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EXPERIMENTAL OBSERVATION OF NONLINEAR VIBRATIONS USING A CLOSED-LOOP VIBRATION SYSTEM

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Abstract

This paper presents experimental observation of nonlinear vibrations in the response of a flexible cantilever beam to transverse harmonic base excitations around its flexural mode frequencies. In the experimental setup, instead of manual control of the signal excitation frequency and amplitude, a closed-loop vibration system is used to keep the excitation amplitude constant during the frequency sweep and to increase confidence in the experimental results. The experimental results show the presence of the third mode in the response when varying the excitation frequency around the fourth mode. The frequency-response curves, response spectrum and Poincaré plots were used for characterization of nonlinear dynamic behaviour of the beam.

Keywords: nonlinear vibrations, closed-loop vibration system, nonlinear structural dynamics, cantilever beam.

1. Introduction

A system is considered linear if the characteristics of its response obey the principle of superposition. Otherwise, the system is classified as nonlinear. In the presence of nonlinearities, the response of a dynamic system may exhibit different nonlinear phenomena, including subharmonic and super-harmonic resonances, the jump phenomenon, quasi-periodic movements, modal interactions and chaos. Nonlinear analysis is discussed in detail in some books such as [1–6].

Used in the context of pendulum models, non-linear springs and beams, the Duffing oscillator is a classical system with one degree of freedom that exhibits interesting nonlinear phenomena. Nonlinear phenomena are observed in many physical systems; examples of nonlinearities in mechanical and electrical systems are reported in [2]. A flexible cantilever beam subjected to the base harmonic excitation also has been examined as a convenient structure in laboratory experiments to exhibit nonlinear phenomena [7–9]. Experimental examinations of a cantilever beam have been successfully conducted by varying one or more system control parameters, such as the frequency or amplitude of input force, and by observing the pattern of periodic response. For this purpose, the frequency-response curves are obtained by keeping the amplitude fixed and varying the frequency of the input excitation, whereas the force-response curves are obtained by varying the amplitude of the excitation and keeping the frequency fixed.

The validity and accuracy of frequency-response curves and force-response curves is directly dependent on the excitation system, sensing mechanism, and data acquisition and processing system. In the experimental examination employing a flexible cantilever beam [10–19], the experiments used a signal generator with a power amplifier providing a harmonic input signal to a shaker, enabling manual control of the signal excitation frequency and amplitude. In

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this setup there are always some difficulties to keep the excitation amplitude constant during the frequency sweep near its resonance value [20]. An alternative, in this case, is the use of a closed-loop vibration system to provide the excitation signals.

To overcome the difficulties associated with keeping the excitation amplitude constant during the frequency sweep and to increase confidence in the experimental results, this paper suggests using a closed-loop vibration system in the experimental setup. The aim of the system is to characterize nonlinear dynamics in the response of a flexible cantilever beam to the harmonic excitation. To validate this setup, we present an experimental characterization of nonlinear vibrations by using the frequency-response curves and phase-plane Poincaré map.

2. Experimental setup

The experimental setup is schematically presented in Fig. 1. An aluminium, 600 mm x 0.8 mm x 20 mm, 2024-T3 flexible cantilever beam, is vertically clamped to an LDS V450 shaker that provides the transverse base excitation of the beam. The LDS PA500 power amplifier provides power in the form of voltage and current to the shaker. The Spectral Dynamics Jaguar closed-loop vibration system ensures that what is measured by the control accelerometer is what has been programmed into the controller.

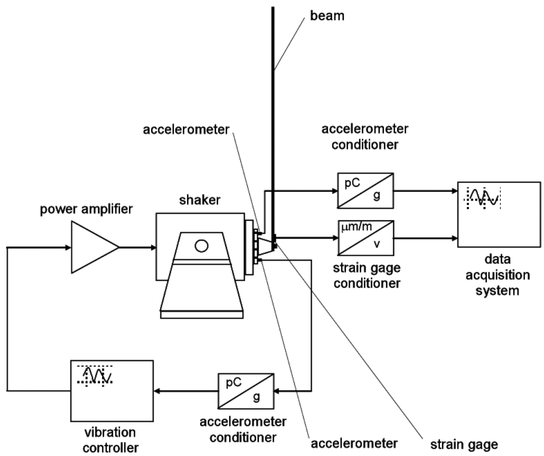

Fig. 1. The experimental setup.

The input acceleration is monitored by two Bruel and Kjaer 4371 accelerometers, used with Bruel and Kjaer 2626 conditioning amplifiers, both positioned on the beam clamping fixture. The first input accelerometer provides the signal for shaker control. The planar response of the beam is measured with a Micro-Measurements CEA-06-187UW-120 strain gage, used with a Yokogawa 3458-10 signal conditioner, positioned on the neutral axis and mounted on the fixed end of the beam, as shown in Fig. 2, where the maximum stress occurs. Considering the mass of the strain gage negligible, the total moving mass is equal to the mass of the beam.

Fig. 2. A photo of the beam clamping device.

3. Application to nonlinear response of beam

As the first part of this work, we have examined the linear natural frequencies of the beam. The measured *frequency response functions* (FRF) were estimated by using random excitation and the data acquisition system. The global excitation method was used to excite all desired modes simultaneously in the frequency range of up to 200 Hz (frequency resolution: 0.3125 Hz). The averaged FRF is shown in Fig. 3.

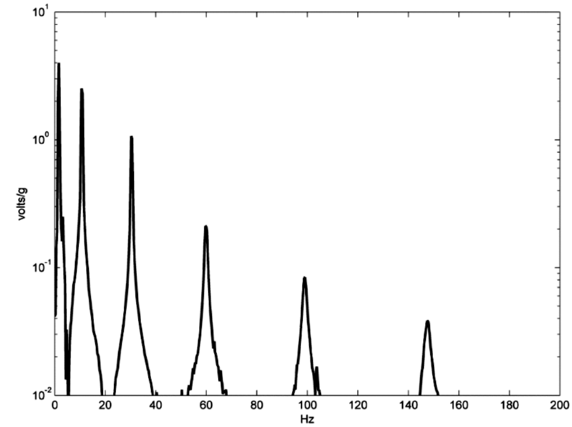

Fig. 3. The averaged FRF.

A finite element model of the beam was developed also. As pointed out in [21], an axial load changes the beam stiffness. This changing in stiffness is compensated in the modelling by adding an initial stress stiffness matrix to the original stiffness matrix. The gravity acts in the same way by adding an axial load to the beam. Its effects in the beam dynamic behaviour were analysed numerically and analytically in [22] and experimentally in [23]. The gravity load changes the values of beam natural frequencies mainly in the first modes. Its effect is as intense as the factor g^* defined as:

$$g^* = \sqrt{\frac{\rho A g L^3}{EI/(1-\nu^2)}} \,. \tag{1}$$

For the case studied in this work (E = 73.1 GPa, ρ = 2780 kg/m^3, ν = 0.33, g = 9.81 m/s^2), g^* = 1.16. In [22] it was shown that for a value close to 1 only the first mode frequency is slightly modified by the gravity effect.

The finite element model used in the analysis of the beam was composed of a total of 3000 quadrilateral linear shell elements with 6 degrees of freedom per node and 3311 nodes. The analysis was performed in ANSYS Multiphysics Release 11 [24]. The beam was clamped in one tip and the gravity load was considered as including a pre-stress effect. In Table 1 the natural frequencies calculated by the *finite element analysis* (FEA) are compared with those obtained from the averaged FRF by using the *experimental modal analysis* (EMA). Only the frequencies of modes identified experimentally are shown.

Examination of the differences between the natural frequencies obtained with FEA and EMA reveals that the differences are higher than expected for all but the first natural frequency (the most affected by inclusion of the gravity effect). The experimentally identified natural frequencies seem to be lower than expected. The non-linear softening behaviour of the structural response in resonance peaks can affect experimental identification of the natural frequencies.

Table 1. The natural frequencies of the beam.

Mode	Natural frequencies [Hz]		Difference [%]
	FEA	EMA	
1	1.65	1.67	−1.17
2	11.36	10.78	5.35
3	32.08	30.50	5.19
4	63.04	59.97	5.11
5	104.35	98.95	5.46
6	156.05	147.56	5.75

Next, we examined the nonlinear response of the beam to a harmonic excitation, by using the frequency-response curves and Poincaré map for characterization of the dynamics of the beam. The excitation frequency (Ω) was the control parameter, and the base acceleration (a_b) was held constant. Extensive experiments were performed for different excitation amplitudes in both forward and reverse sweep directions near their flexural mode frequencies. When varying the excitation frequency in the range between 61 Hz and 64 Hz, at a linear rate of 0.0005 Hz/s, in the neighbourhood of the fourth mode $w4$, we observed activation of the third mode ω_3 when the vibration controller was set to control the acceleration base amplitude at 4 g ± 0,5 dB. The results of this observation are shown in Fig. 4.

The frequency-response curve indicates that the activation of the third mode ω3 occurs for a small range of frequencies, between 61.7 Hz and 63.0 Hz during the forward sweep, and between 61.0 Hz and 63.0 Hz during the reverse sweep.

In [19], the obtained experimental results are similar to ours. The presented results were obtained for a vertically mounted carbon steel cantilever beam with dimensions similar to those used in our research. The first four experimentally measured natural frequencies of the beam are 0.66 Hz, 5.69 Hz, 16.22 Hz and 32.06 Hz. When varying the excitation frequency in the neighbourhood of the fourth mode frequency and holding the transverse base excitation of the beam constant at 0.85 g, the authors observed a motion with the third mode when the excitation frequency was between 32.74 Hz and 31.00 Hz during the downward sweep. In the experimental upward sweep in frequency, the third mode was dominant from 32.20 Hz to 32.74 Hz. In addition, a large first mode component was present in the response in both downward and upward sweeps. The authors interpreted it as a static response of the first mode due to a dynamic response of the third and fourth modes. The fourth mode had a small contribution to the response.

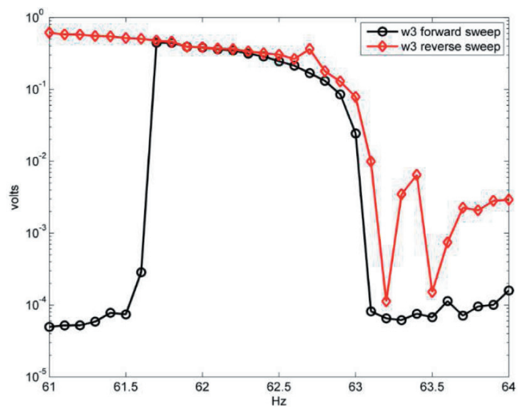

Fig. 4. The frequency-response curve of the third mode ω_3 for a_b = 4 g.

Visual inspection of the beam response indicated that the motion was dominated by the third mode ω_3. The spectrum of this response at Ω = 61.9 Hz is shown in Fig. 5, with a strong peak at 31.0 Hz and a smaller peak at 61.9 Hz, associated with the frequency of the mode ω_3 and the frequency of excitation (near the fourth mode w_4), respectively. This result suggests that the mode ω_3 was excited by the principal parametric resonance and the mode ω_4 was excited by the external resonance [3].

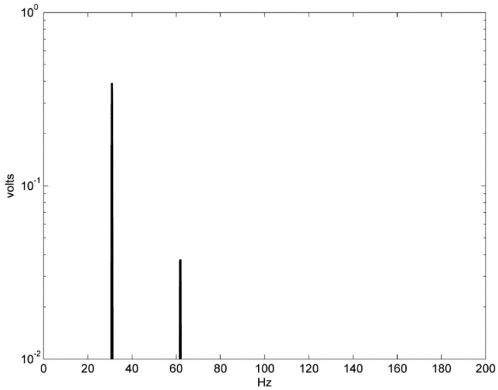

Fig. 5. The motion at Ω = 61.9 Hz for a_b = 4 g.

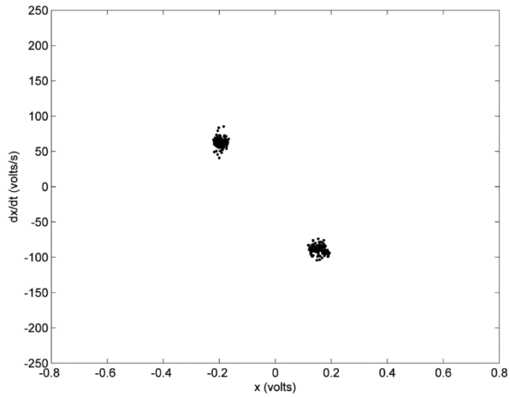

Fig. 6. The Poincaré map of the response at Ω = 61.9 Hz for a_b = 4 g.

To complete the analysis, the Poincaré map for this response is shown in Fig. 6. The map consists of two fixed points indicating that this periodic motion has a period two times greater than the period of excitation ($\omega_3 \approx \Omega/2$).

4. Conclusions

We have presented an experiment to characterize nonlinear vibrations in the response of a flexible cantilever beam to a transverse harmonic excitation, by using a closed-loop vibration system providing accurate amplitude excitation control during the frequency sweep. The experimental observations showed the presence of a periodic motion with a period two times greater than the period of excitation, when the acceleration base amplitude was set at 4.0 g ± 0,5 dB, whereas the excitation frequency was changed around the fourth natural frequency of the beam. During both downward and upward frequency sweeps the third mode was present in the motion. Thus, our experimental observations of such a periodic motion are in agreement with the experimental observations of a flexible beam reported in [19].

In addition, the experimental results indicate that the presented experimental setup can be used in examination of different structural systems to construct frequency-response curves.

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