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DETERMINATION OF THE EXPONENT *n* IN THE EQUATION DESCRIBING A CONSTANT-RESISTANCE ANEMOMETER

WYZNACZENIE WYKŁADNIKA *n* W RÓWNANIU OPISUJĄCYM PRACĘ ANEMOMETRU STAŁOREZYSTANCYJNEGO

In order to measure the velocity of gas flow with a constant-resistance thermoanemometer, prior calibration of anemometer sensor wires is required. In this process three sensor-specific parameters of King's equation are estimated. In the original form of the equation the parameters are not independent from each other. As has been demonstrated by Ligęza, it is possible to rewrite the equation in a dimensionless form, in which the parameters become independent. Here we provide an algorithm to derive the parameters from thermoanemometer measurements.

Keywords: hot-wire anemometer, constant-resistance anemometer, King's law

Przy wzorcowaniu sond termoanemometrycznych np. do pomiarów wentylacyjnych kopalni, często zachodzi konieczność wyznaczenia parametrów krzywych wzorcowania czujników a w szczególności wartości parametrów w równaniach (1) czy (2). W pracy przedstawiono nowe podejście do tego zagadnienia pozwalające na niezależne wyznaczenie wszystkich parametrów.

Słowa kluczowe: anemometr cieplny, anemometr stałorezystancyjny, prawo Kinga

1. Introduction

A constant-resistance anemometer may be described by King's (1914) equation which relates power dissipated by the anemometer sensor wire with the velocity v of the cooling fluid:

$$I_w^2(v) = (a + b\sqrt{v})(1 - \frac{1}{N}) \tag{1}$$

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Recently, a generalized form of the equation is used

$$I_w^2(v) = (a + bv^n)(1 - \frac{1}{N}) \tag{2}$$

where I_w denotes current in the sensor wire, a and b are sensor calibration parameters, $N = R_w/R_g$ is a predefined overheat ratio of sensor resistance at work-temperature R_w to resistance at calibration-temperature R_g, and finally n is an exponent of a value close to 0.5, as reported by different authors.

Ligęza (2005) has proposed another form of the above equations:

$$I_w^2(v) = I_k^2(0)(1 - \frac{1}{N})\left[1 + (\frac{v}{v_k})^n\right] \tag{3}$$

where $I_k^2 = a$ is a critical current at which the sensor resistance grows to infinity given no gas flow (velocity $v = 0$), and

$$v_k = (\frac{a}{b})^{\frac{1}{n}} \tag{4}$$

Then, Eq. (3) might be written in a dimensionless form:

$$\frac{N}{N-1}\frac{I_w^2}{I_k^2} - 1 = (\frac{v}{v_k})^n \tag{5}$$

where the current variables are separated from the flow velocity variables.

The right side of Eq. (5) is always equal 1 when

$$v = v_k \tag{6}$$

and then

$$\frac{N}{N-1}\frac{I_w^2(v)}{I_k^2} = 2 \tag{7}$$

and finally

$$I_w(v = v_k) = I_k\sqrt{2\frac{N-1}{N}} \tag{8}$$

Moreover, Eq. (5) for $v = 0$:

$$I_w^2(0) = I_k^2(1 - \frac{1}{N}) \tag{9}$$

leads to the overheat ratio reciprocal 1/N equal

$$\frac{1}{N} = 1 - \frac{I_w^2}{I_k^2} \tag{10}$$

Using the above equations, the following experiment can be used to calibrate a sensor wire. First, for no gas flow ($v = 0$), the current I_w is measured for multiple different values of the overheat coefficient N. Based on Eq. (10) the relation $f(\frac{1}{N}, I_w^2)$ between the overheat ratio reciprocal and square of wire current is linear. The point at which the line crosses the I_w^2 axis corresponds to I_k^2 (see Fig. 1).

Next, for known N and corresponding value I_w^2, I_k^2 can be derived from Eq. (10):

$$I_k^2 = I_w^2 \frac{N}{N-1} \tag{11}$$

If I_k^2 is known a new function $F(v, N)$ can be constructed out of the left side of Eq. (5):

$$F(v,N) = \frac{N}{N-1} \frac{I_w^2(v)}{I_k^2} - 1 \tag{12}$$

When for given v and N the value of the function is equal 1, then based on Eq. (5) the equation $v_k = v$ must be true independently on the value of n.

The value of the exponent n can be calculated by taking logarithm of Eq. (5):

$$\ln(F(v,N)) = n \ln(\frac{v}{v_k}) \tag{13}$$

and consequently

$$n(v,N) = \frac{\ln(F(v,N))}{\ln(\frac{v}{v_k})} \tag{14}$$

2. Fitting the model to experimental data

In order to verify the model, a constant-resistance anemometer system has been built and installed in a flow tunnel (provided by TSI, model 1129 (TSI Model 1129)). The sensor has been made of a tungsten wire of 2 mm length and 5 µm diameter and displayed room-temperature resistance of $R_0 = 5.49\ \Omega$. The flowing fluid was air of temperature 26°C. Sensor wire currents $I(0)$ were measured for different air flow velocities v and for three different overheat ratios of $N = 2$, 1.8 and 1.6. All measurements are listed in Tab. 1.

TABLE 1

Experimental data. Currents in the sensor $I_w(v)$ were measured for different air speeds v. The experiment was run for three overheat ratios N

N = 2		**N = 1.8**		**N = 1.6**	
v [m/s]	**I [mA]**	**v [m/s]**	**I [mA]**	**v [m/s]**	**I [mA]**
0.075	44.750	0.073	41.943	0.072	37.831
0.222	46.889	0.221	44.063	0.221	40.014
0.411	48.823	0.411	45.990	0.412	41.806
0.610	50.479	0.610	47.571	0.610	43.307
0.809	51.844	0.809	48.891	0.809	44.528
1.011	52.992	1.011	49.999	1.011	45.565
1.215	53.961	1.215	50.942	1.215	46.444
1.416	54.802	1.416	51.754	1.415	47.183
1.616	55.551	1.616	52.471	1.615	47.851
1.826	56.243	1.826	53.138	1.825	48.477
2.029	56.864	2.028	53.728	2.027	49.035
2.230	57.439	2.230	54.288	2.229	49.560
2.436	57.989	2.435	54.810	2.434	50.046
2.635	58.499	2.634	55.300	2.634	50.500
2.841	58.984	2.840	55.773	2.839	50.946
3.039	59.428	3.038	56.203	3.037	51.346
3.236	59.849	3.235	56.624	3.235	51.726
3.442	60.264	3.442	57.025	3.441	52.122
3.637	60.655	3.637	57.409	3.636	52.470
3.842	61.048	3.841	57.787	3.841	52.830
4.047	61.436	4.046	58.170	4.045	53.173
4.251	61.814	4.250	58.518	4.249	53.505
4.444	62.148	4.443	58.854	4.442	53.806
4.651	62.494	4.650	59.189	4.649	54.117
4.848	62.819	4.846	59.489	4.846	54.405
5.064	63.153	5.063	59.808	5.062	54.702
5.268	63.453	5.266	60.090	5.265	54.973

Using the collected data the sensor parameters were estimated. First, based on the sensor current for no air flow $I_w^2(v = 0, N = 2) = 2040$ mA^2 and therefore the first sensor parameter $I_k^2 = a$ is $I_k^2 = 4080$ mA^2.

Next, values of the function $F(v, N)$ were calculated for the overheat ratio $N = 1.8$ (see Tab. 2). Based on a linear fit to the six data points, which were the closest to the point corresponding to the condition $F(v, N) = 1$ (see Fig. 2), the sensor wire calibration velocity was estimated to $v_k = 5.346$ m/s.

Similar calculations were performed for $N = 1.6$ and $N = 2.0$ leading to $v_k(N = 1.6) = 5.468$ m/s and $v_k(N = 2.0) = 5.531$ m/s, respectively. After averaging the second wire parameter $v_k = 5.445$ m/s.

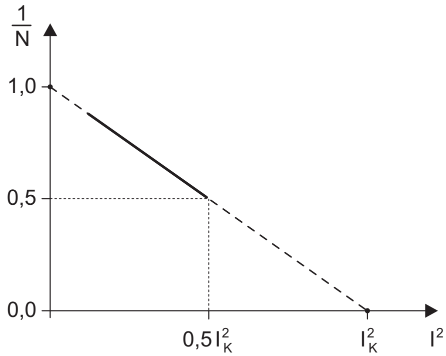

Fig. 1. Graphical interpretation of I_k^2

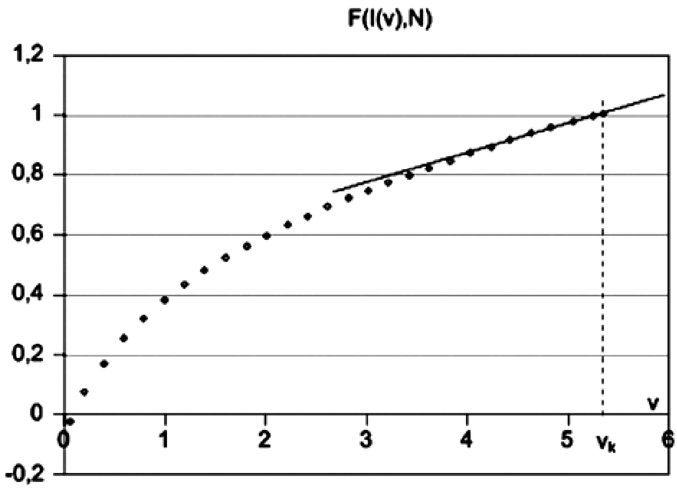

Fig. 2. Parameter v_k is equal to velocity for which $F(v, N) = 1$

3. Calculation of the exponent $n(v)$

Finally, v/v_k, $\ln(I_w(v, N))$ and $\ln(v/v_k)$ were calculated (see Tab. 2). The last column of Tab. 2 provides values of the exponent $n(v, N)$ calculated according to the Eq. (9).

Given v_k for three different overheat ratios the values of the exponent $n(v, N)$ were calculated based on Eq. (11). As shown in Fig. 3, for no air flow the exponent n is approximately equal to 2. This effect has been previously described (DISA Information, 1969; Kiełbasa, 2010; Papierz & Kiełbasa, 2011). For gas speeds in range $0 < v < 2$ m/s the value of the exponent n monotonically decreases to approx. 0.5.

TABLE 2

Details of calculations for the overheat ratio $N = 1.8$

v [m/s]	I_w [mA]	$F(v, N)$	v/v_k	$\ln(F(v, N))$	$\ln(v/v_k)$	$n(v, N)$
0.073	41.943	-0.0298	0.0136		-4.3003	
0.221	44.063	0.0707	0.0412	-2.6494	-3.1896	0.8306
0.411	45.990	0.1664	0.0767	-1.7932	-2.5674	0.6985
0.610	47.571	0.2480	0.1139	-1.3945	-2.1728	0.6418
0.809	48.891	0.3182	0.1509	-1.1451	-1.8911	0.6055
1.011	49.999	0.3786	0.1886	-0.9712	-1.6683	0.5822
1.215	50.942	0.4311	0.2267	-0.8413	-1.4840	0.5669
1.416	51.754	0.4771	0.2641	-0.7401	-1.3315	0.5558
1.616	52.471	0.5183	0.3014	-0.6572	-1.1992	0.5480
1.826	53.138	0.5572	0.3407	-0.5849	-1.0767	0.5432
2.028	53.728	0.5919	0.3783	-0.5244	-0.9720	0.5395
2.230	54.288	0.6253	0.4160	-0.4696	-0.8771	0.5354
2.435	54.810	0.6567	0.4543	-0.4206	-0.7891	0.5330
2.634	55.300	0.6864	0.4914	-0.3762	-0.7104	0.5296
2.840	55.773	0.7154	0.5298	-0.3349	-0.6353	0.5271
3.038	56.203	0.7420	0.5667	-0.2984	-0.5679	0.5255
3.235	56.624	0.7682	0.6036	-0.2638	-0.5049	0.5224
3.442	57.025	0.7933	0.6421	-0.2316	-0.4430	0.5227
3.637	57.409	0.8175	0.6784	-0.2014	-0.3880	0.5193
3.841	57.787	0.8415	0.7166	-0.1725	-0.3332	0.5177
4.046	58.170	0.8660	0.7548	-0.1439	-0.2813	0.5114
4.250	58.518	0.8884	0.7928	-0.1183	-0.2321	0.5096
4.443	58.854	0.9102	0.8288	-0.0941	-0.1877	0.5013
4.650	59.189	0.9320	0.8675	-0.0704	-0.1422	0.4953
4.846	59.489	0.9516	0.9041	-0.0496	-0.1009	0.4915
5.063	59.808	0.9726	0.9445	-0.0278	-0.0571	0.4859
5.266	60.090	0.9912	0.9824	-0.0088	-0.0177	0.4974

4. Conclusions

1. The three parameters of Eq. (5): I_k^2, v_k and the function $n(v, N)$ might be estimated from wire current measurements.
2. The parameters are independent from each other, in contrast to the parameters of the King's equation (2) or (3).
3. It has been demonstrated that the exponent $n(v, N)$ is close to 2 for $v = 0$ (no air flow) (DISA Information, 1969; Kiełbasa, 2010; Papierz & Kiełbasa, 2011). and decreases up to 0.5 with growing air velocities.
4. Based on $n(v, N)$ it is possible to linearize current measurements.

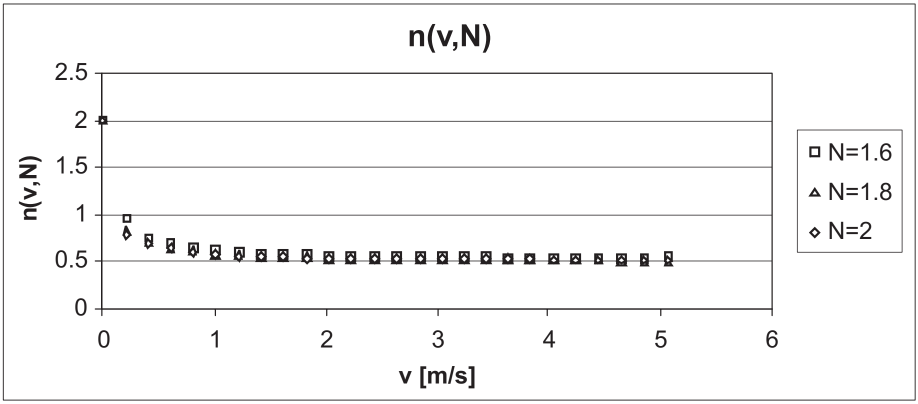

Fig. 3. Dependence of the exponent $n(v, N)$ on air flow velocity v

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