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ANALYZING VARIATIONS IN ROUNDNESS PROFILE PARAMETERS DURING THE WAVELET DECOMPOSITION PROCESS USING THE MATLAB ENVIRONMENT

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Abstract

Signal analysis performed during surface texture measurement frequently involves applying the Fourier transform. The method is particularly useful for assessing roundness and cylindrical profiles. Since the wavelet transform is becoming a common tool for signal analysis in many metrological applications, it is vital to evaluate its suitability for surface texture profiles. The research presented in this paper focused on signal decomposition and reconstruction during roundness profile measurement and the effect of these processes on the changes in selected roundness profile parameters. The calculations were carried out on a sample of 100 roundness profiles for 12 different forms of mother wavelets using MATLAB. The use of Spearman's rank correlation coefficients allowed us to evaluate the relationship between the two chosen criteria for selecting the optimal mother wavelet.

Keywords: mother wavelet, Fourier transform, wavelet transform, surface texture, surface roundness, roundness profile parameters.

1. Introduction

The dynamic development of technology and production processes has given rise to considerable improvements in the surface finish of components and accordingly the quality of finished products. Assessing the surface quality of products requires applying state-of-the-art measurement techniques and equipment, including special computer programs based on advanced algorithms for digital processing and analysis of signals, particularly the fast changing ones [1, 2, 3].

The most common algorithm used for digital signal processing is the discrete Fourier transform, which allows us to convert time domain signals into frequency domain ones, this being known as spectrum analysis. The procedure is well-suited for analyzing stationary signals. Non-stationary signals, however, may require joint time and frequency domain representation. The time resolution of an analysis carried out using the Fourier transform is not sufficient, especially in the case of rapidly-changing signals [3, 4].

A significant improvement in quality can be acquired by applying the wavelet transform. It is important that individual wavelet functions are well localized in time or space represented in the function of time and frequency (scale). There exist many sets of mother wavelets that differ in smoothness and localization in the time domain. Unlike the Fourier transform, where the resolution remains stable over the whole time-frequency domain, the wavelet transform has the so-called time-frequency windows, whose dimensions are dependent on localization. Wavelet transforms are mainly used to approximate physical phenomena. The aim of the approximation is to determine their characteristic features by decomposing the signal describing a given phenomenon.

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The wavelet transform has been used extensively in numerous fields of science and industry to analyze, denoise and compress signals, e.g. for vibration and shock analysis in mechanics, biomedical signal analysis in medicine (ECG, EEG), and acoustic analysis in the music industry [5].
As the wavelet transform is being applied more and more frequently, researchers dealing with surface texture analysis have begun using it mainly for roughness profiles to filter and identify their parameters [6].

The aim of the study presented in this paper was to assess the level of decomposition and approximation of roundness profiles at which there are no significant changes in the geometrical parameters and to evaluate the influence of different forms of the mother wavelet on the approximation process using statistical methods.

2. Wavelet transform for processing measurement signals

Wavelets $\psi(t)$, as the name indicates, are small waves oscillatory in character with time and amplitude boundaries. The independent variable t is referred to as time or spatial variable. Wavelets are a specific set of basis functions applied to describe function space. They are particularly appropriate for discontinuous and irregular functions, which are common in responses of real physical systems. Bases of wavelet functions are generally well localized in the time and frequency domains. They are created by time scaling (with parameter σ) and time shifting or translation (with parameter τ) the mother wavelet $\psi(\sigma t + \tau)$, which results in a hierarchical representation of the analyzed function [7, 8].

The Continuous Wavelet Transform (CWT) of the $x(t)$ signal for a given wavelet, $\psi(t)$, is defined as [3, 9]:

$$W(\tau,\sigma) = \int x(t)\psi^{*}_{t,\sigma}(t)dt \;; \qquad \psi_{t,\sigma}(t) = \frac{1}{\sqrt{\sigma}}\psi(\frac{t-\tau}{\sigma}), \tag{1}$$

where: τ – time shifting (translation), σ –scale (frequency), ψ^{*} - conjugate wavelet.
The Discrete Wavelet Transform (DWT) can be implemented after the $x(t)$ signal is discretized, assuming that:

$$\sigma = 2^{-s}; \qquad \tau = 2^{-s}\cdot l, \tag{2}$$

where: l – translation coefficient, s –scale coefficient.
Considering the above, we get:

$$\begin{aligned} W(\tau,\sigma) &= W(l\cdot 2^{-s}, 2^{-s}) = W(l,s) = 2^{s/2}\int_{-\infty}^{\infty} x(t)\psi(2^{s}\cdot t - l)dt = \\ &= 2^{s/2}\sum_{n=0}^{N-1} x(n)\psi(2^{s}\cdot n - l). \end{aligned} \tag{3}$$

There has been hardly any research on the use of wavelet analysis to assess waviness and form profiles of machine parts. This may be due to that fact that such profiles are frequently assumed to be stationary and the Fourier transform seems sufficient to analyze them. It is also important to find out how the processes of profile decomposition and approximation affect their parameters. Profile approximation may require removing certain information-carrying details but only if their absence does not significantly change the input profile (i.e. its geometrical characteristic).

Many findings show that the mother wavelet should be carefully selected so that the profile is then decomposed and reconstructed appropriately. It is vital that further research should be carried out to determine the criterion for selecting the base wavelet (mother wavelet).

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3. Parameters used in roundness assessment

The basic parameter to assess a roundness profile is roundness deviation, *RONt*, hereafter denoted as Z_t [10]. This parameter is well-suited to evaluate form deviations in regular profiles (e.g. ovals). In practice, however, we frequently deal with complex roundness deviations, and the information provided by this parameter may be insufficient. It is thus essential to use other parameters which carry more information about a roundness profile.
Basing on the selected reference circles, i.e. LSC, MCC, MIC, MZC, one can define and compute different parameters of roundness profiles. Each reference circle is responsible for a different parameter of roundness profiles. The least squares circle (LSC) seems to be most suitable [10, 11].

3.1. Amplitude parameters

The above differences are best described by statistical parameters, i.e. the arithmetic mean deviation, Z_a and the mean square deviation, Z_q. The parameter Z_a is the arithmetic mean of the absolute deviations of a profile measured relative to the reference circle.

$$Z_a = \frac{1}{N}\sum_{i=1}^{N} | y_i |, \tag{4}$$

where: y_i – deviation of the *i-th* point of the roundness profile.

The parameter Z_q is the mean square deviation of a profile measured relative to the mean circle according to the following formula:

$$Z_q = \sqrt{\frac{\sum_{i=1}^{N} y_i^2}{N}}. \tag{5}$$

3.2. Dynamic parameters

These parameters are related to the vibration rate of the measurement tip moving across a surface. Examples include *SVU* and μPC, with the latter defined by the following formula [4]:

$$\mu PC = \frac{V_d}{n_{ov}}, \tag{6}$$

where: V_d – radial vibration rate of the measurement tip, n_{ov} – rotational speed of the object.

3.3. Parameters related to the shape of the roundness profile [4, 10]

The *skewness coefficient* of a roundness profile Z_{sk}, which is a measure of the asymmetrical amplitude distribution, is defined as:

$$Z_{sk} = \frac{1}{Z_q^3}\frac{1}{N}\sum_{i=1}^{N} y_i^2, \tag{7}$$

where: Z_q – mean square deviation of the roundness profile, y_i – deviation of the *i-th* point of the roundness profile, N - number of deviations (intervals) of the roundness profile.

The *flattening (excess) coefficient* Z_{ku} is given as:

$$Z_{ku} = \frac{1}{Z_q^4}\frac{1}{N}\sum_{i=1}^{N} y_i^4, \tag{8}$$

where Z_q, y_i and N are as defined in Eq. (7).

4. Parameters used in roundness assessment

Applying wavelet analysis to assess measurement signals requires determining the location and character of signal discontinuities and approximating the signal after total or partial removal of details selected in the decomposition process.

The approximation leads to profile smoothing and, consequently, a change in parameter values. This study aimed at determining how far, i.e. to which decomposition level, we can approximate each irregularity profile, not causing significant changes in their amplitude parameters. It was vital to assess the influence of the form of the mother wavelet on the scale of changes in the amplitude parameters.

Since surface irregularity profiles are not identical over the whole area and are measured at random, the tests were conducted using a larger sample size. Five different surfaces were measured to obtain 20 roundness profiles each (100 profiles in total).

The suitability of the mother wavelet was assessed using a special program developed in MATLAB, applying the minimum Shannon entropy (Shn) and the white noise test (WNT) [9, 12, 13]. The variation in geometry of decomposed profiles was evaluated by means of the following parameters:

- the arithmetic mean of the profile ordinates, (Z_a),
- the mean square root of profile ordinates, (Z_q),
- the total profile height (Z_t),
- the skewness coefficient of the profile ordinates (Z_{sk}),
- the kurtosis coefficient of the profile ordinates (Z_{ku}).

Five different surfaces were measured. The characteristics of the main parameter, i.e. the roundness deviation, Z_t, (*RONt* according to ISO/TS 12181-1:2003) are:

- Surface number 1: Z_t = <2.391; 7.689> µm,
- Surface number 2: Z_t = <2.112; 3.356> µm,
- Surface number 3: Z_t = <24.063; 30.195> µm,
- Surface number 4: Z_t = <2,589; 3.488> µm,
- Surface number 5: Z_t = <0.375; 1.997> µm.

Twenty profiles were measured in different sections of the surface. Each profile was decomposed and approximated with twelve forms of mother wavelets using the MATLAB environment. The variation in the selected parameters (Z_a, Z_q, Z_{sk}, Z_{ku}, Z_t) was determined with respect to the original profile. The mother wavelet was selected using two criteria: the minimum Shannon entropy (Shn) and the white noise test (WNT) [7, 9, 13].

The calculation results are presented in Table 1. The rank of each wavelet is given according to the criteria. The wavelet ranking was used to statistically compare the two criteria for wavelet selection (the minimum entropy and the white noise test) for twelve forms of mother wavelets by applying Spearman’s rank correlation coefficient. The results summarized in Table 1 indicate that the correlation between the two methods for selecting wavelets for roundness profiles are moderate according to the Guilford scale [10, 14].

Considering the scale of variation in the amplitude parameters of roundness profiles, one can state that the decomposition and approximation processes can be conducted up to level six, with no significant changes in the character of the approximated profile. This high decomposition level with no considerable changes in its basic parameters is attributable to the previous filtration of harmonic components from the original (measured) profile to a high-frequency profile. Based on five selected mother wavelets, in Figures 1–5, we present how profile approximation’s characteristics deviate from the original profile, as the level of decomposition progresses.

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Table 1. Ranking of wavelets and Spearman's correlation coefficients.

Wavelet	Number of surfaces measured										Total	
	1		2		3		4		5			
	Shn	WNT	Shn	WNT	Shn	WNT	Shn	WNT	Shn	WNT	Shn	WNT
db1	12	12	12	12	12	12	12	12	12	12	12	12
db2	7	7	7	5	8	9	8	1	7	1	5	7
db3	8	2	8	2	9	2	7	4	9	3	2	8
db4	9	5	9	6	7	10	10	6	8	6	9	9
db5	10	6	10	7	10	6	9	8	10	7	6	10
coif3	3	9	6	9	4	5	6	9	4	9	8	4
coif5	2	10	2	10	6	8	4	10	2	10	10	2
sym4	5	4	4	4	3	4	2	5	5	5	4	5
sym6	6	8	5	8	5	7	5	7	6	8	7	6
bior2.4	1	3	3	3	2	3	1	2	1	4	3	1
bior4.4	4	1	3	1	1	1	3	3	3	2	1	3
bior1.5	11	11	11	11	11	11	11	11	11	11	11	11
Spearman	0.35		0.36		0.70		0.53		0.34		0.49	

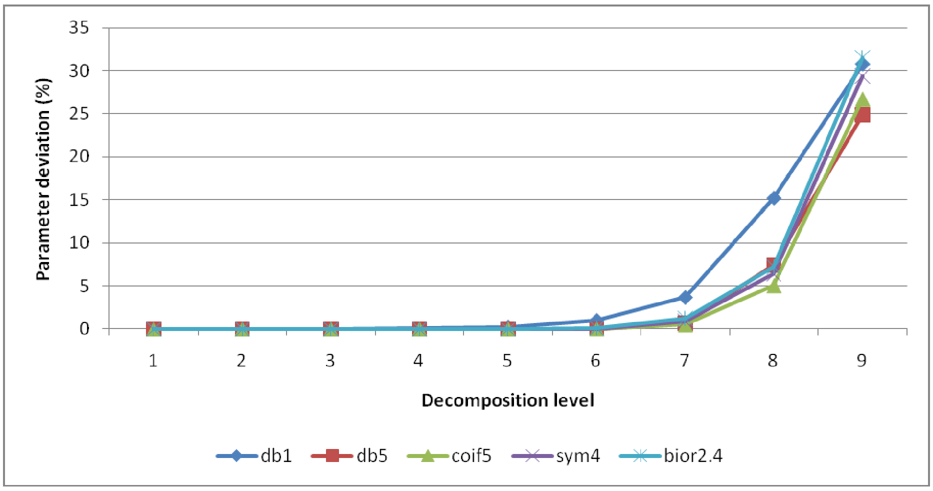

Fig. 1 Relative changes in Z_a during wavelet approximation for roundness profiles (sample: n = 100 profiles).

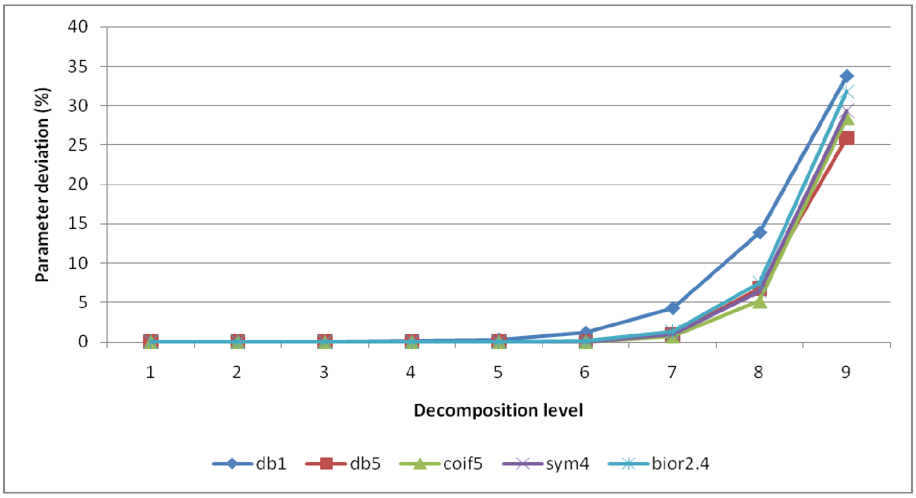

Fig. 2 Relative changes in Z_q during wavelet approximation for roundness profiles (sample: n = 100 profiles).

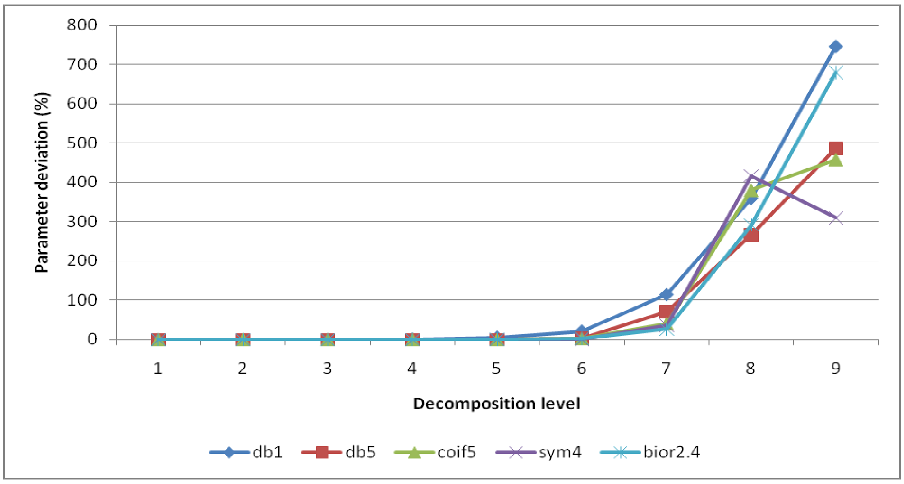

Fig. 3 Relative changes in Z_{sk} during wavelet approximation for roundness profiles (sample: n = 100 profiles).

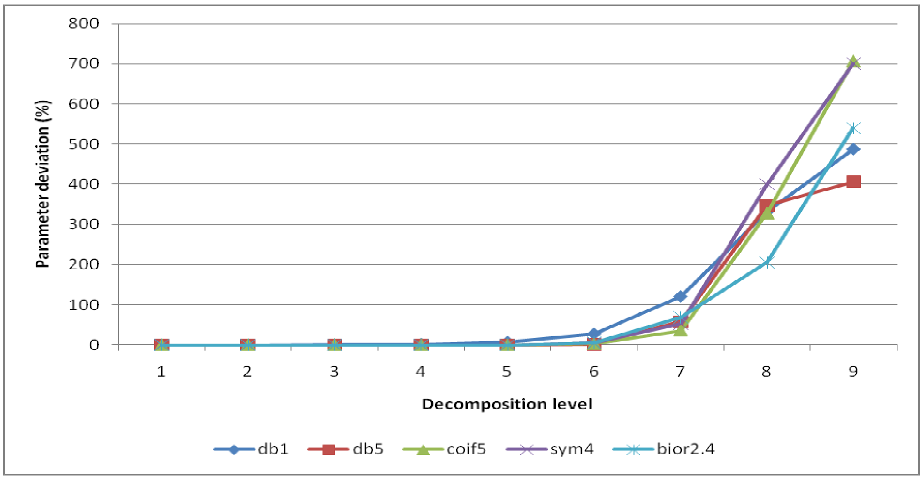

Fig. 4 Relative changes in Z_{sk} during wavelet approximation for roundness profiles (sample: n = 100 profiles).

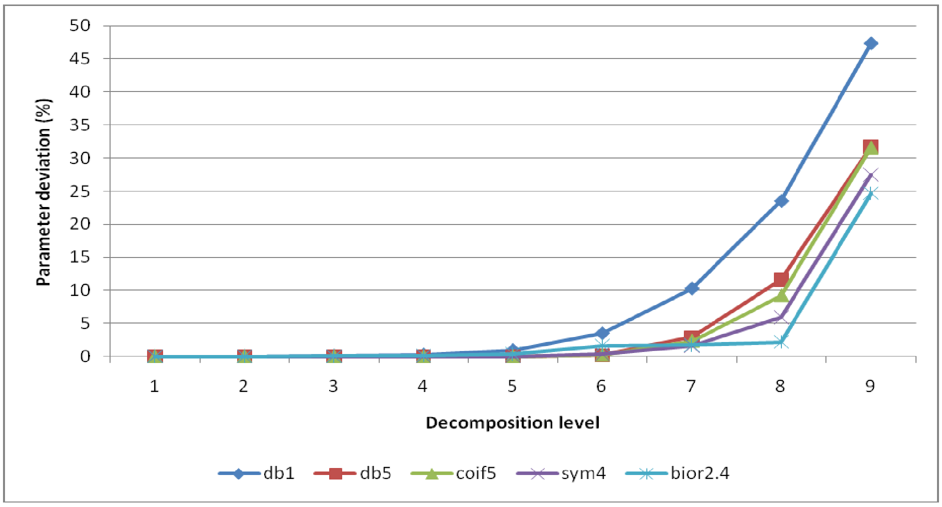

Fig. 5 Relative changes in Z_t during wavelet approximation for roundness profiles (n=100 profiles).

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5. Discussion of results

As suggested in Refs. [6, 7, 8, 9, 13], the discrete wavelet transform can be used to assess surface irregularities of machined parts, and especially to:

– identify profile deviations and establish their causes by determining the localization and character of signals (frequency and amplitude),
– identify the noise and reconstruct the input signal according to the method applied (by determining the noise form and soft or hard thresholding),
– approximate the recorded irregularity profiles using a selected mother wavelet.

In a majority of cases, the optimal wavelet is selected according to the minimum entropy criterion; no consideration is given to the way in which the characteristic parameters change during the decomposition process.

This paper reviews the findings on the selection of the form of the mother wavelet using the minimum entropy criterion. Changes in the basic parameters resulting from the decomposition process have not been analyzed yet.

Because of the stochastic nature of surface irregularities and differences in the surface texture, both closely related to the method of finishing, the analysis was conducted using a sample of 100 profiles, grouped in 5 series, obtained from five different surfaces with 20 profiles per surface.

Assuming that the absolute value of the change in each parameter does not exceed 10%, we can establish the level of decomposition. A special program was developed using MATLAB software to calculate the optimal form of the mother wavelet, which was applied to approximate roundness profiles. The results are shown in Table 2.

Table 2. Maximum decomposition level for which the change in the parameters does not exceed 10%.

Profile parameters	**Maximum decomposition level**
Arithmetic mean deviation (Z_a)	8
Mean square deviation (Z_q)	8
Total profile height (Z_t)	8
Skewness coefficient (Z_{sk})	6
Kurtosis coefficient (Z_{ku})	6

The results summarized in the above table show that the parameters which are most likely to change during the approximation process are skewness and excess.

Further analysis indicates that the quality relationships are slightly dependent on the form of the mother wavelet. The mother wavelet is responsible for the changes in the particular parameters but the differences are not significant. One can, thus, conclude that some mother wavelets are more suitable for the wavelet transform of a given profile than others. This may be due to the fact that there are many similar mother wavelets.

In order to accurately (quantitatively) assess the influence of a given form of the mother wavelet on the decomposition and approximation processes, Spearman’s rank correlation coefficient was used. This allowed us to answer the following questions:

– Is it possible to minimize the changes in the amplitude parameters by selecting an appropriate form of the mother wavelet?
– What is the correlation between the suggested method of selection of the appropriate mother wavelet aiming at the minimization of parameter changes and the standard methods based on the minimum entropy (Shn) and the white noise test (WNT)?

The answer to the question concerning the correlation between the two classic criteria for wavelet selection was provided in the previous section of this paper. According to the Guilford scale, the correlation is moderate or low.
Spearman's rank correlation coefficients are presented in Table 3.

Table 3. Spearman's rank correlation coefficients for selecting the mother wavelet for roundness profiles.

		Profile parameters					Previous criteria	
		Z_a	Z_q	Z_t	Z_{sk}	Z_{ku}	Shn	WNT
Profile parameters	Z_a	X	0.94	0.32	0.85	0.99	0.40	0.55
	Z_q	0.94	X	0.31	0.96	0.90	0.43	0.65
	Z_t	0.32	0.32	X	0.23	0.31	0.37	0.07
	Z_{sk}	0.85	0.96	0.23	X	0.78	0.45	0.78
	Z_{ku}	0.99	0.90	0.31	0.78	X	0.33	0.49

It can be concluded that for the roundness profiles analyzed here:
- there exists a significant positive correlation between Z_a, Z_q, Z_{sk}, and Z_{ku},
- there exists a low correlation between Z_t and the other parameters.

6. Conclusions

From the analytical calculations performed on the actual profiles of surface irregularities, we can conclude the following:

1. The processes of decomposition and approximation of roundness profiles should be performed up to level six to guarantee that the surface texture parameters do not change significantly.
2. It is recommended that this principle be used as a preliminary guideline; each approximation process needs to be combined with the monitoring of changes in the profile parameters.
3. The program developed at the Kielce University of Technology using the MATLAB programming environment enables quick selection of the optimal form of the mother wavelet and simultaneous monitoring of changes in the parameters characterizing the surface texture of machine parts.

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