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Metrol. Meas. Syst., Vol. XXIII (2016), No. 1, pp. 47–58.

METROLOGY AND MEASUREMENT SYSTEMS

Index 330930, ISSN 0860-8229

www.metrology.pg.gda.pl

METROLOGICAL ANALYSIS OF PRECISION OF THE SYSTEM OF DELIVERING A WATER CAPSULE FOR EXPLOSIVE PRODUCTION OF WATER AEROSOL

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Abstract

In this paper precision of the system controlling delivery by a helicopter of a water capsule designed for extinguishing large scale fires is analysed. The analysis was performed using a numerical method of distribution propagation (the Monte Carlo method) supplemented with results of application of the uncertainty propagation method. In addition, the optimum conditions for the airdrop are determined to ensure achieving the maximum area covered by the water capsule with simultaneous preserving the precision level necessary for efficient fire extinguishing.

Keywords: water capsule airdrop control system, uncertainty analysis, GUM uncertainty framework, propagation of distributions.

1. Introduction

For more than a dozen years the use of water aerosol for quenching small-scale fires proved to be extremely efficient [1−3]. Application of such aerosol for extinguishing large-scale fires, however, is a problem with a shorter history. Explosive production of aerosol necessary for such a purpose seems to be the only realistic method since a large amount of aerosol has to be generated in a short time.

The method of explosive production of water aerosol is based on detonating a pyrotechnic charge fixed inside a water bag [4]. Such a water capsule can be delivered in a short time to the location of fire by a helicopter. The system controlling the capsule's airdrop enables to release the capsule automatically at a distance which secures reaching by it the optimum point above the target at which it explodes producing aerosol [5, 6]. Such a method can be used for extinguishing large-scale forest fires or fires of objects poorly accessible for conventional firefighting equipment, like high industrial structures.

In principle, the problem of delivering a water capsule to a given point above the ground is similar to the problem of hitting a target by a bomb. Unfortunately, many papers on this subject are not easily available to a broader scientific community, especially those containing description of implementation of the weapon delivery systems and their verification. Therefore, in the paper data from experimental research of flight of containers made of flexible materials and subjected to considerable deformations are taken into consideration. The existing literature contains studies of theoretical foundations, simulations and analysis of experimental data concerning flight of practically rigid objects (bombs, fuel tanks *etc.*) [7−10]. Besides, none of

Article history: received on Jan. 26, 2015; accepted on Oct. 26, 2015; available online on Feb. 26, 2016; DOI: 10.1515/mms-2016-0006.

the available studies contains complete analysis of uncertainties of hitting the target by a released object.

The airdrop control system consists of, among others, an onboard computer that computes the capsule's trajectory and communicates with a GPS receiver registering the current position and velocity, and with a programmable fuse allowing to predetermine the moment of explosion [5, 6].

The hitting precision of a capsule in the horizontal axis of about 10 m enables to extinguish efficiently (with one airdrop) any fire with a diameter of up to 20 m as the aerosol cloud diameter is about 40 m. This guarantees immediate quenching of fires in the initial stage. Because the acceptable – from the point of view of the fire extinguishing efficiency – height of explosion above the ground is between 8 m and 16 m, it is assumed that the necessary vertical axis precision is 4 m for the optimum height of explosion equal to 12 m above the ground [6].

The experiments carried out in 2008 [5] gave an introductory confirmation of efficiency and precision of the system. The results of experiments conducted in 2009 again gave satisfactory results. A limited number of trials, however, did not enable to determine the exact motion (position and velocity) parameters of the aircraft at the airdrop moment that would guarantee determining the explosion point with sufficient precision. In this context it was decided to simulate a larger number of trials using the Monte Carlo method to estimate the uncertainty of hitting the target by a capsule (in both the horizontal and vertical axes) and to examine the influence of aircraft motion parameters on the eventual hitting precision.

2. Theoretical background

The flight of the capsule is a case of a motion of a material body in the horizontal projection in the presence of drag and horizontal ($\boldsymbol{v}_1$) and vertical ($\boldsymbol{v}_2$) air currents (Fig. 1).

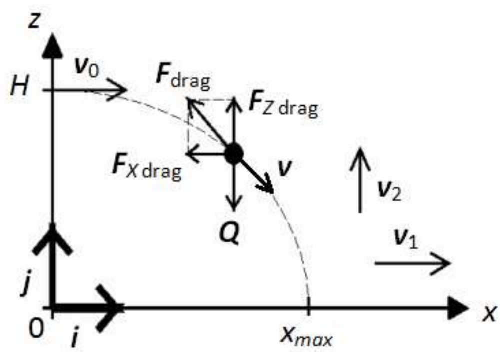

Fig. 1. The horizontal projection from the height H with the initial velocity v_0.

The drag force is determined by the following formula:

$$\boldsymbol{F}_{drag} = -\frac{c\rho A}{2}\sqrt{v_x^2 + v_z^2}\,\boldsymbol{v}, \tag{1}$$

where: c – is the aerodynamic drag coefficient; ρ – is the air density [kg/m^3]; A – is the transverse cross-section area of the capsule [m^2]; $\boldsymbol{v}$ – is the capsule's velocity vector; v_x – is the capsule's horizontal velocity component; v_z – is the capsule's vertical velocity component.

The equations of motion for the horizontal (x) and vertical (z) coordinates of force, acceleration and velocity are as follows:

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$$\begin{cases} OX: ma_x(t) = -b\sqrt{v_x^{\ 2}(t)+v_z^{\ 2}(t)}v_x(t) \\ OZ: ma_z(t) = -mg - k\sqrt{v_x^{\ 2}(t)+v_z^{\ 2}(t)}v_z(t) \end{cases}, \tag{2}$$

where: m − is the mass of the capsule [kg]; g − is Earth's gravity [m/s^2]; a_x − is the capsule's horizontal acceleration component [m/s^2]; a_z – is the capsule's vertical acceleration component [m/s^2].

The variable b denotes the generalized drag coefficient (in [kg/m]) for a body in the horizontal motion, determined from the formula $b = c_x\rho A_x/2$, where A_x is the cross-section area of the body and c_x is the drag coefficient for the body for its motion along the X (horizontal) axis; the generalized drag coefficient k for the same body in the vertical motion is determined from the same formula with parameters A_z instead of A_x and c_z instead of c_x.

The motion of the body is influenced by both the horizontal and vertical air currents characterized by their velocities (*cf.* Fig. 1) v_1 and v_2, respectively. Taking them into account one has to modify (2) thus obtaining:

$$\begin{cases} OX: ma_x(t) = -b\sqrt{v_{xx}^{\ 2}(t)+v_{zz}^{\ 2}(t)}v_{xx}(t) \\ OZ: ma_z(t) = -mg - k\sqrt{v_{xx}^{\ 2}(t)+v_{zz}^{\ 2}(t)}v_{zz}(t) \end{cases}, \tag{3}$$

where: $v_{xx}(t) = v_x(t) - v_1$ and $v_z(t) - v_2$ are the velocity differences.
The initial conditions of (3) are given as:

$$\begin{cases} v_x(0) = v_0 \\ x(0) = 0 \\ v_z(0) = 0 \\ z(0) = H \end{cases}. \tag{4}$$

3. Experimental research

Creating a system enabling to determine the right airdrop moment required working out a numerical procedure for determination of the capsule's trajectory for the measured parameters of the flight – the velocity and height above the target, as well as the generalized drag coefficients b and k [5] that had to be determined experimentally. For the latter purpose it was necessary to arrange an experimental setup for determining the position of the capsule in a two-dimensional frame of reference.

In order to attain this, a number of reference points have been chosen to guide the helicopter pilot on approaching the target. Four color flags (A–D) were used as "ticks" for marking the horizontal length scale interval (x_{scale}); for the vertical coordinate the identical role was played by the rope of the known length (z_{scale}) used for hanging the capsule under the helicopter (Fig. 2).

The trials were registered with a Phorton Ultima 1024 video-camera working in the mode of 250 frames per second. In the course of the films' analysis the position of the capsule in the two-dimensional coordinate system was determined as a discrete function of time. The analysis was performed with the VIANA computer program [11].

The measurement method was based on inspecting the film's frames one by one and determining the capsule's coordinates measured in the numbers of pixels – each frame had the form of a 1024 x 1024 bit-map.

The experimental data were used for determining, with the least square method, the third-order polynomial to be used as the analytical model for the data representation. The capsule's velocity in time was determined as the derivative of this model dependence of its position on time. In a similar way model functions for the capsule's acceleration were determined.

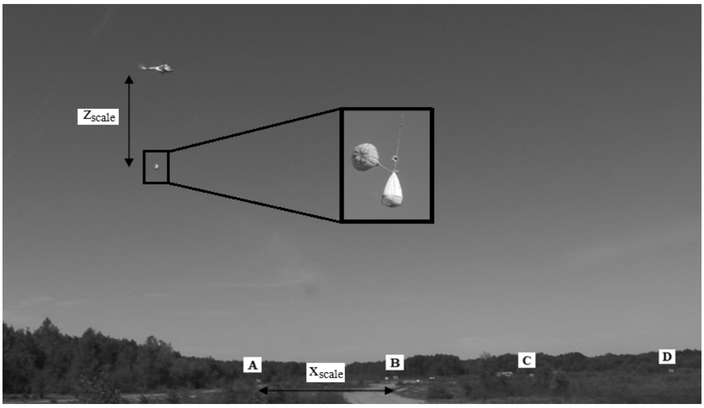

Fig. 2. Arrangement of the experimental setup during helicopter's flight. The helicopter carrying a water capsule hanging on a rope is visible on the left. The capsule is equipped with a stabilizing parachute.

At this stage of the data analysis one deals with the following set of formulae representing parameters of the capsule's trajectory as functions of time:

$$\begin{cases} x(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0 \\ v_{xx}(t) = 3a_3 t^2 + 2a_2 t + a_1 \\ a_x(t) = 6a_3 t + 2a_2 \end{cases}, \tag{5}$$

$$\begin{cases} z(t) = b_3 t^3 + b_2 t^2 + b_1 t + b_0 \\ v_{zz}(t) = 3b_3 t^2 + 2b_2 t + b_1 \\ a_z(t) = 6b_3 t + 2b_2 \end{cases}. \tag{6}$$

In Fig. 3 an example of the capsule's trajectory reconstructed from a film is shown.

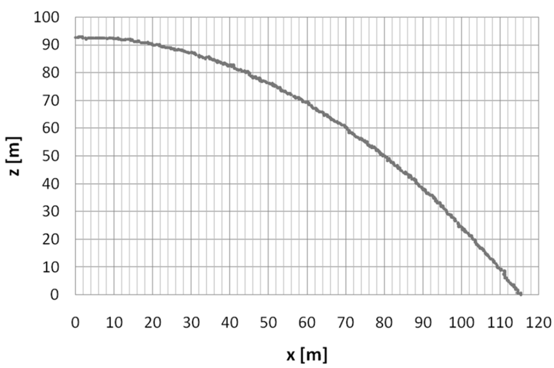

Fig. 3. A trajectory of the capsule's flight. In fact the plot represents a discrete set of points not visible at the figure's resolution.

In Fig. 4 dependencies on time of the horizontal and vertical coordinates of the capsule's position for the same trial are shown.

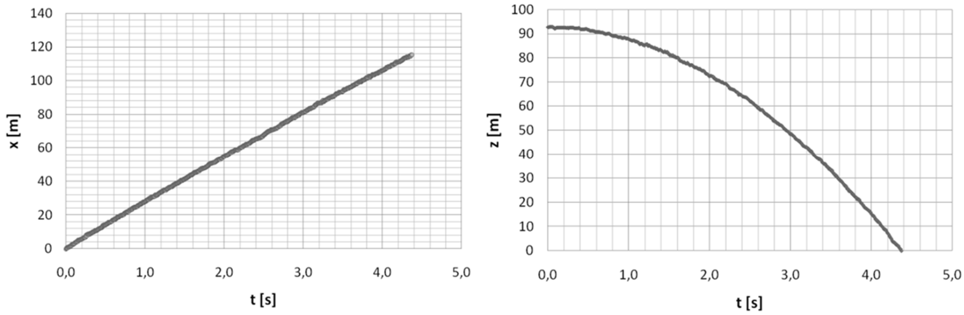

Fig. 4. Dependence on time of the capsule's horizontal and vertical coordinates.
The same comment as made for Fig. 3 applies.

For this trial the following particular third-order polynomials of time, approximating sets of points from the left and right part of Fig. 4, respectively, are obtained:

$$\begin{cases} x(t) = -0.036t^3 - 0.16t^2 + 28t + 0.61 \\ z(t) = 0.021t^3 - 4.9t^2 - 0.33t + 92 \end{cases} . \tag{7}$$

The determined coefficients R^2 of the above equations are very close to 1. All of the coefficients of the regression equations are significant because the probability of exceeding the calculated F-Snedecor value is equal to 0 (Table 1).

The drag coefficients b and k were determined using (3) after taking into account the inequalities:

$$\begin{cases} a_z < 0 \\ g < 0 \\ a_x < 0 \end{cases} \tag{8}$$

that follow from assuming the coordinate frame shown in Fig. 1.

Table 1. The parameters of the regression functions (obtained from Minitab [12]).

	Coefficient	Value of coefficient	Standard error coefficient	Probability of exceeding of calculated F-Snedecor value p(*F*)	Determination Coefficient R^2
x(*t*)	a_3	–0.0359	0.0049	0.000	0.9999
	a_2	–0.1557	0.0327	0.000	
	a_1	27.6024	0.0615	0.000	
	a_0	0.6080	0.0311	0.000	
z(*t*)	b_3	0.0210	0.0049	0.000	0.9999
	b_2	–4.8535	0.0327	0.000	
	b_1	–0.3299	0.0616	0.000	
	b_0	91.8120	0.0311	0.000	

The formulae giving the drag coefficients are as follows:

$$b = \frac{m \cdot |a_x|}{\sqrt{v_{xx}^2 + v_{zz}^2} \cdot v_{xx}}, \tag{9}$$

$$k = \frac{m\left(-\left|a_z\right| + \left|g\right|\right)}{\sqrt{v_{xx}^2 + v_{zz}^2} \cdot v_{zz}}. \tag{10}$$

The coefficients determined by such a method include information on the influence of horizontal and vertical air-currents on the capsule's motion.

After taking into account (5) and (6) that approximate dependence of the position, velocity and acceleration of the capsule on time, the formulae defining the drag coefficients assume the form:

$$b(t) = \frac{m \cdot \left|6a_3 t + 2a_2\right|}{(3a_3t^2 + 2a_2t + a_1)\sqrt{(3a_3t^2 + 2a_2t + a_1)^2 + (3b_3t^2 + 2b_2t + b_1)^2}}, \tag{11}$$

$$k(t) = \frac{m\left(-\left|6b_3t + 2b_2\right| + \left|g\right|\right)}{\left(3b_3t^2 + 2b_2t + b_1\right)\sqrt{(3a_3t^2 + 2a_2t + a_1)^2 + \left(3b_3t^2 + 2b_2t + b_1\right)^2}} \tag{12}$$

and obviously give explicitly time-dependent coefficients. For further computations, however, values of the coefficients averaged first over time for a single flight, and then over subsequent flights, were used.

4. Determination of target hitting uncertainties

For the sake of determination of values of the target hitting uncertainties and determination of the optimum values of the capsule release parameters an application working in the LabVIEW environment was created, with which numerical computations using the Monte Carlo method were executed [13–19].

The scheme of work of the application is shown in Fig. 5. In the first step the values of v_x and H are picked at random from the assumed intervals of values (to be specified in Subsection 4.1) of both the capsule's velocity and height above the ground at the moment of release. Those random values are used as estimates of the expectation values for the normal distributions with the standard uncertainties given in Table 2.

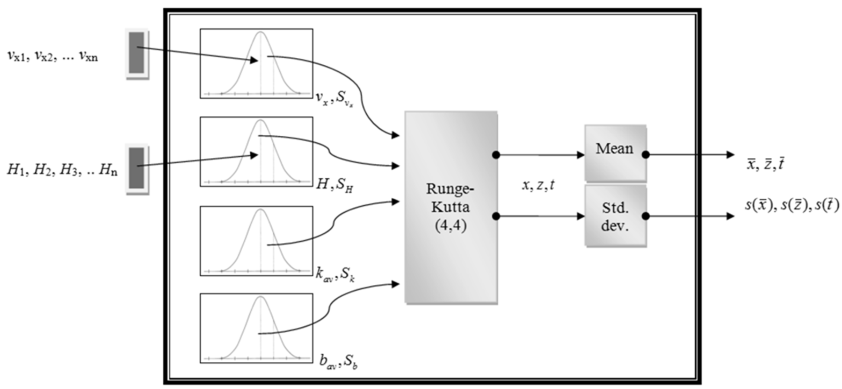

Fig. 5. A scheme of LabVIEW applications action.

In the second step, values of v_x, H, b and k are picked at random according to the corresponding distributions, and computations of the horizontal and vertical distance and of the flight's time are executed using a specific numerical procedure.

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This step is repeated 10^5 times for each pair of values picked randomly in the step one. This enables to determine standard uncertainties of the horizontal and vertical distance for given initial conditions using LabVIEW statistical functions. The application enables also to check whether the computed uncertainty values fulfill the assumptions that the value of expanded uncertainty of hitting the target in the horizontal axis $U(x)$ does not exceed 10 m and the value of expanded uncertainty of hitting the target in the vertical axis $U(z)$ is not larger than 4 m.

The expanded uncertainties are determined by the formula:

$$U(y) = k_p\, u(y), \tag{13}$$

where: k_p – is the coverage factor; $u(y)$ – is the measurement uncertainty of the variable y (determined by its standard deviation).

For the coverage probability of $p = 95\%$ and 10^5 Monte Carlo trials the determined $k_p = 1.96$ [14]. The data used in the Monte Carlo computations are shown in Table 2.

The uncertainty results of velocity and height (the vertical position coordinate) measurements are related mainly to the accuracy of the measuring apparatus – the GPS receiver. To increase the accuracy of positioning the helicopter, two GPS receivers GX1230GG from Leica, working in the difference regime, are used. One of them, serving as the reference, is located on the ground, and the moving one (rover) is installed on the board of the helicopter [5]. A typical value of the expanded uncertainty for the GPS receiver velocity is 0.2 m/s with the confidence probability of 95% [20]. Since for such a confidence probability the divisor is 2, the standard uncertainty is 0.1 m/s. The standard uncertainty value for kinematic difference measurements determined in the technical specification of the Leica 1230GG receiver is 0.02 m [21].

The average drag coefficients b_{av} and k_{av} (and their standard uncertainties) were computed from the experimental data.

Table 2. The data used in the Monte Carlo computations.

Source of uncertainty	Estimate	Probability distribution	Value of standard uncertainty
Helicopter velocity	v_x	Normal	0.1 m/s
Generalized drag coefficient (vertical component)	b_{av} = 1.14 kg/m	t-Student	0.36 kg/m
Generalized drag coefficient (horizontal component)	k_{av} = 0.86 kg/m	t-Student	0.2 kg/m
Height	H	Normal	0.02 m

In the considered case one has to do with uncertainties of types A and B. The type A uncertainty was estimated by analyzing the series of 8 experiments. The type B uncertainty results from the method of measuring the capsule's position in space and time. The estimated uncertainty of space coordinates was 0.5 m, whereas the time uncertainty was 0.004 s (the time interval between two subsequent frames). The uncertainties of velocity $u(v)$ and acceleration $u(a)$, and finally the uncertainties $u(b)$ and $u(k)$ have been estimated using formulae for the total uncertainty.

Because the type A and B uncertainties had comparable values, and due to a small number of observations, the Student's t-distribution was used, and the total uncertainty values shown in the table were determined as the geometric sum of the type A and B uncertainties [13, 16].

4.1. Determination of the optimum values of capsule release parameters

The computations have been carried out for the range of v_x values from 1 m/s to 100 m/s and values of height from H = 20 m to H = 250 m from which 5000 pairs of values were picked at random and used as the input data. The overall time of computations with a Pentium i5 processor was about 40 hours.

The plot of pairs of variables v_x and H for which the imposed precision conditions described in section 1 and 4 are fulfilled, is shown in Fig. 6. As is clear, the limiting value of H is close to 180 m. The largest value of v_x for the minimum assumed height can reach about 72 m/s.

The straight line connecting the two limiting points determines a simplified limitation for the flight parameters of the aircraft (Fig. 6). The limitation is not absolute as some (v_x, H) pairs fulfilling criteria of acceptable uncertainties are located above the line. The exact limitation, however, cannot be described by a simple formula and would not be practically applicable for correcting flight parameters before the airdrop.

The equation of the limiting straight line has the form:

$$H = -2.36 \cdot v_x + 189 . \tag{14}$$

The set of points (velocities and corresponding heights of the release point) obtained by such a procedure was used to determine the capsule's flight range (triangular points in Fig. 6 forming a parabola-like "curve"). The maximum value can reach x = 163 m for a certain velocity from the interval between v_x = 47 m/s (H = 78.08 m) and v_x = 53 m/s (H = 63.92 m).

For the points lying on the straight line represented by the (14) the capsule's position uncertainty have been computed using two alternative methods. The uncertainty computations were executed using a modified version of the application shown in Fig. 6.

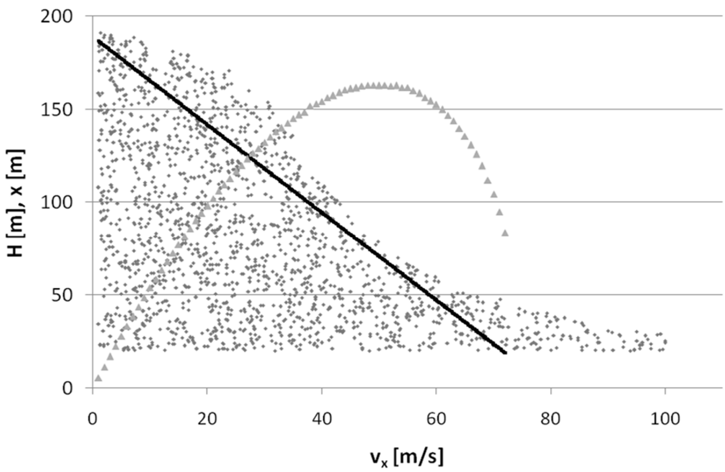

Fig. 6. A cloud of points fulfilling the accuracy criterion of the capsule airdrop system (light-grey diamonds), the straight line described by (14) (black line), and the dependence of the airdrop range on the flight velocity (as well as the corresponding height of the release point) at the moment of airbag release for the points described by (14).

4.2. Estimating the capsule's position uncertainty using the method of propagation of probability distributions

For each pair of points (v_x, H) (lying on the straight line given by (14)) 10^6 points were computed using the Monte Carlo method, which gave a set of values of $\bar{x}, \bar{z}, \bar{t}$, as well as $U(x)$

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and $U(z)$. The obtained values of uncertainty $U(x)$ varied between 1.2 m and 9.5 m, whereas those of $U(z)$ – between 0.1 m and 3.9 m, depending on the capsule's initial velocity and the height of the release point. The uncertainty $U(x)$ increases with the initial velocity and $U(z)$ with the height of the airdrop.

4.3. Estimating the uncertainty of the capsule's position at the target using the method of uncertainty propagation

It is impossible to compute the uncertainty of hitting the target by the capsule according to the recommendations given by the GUM guidebook [13], due to the lack of analytical solutions of the system of (3). The uncertainties can be estimated, instead, using the set of approximate equations:

$$\begin{cases} ma_x(t) = -b(v_x(t))^2 \\ ma_z(t) = -mg + k(v_z(t))^2 \end{cases}. \tag{15}$$

Since variables for the x and z coordinates in (15) are not coupled, each of the equations can be integrated separately, thus obtaining:

$$\begin{cases} x(t) = \dfrac{m}{b}\ln\left(\dfrac{bv_x}{m}t+1\right) \\ z(t) = \sqrt{\dfrac{gm}{k}}\left(-t+\sqrt{\dfrac{m}{kg}}\ln\left(\dfrac{2e^{\sqrt{\frac{4kg}{m}}t}}{e^{\sqrt{\frac{4kg}{m}}t}+1}\right)\right)+H \end{cases}. \tag{16}$$

The estimates of the position uncertainty for the axes 0X and 0Y, respectively, can be obtained by computation of corresponding composite standard uncertainties according to the formulae:

$$u_c(x) = \sqrt{\left(\frac{\partial x}{\partial v_x}\right)^2 u^2(v_x) + \left(\frac{\partial x}{\partial b}\right)^2 u^2(b)}, \tag{17}$$

$$u_c(z) = \sqrt{\left(\frac{\partial z}{\partial H}\right)^2 u^2(H) + \left(\frac{\partial z}{\partial k}\right)^2 u^2(k)}, \tag{18}$$

where the sensitivity coefficients are defined by:

$$\frac{\partial x}{\partial v_x} = \frac{t}{\left(\dfrac{bv_x}{m}t+1\right)}, \tag{19}$$

$$\frac{\partial x}{\partial b} = -\frac{m}{b^2}\ln\left(\frac{bv_x}{m}t+1\right) + \frac{v_x t}{b\left(\dfrac{bv_x}{m}t+1\right)}, \tag{20}$$

$$\frac{\partial z}{\partial H} = 1, \tag{21}$$

$$\frac{\partial z}{\partial k}=t\frac{1}{2\sqrt{\frac{gm}{k}}}\frac{gm}{k^2}-\frac{m}{k^2}\ln\left(\frac{2e^{\sqrt{\frac{4kg}{m}}t}}{e^{\sqrt{\frac{4kg}{m}}t}+1}\right)+\frac{1}{k\left(\frac{2e^{\sqrt{\frac{4kg}{m}}t}}{e^{\sqrt{\frac{4kg}{m}}t}+1}\right)}\frac{4gte^{\sqrt{\frac{4kg}{m}}t}}{\sqrt{\frac{4kg}{m}}\left(e^{\sqrt{\frac{4kg}{m}}t}+1\right)^2}. \tag{22}$$

The expansion coefficients were determined on the basis of the number of effective degrees of freedom computed using the Welch-Satterthwait formula [16] for the confidence level 95%. The results obtained in this way served as the basis for computing the expanded uncertainties. Depending on the initial velocity of the capsule and the airdrop height the $U(x)$ values range from 1.7 m to 15.1 m, whereas $U(z)$ – from 0.1 m to 3.8 m.

The maximum values of $U(x)$ and $U(z)$ obtained with the Monte Carlo method fulfill the accuracy criteria determined in Section 1, while those obtained with the second method fail to fulfill those criteria. One should remember, however, that the latter uncertainties have been determined from the approximate equations of motion for the capsule's flight.

Below the results of computations of expanded uncertainties performed with both methods for a typical cruise speed of the W3 Sokół helicopter used during practical field tests, *i.e.*, v = 29 m/s and the corresponding airdrop height obtained from (14), *i.e.*, H = 120.6 m, are shown. Computations with the Monte Carlo method (for 10^6 trials) gave the following results: x = 129.1 m, z = 112.9 m, t = 4.799 s, $U(x)$ = 6.6 m and $U(z)$ = 2.3 m.

For the purpose of computing the analogous values with the distribution propagation method the uncertainty budgets for the flight range for both the horizontal and vertical axes, obtained from the data contained in Table 2, were exposed. All the data, as well as the dependences given by (16–22) were inserted into the application code LabVIEW.

Table 3. The uncertainty budget for position along the horizontal axis.

Source of uncertainty	**Value of uncertainty**	**Type of probability distribution**	**Divisor**	**Sensitivity coefficient**	**Degrees of freedom**	**Standard uncertainty**
Helicopter velocity	0.2 m/s	Normal	2	4.24 s	∞	0.42 m
Generalized drag coefficient (horiz. comp.)	0.36 kg/m	t-Student	1	–6.83 m²/kg	7	2.46 m
Combined standard uncertainty						$u_c(x)$ = 2.50 m
Effective degree of freedom					7.42	
Coverage factor					2.36	
Expanded uncertainty					$U(x)$ = 2.36 × 2.50 m = 5.9 m	

Table 4. The uncertainty budget for position along the vertical axis.

Source of uncertainty	**Value of uncertainty**	**Type of probability distribution**	**Divisor**	**Sensitivity coefficient**	**Degrees of freedom**	**Standard uncertainty**
Height	0.02 m	Normal	1	1	∞	0.02 m
Generalized drag coefficient (vert. comp.)	0.2 kg/m	t-Student	1	3.26 m²/kg	7	0.65 m
Combined standard uncertainty						$u_c(z)$ = 0.65 m
Effective degree of freedom					7.01	
Coverage factor					2.36	
Expanded uncertainty					$U(z)$ = 2.36 × 0.65 m = 1.5 m	

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The computations using the distribution propagation method gave the following values (Tables 3 and 4): x = 130.7 m, z = 110.1 m, $U(x)$ = 5.9 m, $U(z)$ = 1.5 m. They are close to the values obtained with the Monte Carlo method, and the discrepancies result from the inaccuracy of the mathematical model (15). Nonetheless, the values of expanded uncertainties of the capsule's position obtained with both methods match the accuracy criteria determined in Section 1.

5. Summary

The above discussed computations enabled to determine parameters of approaching the target by the helicopter. To hit the target the flight velocity and its height above the target have to fall into the set shown in Fig. 6 (cloud of grey points). Since verification of fulfillment of the exact criterion is difficult, a simplified method, based on the equation of the approximating limiting straight line (14) can be used. Such a method can be very easily implemented for computer operations and its principal virtue is its speed. If the information indicating that the input conditions are not fulfilled appears in the course of flight at the console of either the pilot or the airdrop system operator, the pilot is expected either to reduce the flight's height and/or velocity.

It should be stressed that the obtained values of parameters are based on earlier testing-ground trials. The target hitting accuracy is determined mainly by the drag coefficients b and k that are estimated with considerable uncertainties (Table 2). Had these uncertainties been smaller the acceptable heights and velocities of flight and the set of acceptable points would have become larger and a substantial increase of the range of the capsule's flight would have been an important consequence.

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