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THE INFLUENCE OF A RAIL LATERAL BENDING ON THE STRESS – STRAIN STATE OF A WHEEL - RAIL CONTACT

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Summary. The aim of the article is to evaluate the influence of a rail lateral bending on wheel – rail contact interaction. At first the rail lateral bending is modeled using FEM and then the normal contact problem is solved with and without results obtained; the simulation results are given.

Key words: rail lateral bending, wheel – rail contact, normal problem

INTRODUCTION

Traditionally the wheel – rail contact is divided into the normal and the tangential problems. The aim of solving the normal problem is contact patch shape and size detection, and also normal pressure distribution within, while the aim of solving the tangential problem – wheel – rail coupling force definition using data achieved from normal program solution. This division is usually justified because the friction has negligible influence on contact patch size and pressure distribution if the bodies are treated as elastic ones. The division is necessary for the simplification of the solution since the treatment of the contact problem in general case, when the contact area is not known a priori, is still not achieved.

Only a finite number of treatises is known, that describe the approximative analytical solutions of different contact problem classes [Hertz 1881], [Carter 1926], [Cattaneo 1938], [Mindlin 1949], [Mossakovskiy 1956]. Therefore the various numerical methods are used to solve the wheel – rail contact problem today: variational and non-variational methods, and also finite and boundary elements methods.

The variational principle use the modern variations calculus ideas and methods. Its foundations were laid by Signorini [Signorini 1955] and for elastic bodies in contact are advanced by Kalker [Kalker 1990], Golubenko [Golubenko 1993], Boucly and Nelias [Boucly 2007]. Despite the variational inequalities theory progress, the solution of the contact problem entails great difficulties: the problem is posed in a three-dimensional formulation; when replacing the covariance inequalities by the sequence of variational ones, that have an equivalent extremal formulation, one need to solve the linear programming task (which is complicated itself) for several times to obtain solution

The nonvariational principle has a basic in a classical contact problem formulation in the form of equality and inequality constraints on contact surface. A search of solution for a contact problem may represent a sequence of elasticity theory problems with qualifying boundary conditions, that defines the terms of contact interaction (works [Johnson 1985.], [Kostyukevich 1991], [Yazykov 2004], [Bokiy 2006]). The disadvantage of this approach is that to obtain a solution one need to solve the elasticity theory problem for several times. A convergence of the iterative process for obtaining a solution is not theoretically proved, though it turns out well to get a numerical result with a desirable precision in practice.

Due to the increase in efficiency of modern computing machinery during the last two decades the finite elements method (FEM) is wide used for simulating a wheel – rail contact (works [Telliskivi 2001], [Damme 2006], [Zhao 2009]). The main advantages of FEM are: highly realistic results can be obtained; no restrictions on geometry of contact surfaces; complex material behaviour models. However grids in FEM models contain tens and even hundreds thousand of nodes, that make calculations sufficiently time consuming.

Boundary elements method (BEM) is extremely suitable for contact modeling, because unlike FEM only surfaces of contacting bodies have to be discretized. Besides that BEM is semy-analytical, that make it more accurate, especially for high stress concentration provlems. However even that the quantity of computational nodes in the grid is much smaller than in FEM, the matrixes are non – symmetrical and dense, that makes calculations time consuming too. An application of BEM for wheel – rail contact problems is studied in [Rudas 2000], [Abascal 2010].

As it can be seen, a wide range of contact models exist to define the wheel-rail interaction. Having the aim to compare accuracy and efficiency of existing theoretical models of wheel – rail contact and those that will be developed, a group of researchers of Manchester Metropolitan University have proposed contact benchmark [Iwnicki 2006]. The benchmark consists of prescribed single wheel or wheelset contact study and dynamical vehicle simulation. According to benchmark, normal and tangential contact problems are considered. For normal contact problem the inputs are the wheel and rail profiles and their mutual orientation (lateral displacement and yaw angle), and vertical load on wheelset. However the rail lateral bending when the wheelset balances in track gauge is not provided in. At the same time it is well known from literature that the lateral load from wheel to rail can obtain values 30 – 40 kN even on straight track. This paper aims to evaluate the rail lateral bending influence on wheel – rail contact.

WHEEL – RAIL CONTACT MODELING

Rail lateral bending was simulated using Ansys FEM software [Ansys]. The 3D model of UIC60 rail having length 1m was developed. The obtained value was meshed with 3-D 10-node tetrahedral structural solid elements Solid92. To avoid rail plastic deformation , in the area of load application (middle of the rail) the mesh was refined.

The lateral force of 30 kN was uniformly distributed on small part of rail surface (see fig.1).

The structural FEM analyses was performed with created model. The maximal lateral displacement was obtained in the top point of middle section and has value of 0,2435 mm.

The normal contact problem was solved using the modified method [Bokiy 2006], assuming frictionless contact.

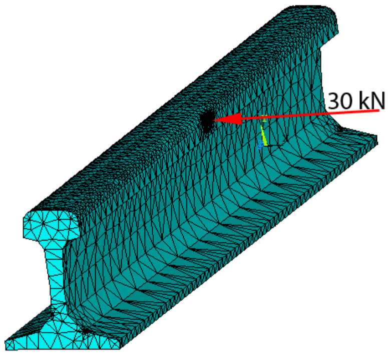

Fig. 1. 3D FEM model of the rail

Let's consider contact interaction of two elastic bodies, each of them is connected with rigid body – rigid support. It is accepted that we can assume the contact surface is flat at any moment t of interaction process and lays in a common tangent plane π, which passes through the initial contact point O. It is assumed that wave and inertial effects are negligible. The interaction is defined with $\Delta_z(t)$ function, which represents forward approach of rigid supports.

Let's introduce $Oxyz$ Cartesian coordinate system, which is linked to lower body ($i=1$). Let's put the origin to O, Ox and Oy axes are placed in π, Oz axis points inside the lower body..

Let's denote normal contact pressure as $P_z(s,t)$; and $w(s,t)$ is a relative displacement function of interacting bodies along z axis, defined in s point:

$$w(s,t) = w_1(s,t) - w_2(s,t) + f(s) - \Delta_z(t)\,, \tag{1}$$

where: $w_i(s,t)$ - elastic displacements of bodies surfaces; $f(s)$ - initial gap between the bodies. Then the contact interaction condition have the form:

$$w(s,t) \geq 0\,,\; P(s,t) \geq 0\,,\; P(s,t)w(s,t) = 0\,,\; s \in \Omega\,,\; t \in [0,T]\,. \tag{2}$$

Here Ω is assumed contact area.

Let's assume that following relationship takes place:

$$w_1 - w_2 = AP_z \tag{3}$$

A is a linear integral operator with integration domain W . If we approximate bodies with elastic half-spaces , then kernels are defined with Boussinesque-Cerrutti formulas for unit load acting upon the elastic half-space. Then (1) takes form:

$$w(s,t) = AP_z + f(s) - \Delta_z(t) \ . \qquad (4)$$

If we substitute the above expression of $w(s,t)$ in (2), we will get the relationship, which $P_z(s,t)$ must be satisfied. This relationship is equal to linear operator equation relative to $P_z(s,t)$:

$$P_z(x,y) = h(P_z - ED(P_z)) \ ,$$
$$D(P_z) = AP_z + f(x,y) - \Delta_z(t) \ , \qquad (5)$$
$$h(\gamma) = \begin{cases} \gamma, & \gamma \geq 0 \\ 0, & \gamma < 0 \end{cases}$$

where: $x, y \in W$, $E(x,y)$ - arbitrary positive function.

The contact pressure determination came to finding $P_z(x,y,t)$ function, defined on set $\Omega \times [0,T]$, which satisfies (5) and initial conditions: $P_z(x,y,0) = 0$ for all $(x,y) \in \Omega$; $\Delta(0) = 0$.

To get the approximate solution of (5) let's proceed to it's discrete analogue. Let's divide the loading process $[0,T]$ into l intervals $(t_0,t_1),(t_1,t_2)$, …, (t_{l-1},t_l) . The assumed contact area Ω is covered with grid which consists of N similar quadric elements Ω_i ($i = \overline{1,N}$) with sides parallel to Ox , Oy axes. The normal contact pressure $p_i(t_m)$ and also the corresponding elastic displacements on every boundary element Ω_i in time t_m are constant within the element and equal to values in (x_i, y_i) - Ω_i elements center.

Based on the discretization made and taking into the account that the normal problem solution under continuous loading doesn't depend on loading history, for contact pressure definition in time t_m we arrive to the next system of equations :

$$p_i(t_m) = h(\gamma_i(t_m)) \ ,$$
$$\gamma_i(t_m) = p_i(t_m) - E_i\left(\sum_{k=1}^{N} a_{i,k}\, p_k(t_m) + g_i(t_m)\right) \ , \qquad (7)$$
$$g_i(t_m) = f(x_i, y_i) - \Delta_z(t_m) \ ,$$

where: $i = \overline{1,N}$, $m = \overline{1,l}$, $E_i > 0$, a_{kd} are the coefficients of flexibility matrix, defined according to A kernel formulas. If $i = j$ and quadric boundary element Ω_i with side h :

$$a_{i,i} = 4c_1 \ln(1 + \sqrt{2})$$

If $i \neq j$ then the distributed load on element Ω_i is replaced with resultant force acting on the element's center:

$$a_{i,j} = \frac{c_1 \omega}{\rho_{ij}} \ ,$$

where: $i, j = \overline{1, N}$, $w = mes(\Omega_i)$, $\rho_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, $c_1 = 2\dfrac{1-\nu^2}{\pi E}$.

For solving the system of equation (7) we can apply nonlinear analogue of Seidel method for linear equations system. Let’s assume that on $(m-1)$ step the contact pressures are known and equal to $p_k(t_{m-1})$, $(k = \overline{1, N})$, and $E_i = 1/a_{i,i}$ ($i = \overline{1, N}$), then on m step contact pressures $p_k(t_m)$ can be found using the following iterative process:

$$p_i^{n+1}(t_m) = h(\gamma_i^{n+1}(t_m))$$

$$\gamma_i^{n+1} = -\frac{1}{a_{i,i}}\left(\sum_{k=1}^{i-1} a_{i,k}\, p_k^{n+1}(t_m) + \sum_{k=i+1}^{N} a_{i,k}\, p_k^{n}(t_m) + g_i(t_m)\right)$$

$$g_i(t_m) = f(x_i, y_i) - \Delta_z(t_m)$$

As a criterion of stopping the iteration process on each load step is suitable to use rms difference

$$\sqrt{\frac{1}{N}\sum_{k=1}^{N}(p_k^{n+1}(t_m) - p_k^{n}(t_m))} \le \varepsilon$$

The given algorithm of solving the normal contact problem was realized as a software in C++ Buider 6.0 programming environment.

For a numerical simulation the wheel S1002 and rail UIC60 profiles from Manchester Contact Benchmark were used. Those profiles are depicted on fig.2. The other inputs are: Wheel rolling radius=460 mm, Gauge width=1435 mm, Flange-back spacing=1360, Vertical load=100 kN, Young’s modulus E = 2.1×10^{11} Па, Poisson ratio ν =0.28.

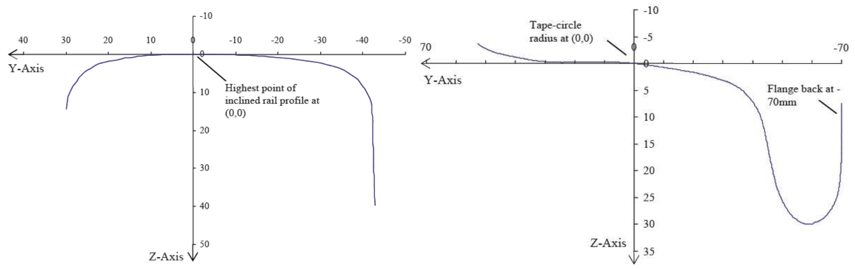

Fig. 2. Wheel and rail profiles [Iwnicki 2006]

The initial contact points locations were defined using algorithm introduced in [Kostyukevich 1991].

The simulation results are shown on fig.3. The wheel and rail profiles without rail bending are drawn with a solid green line, and the one with rail bending with a dashed gray line.. It must be admitted that the changing in position of wheel profile is connected with the lateral rolling motion of the wheelset. The points of initial contact with and without bending are marked with maroon circles.

As it can be seen from the figure, a rail bending has a significant impact upon the size and a shape of a contact patch. In Case 1 (without rail bending) the maximum pressure is 1175 MPa, the contact patch area - 185 mm^2. In Case 2 (with rail bending) the maximum pressure is 1330 MPa, , the contact patch area - 127 mm^2. Hence, the difference between the contact patches' area exceeds 30%.

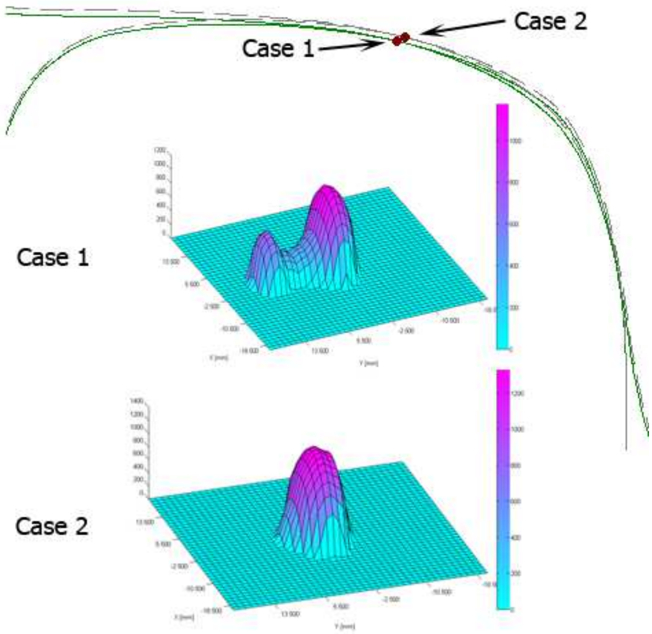

Fig.3. The normal problem solution results (1 –without rail bending, 2 – with one)

CONCLUSIONS

The mathematical model of normal contact between the wheel and the rail is developed. It is shown that the solution of the normal contact problem without rail lateral bending may lead to significant (over 30%) errors in contact area detection.

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ВЛИЯНИЕ БОКОВОГО ОТЖАТИЯ РЕЛЬСА НА НАПРЯЖЕННО – ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ В КОНТАКТЕ «КОЛЕСО - РЕЛЬС»

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Илья Цыгановский, Владимир Ноженко

Аннотация. Целью данной статьи являтся оценка бокового отжатия рельса на процесс взаимодействия колеса с рельсом. Сначала боковое отжатие рельса моделируется с помощью метода конечных элементов, а затем решается нормальная контактная задача с учетом полученных результатов и без; приведены результаты численного моделирования.

Ключевые слова: боковое отжатие рельса, контакт «колесо - рельс», нормальная задача