θ occurred and it is equal now θ_1 , (i.e. in our case a step change of the mass occurred).

The probability density distribution $P_\theta(y)$ means the probability of obtaining the measurement of y under condition that θ is the real parameter. If $P_{\theta_1}(y) > P_{\theta_0}(y)$, it means that the observed y is more probable at $\theta = \theta_1$. In case, when the parameter θ can take on only two different values: θ_0 or θ_1 , in order to decide which one is more probable the following likelihood ratio is defined:

$$\frac{P_{\theta_1}(y_i)}{P_{\theta_0}(y_i)} \quad (13)$$

Due to practical reasons, for calculating the *decision rule* the logarithm of this ratio is used:

$$s_i = \ln \frac{P_{\theta_1}(y_i)}{P_{\theta_0}(y_i)} = \ln P_{\theta_1}(y_i) - \ln P_{\theta_0}(y_i) \quad (14)$$

In case of N measurements the probability of their obtaining is given by the probabilities product:

$$P_\theta(y_1^N) = \prod_{i=1}^N P_\theta(y_i) \quad (15)$$

which is called the *likelihood function*. Thus, the decision rule takes a form of the *likelihood ratio*:

$$Q_N = \ln \frac{\prod_{i=1}^N P_{\theta_1}(y_i)}{\prod_{i=1}^N P_{\theta_0}(y_i)} \quad (16)$$

which can be simpler written in a form of cumulative sum:

$$S_N = Q_N = \sum_{i=1}^N s_i \quad (17)$$

In case of the GLR algorithm, the logarithm of the likelihood function for observations from y_j to y_k is written as:

$$S_j^k(\theta_1) = \sum_{i=j}^k \ln \frac{P_{\theta_1}(y_i)}{P_{\theta_0}(y_i)} \quad (18)$$

Since the θ_1 parameter is not known, the above value is a function of two independent parameters: time of the signal change and parameter θ_1 value after this change. The standard statistical approach is in such case the application of the estimate of the highest likelihood of these parameters, it means the double maximisation [12]:

$$g_k = \max_{1 \leq j \leq k} \sup_{\theta_1} S_j^k(\theta_1) \quad (19)$$

In practice, if the signal disturbance is the Gaussian white noise, the average value μ_0 before the change is known, the average value μ_1 after the change is unknown and the constant variance σ^2 is known, the cumulative sum equals:

$$S_j^k = \frac{\mu_1 - \mu_0}{\sigma^2} \sum_{i=j}^k \left(y_i - \frac{\mu_1 + \mu_0}{2} \right) \quad (20)$$

After introducing $v = \mu_1 - \mu_0$, the decision function can be written in a form:

$$g_k = \max_{1 \leq j \leq k} \sup_v \sum_{i=j}^k \left[\frac{v(y_i - \mu_0)}{\sigma^2} - \frac{v^2}{2\sigma^2} \right] \quad (21)$$

Since the best estimate of the step change is, in this case, the arithmetic mean, the above equation can be written as:

$$g_k = \max_{1 \leq j \leq k} \sum_{i=j}^k \left[\frac{\tilde{v}_j(y_i - \mu_0)}{\sigma^2} - \frac{\tilde{v}_j^2}{2\sigma^2} \right] \quad (22)$$

where:

$$|\tilde{v}_j| = \frac{1}{k-j+1} \sum_{i=j}^k |y_i - \mu_0| \quad (23)$$

The decision function assumes a final form:

$$g_k = \frac{1}{2\sigma^2} \max_{1 \leq j \leq k} \frac{1}{k-j+1} \left[\sum_{i=j}^k (y_i - \mu_0) \right]^2 \quad (24)$$

The algorithm principle of operation is presented in Fig. 7.

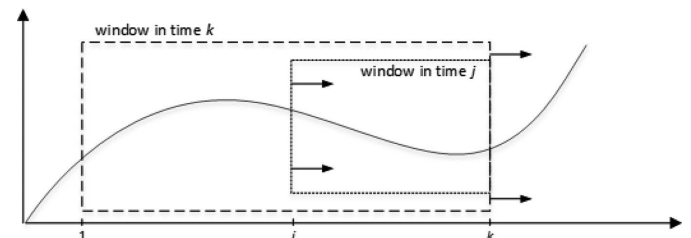


Fig. 7. Operation principles of the GLR algorithm. A signal is analysed in an expanding window from 1 to the current moment k . To shorten calculation time, the window can be movable, of the constant width. In the inner, narrowing window for each $j = 1, \dots, k$ the average value from equation (24) is calculated

²⁾ The subject of analysis are residues, it means the difference between the measured signal, originated from the sensor, and the corresponding variable analytically calculated on the model basis.

5. Simulation results

The proposed control algorithm consists of the following stages (shown in the upper diagram of Fig. 8):

1. **Stabilisation** – the delay needed for a stabilisation of the force transmitted to the foundation.
2. **Detection** – the phase of the GLR algorithm operation aimed at detection the essential change of the feed mass. The detection of this change is signalled by exceeding the assumed threshold value by the decision function (in the middle diagram of Fig. 8, it is seen in the instant of 2.165 s, while the change occurred really in the instant of 2 s).
3. **Stabilisation** – the delay needed for a stabilisation of the force after the step change of the feed mass. The delay time is countdown from the time the change was detected in the previous stage.
4. **Averaging** – the stage needed for the feed mass estimation on the bases of the average force calculated at this stage – acc. to the regression model (11).
5. **Control changing** – at this stage the control value is calculated (i.e. optimal frequency) on the base of the actual feed mass calculated in the previous stage – acc. to the regression model (12). Then, this new control is fed to the object in a form of a smooth polynomial trajectory in order to minimise overshoot, (shown in the lower diagram of Fig. 8).
6. **New control** – the stage in which the force is already stabilised after the control change and its effects can be assessed.

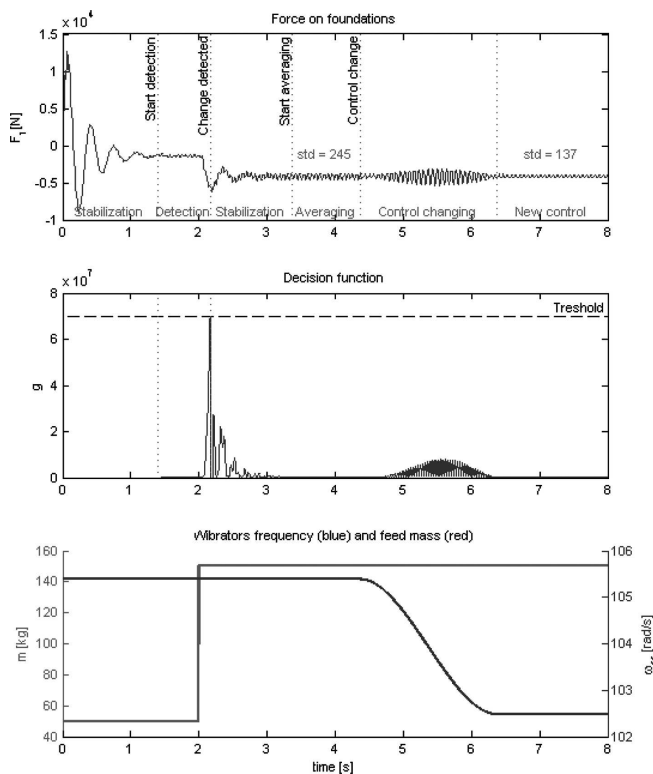


Fig. 8. Results of the numerical simulation. Upper Fig.: time-series of the force transmitted to foundations with the description of individual control stages. Middle Fig.: time-series of the decision function of the GLR algorithm. Lower Fig.: time-series of the feed mass and of the control (excitation frequency)

Duration times of the above stages were experimentally selected. In a similar fashion, the detection threshold for the decision function of the GLR algorithm was individually adjusted for the analysed case by the trial-and-error method.

The positive effect of the controller intervention can be assessed by comparing the standard deviation of the force time-series (*std*) in stage 4 (averaging) and in stage 6 (new control). As it was marked in the upper diagram of Fig. 8, before the controller intervention the *std* was equal 245, while after implementing the new control the *std* decreased to 137. Which means the profit was 108 (by the means of the amplitude of the force transmitted to foundations) when the mass of the feed increased by 100 kg, which constitutes decreasing the force amplitude by app. 56%. The control profit is directly proportional to the mass change values. The control algorithm operates in the same way when the mass is abruptly decreased.

6. Conclusions

Simulation investigations confirmed the usefulness of the rule-based control of the vibrators excitation frequency in the vibratory conveyor operating on the basis of the Frahm's eliminator. The event driven control interferes in the system only when it is justified by the current process situation (e.g. in case of the essential feed mass change), ensures the stable machine operation and limits transient states. Simultaneously, it allows to apply this type of conveyors for transporting significantly larger feed masses than so far.

Further research should be focused on the case when the change of the feed mass is not abrupt but continuous (e.g. linear), which is more difficult for the detection (especially in case of large measuring disturbances), but more probable in the real industrial installation.

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