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ANALYTICAL DESIGN OF WATER-FREE PRODUCTION IN HORIZONTAL WELLS USING HODOGRAPH METHOD

ZASTOSOWANIE METODY HODOGRAFU DO OKREŚLENIA KRYTYCZNEGO WYDATKU POZIOMYCH OTWORÓW PRODUKCYJNYCH

Horizontal well has been widely used as a solution for oil reservoir with underlain strong water drive. The advantage of horizontal well over vertical well is to increase the reservoir contact and thereby enhance well productivity. Because of that, horizontal well can provide a very low pressure drawdown to avoid the water coning and still sustain a good productivity. However, the advantage of the large contact area with reservoir will soon become the disadvantage when the water breakthrough into the horizontal well. The water cut will increase rapidly due to the large contact area with reservoir and it may cause the productivity loss of the whole well. Therefore, keeping the horizontal well production rate under critical rate is crucial.

However, existing models of critical rate either oversimplify or misrepresent the nature of the WOC interface, resulting in misestimating the critical rate. In this paper, a new analytical model of critical rate is presented to provide accurate calculations on this subject for project design and performance predictions.

Unlike the conventional approach, in which the flow restriction due to the water crest shape has been neglected; including the distortions of oil-zone flow caused by the rising water crest, the new analytical model gives an accurate simultaneous determination of the critical rate, water crest shape and the pressure distribution in the oil zone by using hodograph method combined with conformal mapping. The accuracy of this model was confirmed by numerical simulations. The results show that neglecting the presence of water crest leads to up to 50 percent overestimation of critical rates.

Keywords: critical rate, horizontal well, hodograph method, analytical method

Typową metodą eksploatacji złóż ropy naftowej z naporową wodą podścielającą są otwory poziome. Ich zaleta w porównaniu z otworami pionowymi jest wysoki wskaźnik produktywności dzięki większej powierzchni kontaktu ze złożem. Otwór poziomy jest produktywny przy bardzo małej depresji która pomaga uniknąć stożków wodnych prowadzących do zawodnienia otworu. Jednakże duża powierzchnia

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kontaktu ze złożem staje się wadą otworów poziomych gdy stożek wodny dostanie się do otworu. Następuje wtedy gwałtowne zawodnienie otworu i szybka utrata produktywności. Z tego powodu wydatek otworu musi być utrzymany poniżej wartości wydatku krytycznego, tzn. maksymalnego wydatku bez udziału stożka wodnego.

Istniejące modele analityczne wydatku krytycznego są albo zbyt uproszczone lub też niedokładne w opisie lokalnej powierzchni kontaktu między ropą naftową i wodą podścielającą co prowadzi do błędnej oceny wydatku krytycznego. W tym artykule prezentujemy nowy model matematyczny wydatku krytycznego który jest bardziej dokładny przez co lepiej nadaje się do obliczeń projektowych.

W przeciwieństwie do istniejących modeli, nasz model uwzględnia ograniczenie dopływu ropy do otworu spowodowane wzrostem stożka wodnego ponad statyczną powierzchnię kontaktu ropy z wodą podścielającą oraz pozwala dokładnie obliczyć wydatek krytyczny oraz opisać kształt powierzchni stożka i zmianę ciśnienia w złożu z odległością od otworu poziomego. Równania modelu zostały wyprowadzone z teorii hodografu połączonej z metodą odwzorowań konforemnych. Wyniki obliczeń przy użyciu równań modelu wykazują zgodność z wynikami symulatora złoża. Stwierdzono również, że typowe dla innych modeli założenie płaskiej powierzchni kontaktu ropa/woda i zaniedbanie efektu kształtu stożka wodnego może prowdzić do 50-procentowej przeceny wartości wydatku krytycznego

Słowa kluczowe: stożki wodne, wydatek krytyczny, otwory horyzontalne, metoda hodografu

Nomenclature

t	–	is a parameter and $0 < t < \infty$,
g	–	gravitational acceleration, $g = 9.81$ m/s^2,
h_o	–	oil zone thickness, ft [m],
h_w	–	water zone thickness, ft [m],
h_{pc}	–	capillary transition zone, ft [m],
$\mathrm{k_o}$	–	oil zone permeability, md [m^2],
$\mathrm{k_w}$	–	water zone permeability, md [m^2],
$\mathrm{k_{ro}}$	–	oil zone relative permeability, md [m^2],
k_{rw}	–	water zone relative permeability, md [m^2],
q_c	–	critical oil rate per unit length, stb/d/ft [m^2/s],
q_{cD}	–	dimensionless critical rate, stb/d/ft [m^2/s],
x	–	x direction or dimension, ft [m],
x_{eD}	–	dimensionless reservoir extent,
y	–	y direction or dimension, ft [m],
z	–	z direction or dimension, ft [m],
μ_o	–	oil viscosity, cp [kg/m.s],
μ_w	–	water viscosity, cp [kg/m.s],
ρ_o	–	oil density, lb/cu.ft [kg/m^3],
ρ_w	–	water density, lb/cu.ft [kg/m^3],
Φ_o, Φ_w	–	flow potential of oil and water, respectively, psi [Pa],
ln	–	Natural logarithm,
cos$h$	–	Hyperbolic cosine.

1. Introduction

An analytical method and numerical simulation can both be used to determine the critical rate for horizontal well. However, for practical purposes, the problem should be solved by the simplest and least costly method that will yield an adequate answer. In practice, compared with the long period of computational time required for numerical simulation, the time and cost factors favor the use of analytical solutions, because a solution may take a simple form and can be easy to apply. They are accurate when all the assumptions are valid.

Several analytical models of critical rate calculation in horizontal wells have been presented by various authors. Chaperon (1986) derived the critical rate analytical solution for horizontal well by extending the classic treatment of the water coning phenomena as presented by Muskat and Wyckoff (1935) to horizontal wells. The key approximation made in her model is that the pressure distribution in the oil zone in the presence of a water cone is effectively the same as the pressure distribution without the water cone. Because the solution neglected the flow restriction due to the presence of the water cresting, the Chaperon approximation may overestimate the critical rate. Giger (1989) provided a general solution of the shape of the water crest and critical rate by using the hodograph method presented by Efros (1963). Because the crest shape is estimated at a large distance from the well, Giger (1989) suggested that these solutions not be used for small values of the dimensionless drainage radius. McCarthy (1993) solved the interface problem using the hodograph method by assuming that the crest shape intersected with the original WOC at a zero angle. This zero angle theory is true in vertical wells. For vertical wells, the pressure distribution in the oil zone is a logarithmic function. However, matters are different for the horizontal wells. At a large distance from the well, the flow is nearly linear. We can no longer assume that a water level has a horizontal asymptote.

In this study, the improved model for critical rate in horizontal wells is built using the hodograph method. The model can improve the accuracy of the critical rate calculation with less restriction. The model is verified with numerical simulation.

2. Analytical model of critical rate in horizontal well

The well is modeled as a line source that assumes that the wellbore radius is negligibly small and the wellbore can be treated as a line. The well is assumed to be parallel to the top and bottom boundaries. The reservoir boundary at the top of the oil pay zone is assumed to be a no-flow boundary. Also, there is constant flow potential at the lateral boundary of the drainage area, x_e representing the condition where potential is maintained in the reservoir due to the natural water influx from underlain aquifer. It is also assumed that fluids are immiscible, incompressible, and have constant viscosities. The permeability is homogenous and isotropic, and porosity is uniform. There is a sharp interface between the water and oil – the transition zone is neglected. Friction loss along the wellbore is negligible.

We use the hodograph method presented by Bear and Dagan (1964) to develop the improved model for critical rate solution. The derivation of the analytical solution is given in Appendix A. The equations for calculating critical rate in a reservoir with thickness at h_0 and bounded by outer distance at x_e:

$$\frac{q_c\,\mu_o}{k_o(\rho_w-\rho_o)g\pi}\ln 2+\frac{q_c\,\mu_o\ln\cosh t}{k_o(\rho_w-\rho_o)g\pi}=h_o \tag{1}$$

$$\frac{2q_c\,\mu_o}{k_o(\rho_w-\rho_o)g\pi^2}\int_0^t t\tanh t dt=x_e \tag{2}$$

Equations 1 and 2 can be solved simultaneously to obtain the critical rate. The solution will be expressed in dimensionless form for simplicity and generality (Table 1).

TABLE 1

Dimensionless group defined

Dimensionless group	Symbol	Equation
Dimensionless critical rate	q_{cD}	$q_{cD}=\dfrac{q_c}{\dfrac{k_o}{\mu_o}(\rho_w-\rho_o)gh_o}$
Dimensionless reservoir extent	x_{eD}	$x_{eD}=\dfrac{x_e}{h_o}$

If the effects of relative permeability and capillary pressure are not negligible, the effective permeability k_o and oil zone thickness h_0 in Equations 1 and 2 can be replaced by end-point relative permeability k_ok_{ro} and an oil zone thickness above the capillary transition zone of h_o-h_{pc}, respectively.

The new dimensionless groups now become:

$$q_{cD}=\frac{q_c}{\dfrac{k_ok_{ro}}{\mu_o}(\rho_w-\rho_o)g(h_o-h_{pc})} \tag{3}$$

$$x_{eD}=\frac{x_e}{h_o-h_{pc}} \tag{4}$$

where h_{pc} is the thickness of the capillary transition zone.

The relationship between q_{cD} and x_{eD} is shown in figure 1. The relationship appears to be linear with the (-1) slope implying that the dimensionless critical rate is proportional to the reciprocal of dimensionless reservoir extent ($1/x_{eD}$) on a log-log scale. Figure 1 can be used as a simple and easy tool to predict critical rate in horizontal wells.

3. Verification

In this section, the validity the hodograph model will be tested using a commercial numerical reservoir simulator. The numerical model (IMEX) is a black-oil model developed by CMG. Figure 2 shows a sketch of a reservoir model with a bottom aquifer and a well perforated at the

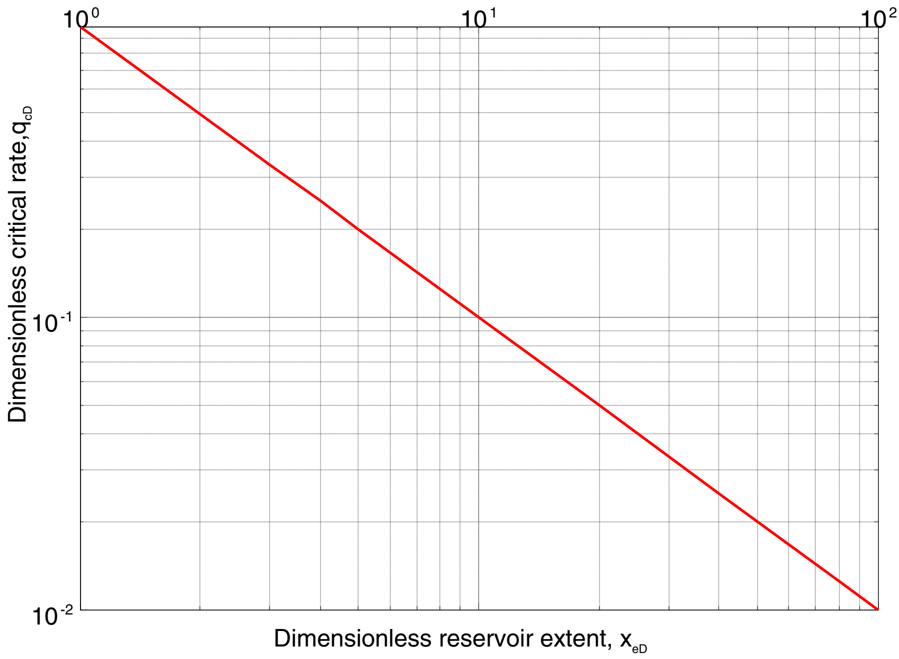

Fig. 1. Dimensionless critical rate in log-log scale

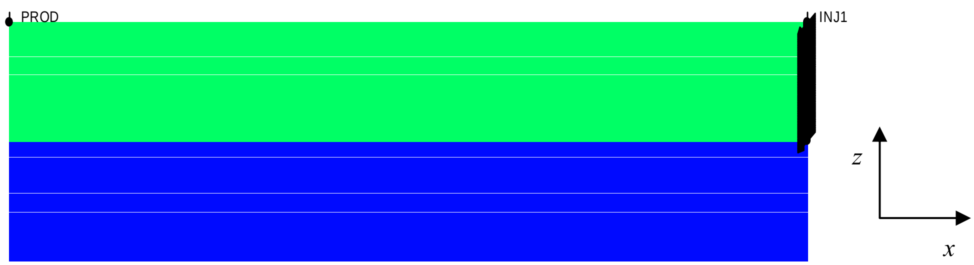

Fig. 2. Numerical model schematics

top of the reservoir. This model has a constant potential boundary obtained by injecting produced fluid into the reservoir through a vertical well. A 2D *x-z* model is used in this simulation.

As two dimensionless variables relating the shape of the relative permeability curves, exponents n_o and n_w are included to study the effect of the shape of the water/oil relative permeability curves between end points in the simulation. Figure 3 shows different permeability curves with the different exponents of n_o and n_w. Capillary pressure is included in some of these cases. Figure 3 also gives the capillary pressure curve.

The input data for all these cases are summarized in Table 2. All simulations were performed above the bubble point pressure and assume incompressible fluids.

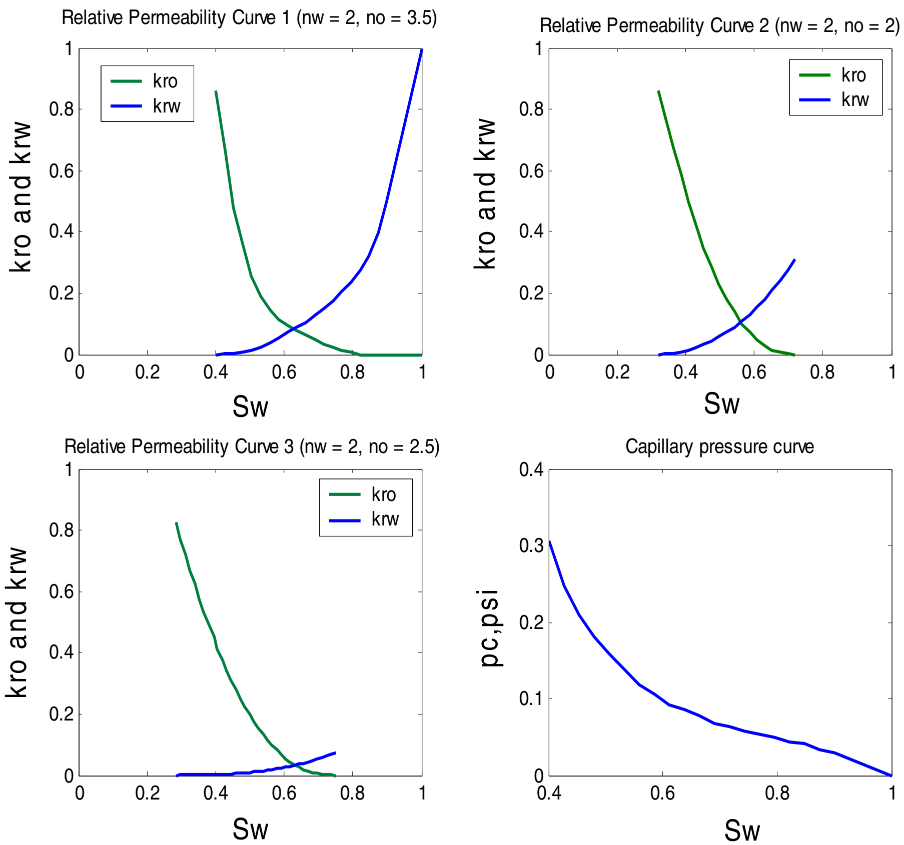

Fig. 3. Input relative permeability curves and capillary pressure curve

TABLE 2

Summary of selected simulation cases

Cases	Oil type	k (md)	μ (cp)	ϕ (%)	API°	x_{eD}	n_o	n_w	p_c (psi)
1	2	3	4	5	6	7	8	9	10
1	Light oil 1	10	1.5	12	35	1.39	1	1	0
2		10	1.5	12	35	2.57	1	1	0
3		10	1.5	12	35	4.95	1	1	0
4		10	1.5	12	35	10.82	1	1	0
5		10	1.5	12	35	23.56	1	1	0
6	Light oil 2	200	2	20	26	1.39	1	1	0
7		200	2	20	26	1.39	2	3.5	0
8		200	2	20	26	1.41	1	1	0
9		200	2	20	26	1.42	2	2	0
10		200	2	20	26	1.45	2	2.5	0
11		200	2	20	26	1.83	2	3.5	0.3057

1	2	3	4	5	6	7	8	9	10
12	Light oil 2	200	2	20	26	2.61	1	1	0
13		200	2	20	26	2.75	1	1	0
14		200	2	20	26	5.00	1	1	0
15		200	2	20	26	5.37	1	1	0
16		200	2	20	26	5.65	2	2	0
17		200	2	20	26	5.75	2	3.5	0
18		200	2	20	26	6.13	2	2	0.3057
19		200	2	20	26	10.82	1	1	0
20		200	2	20	26	10.88	1	1	0
21		200	2	20	26	21.20	1	1	0
22	Heavy oil 1	870	20	30	20	1.36	1	1	0
23		870	20	30	20	1.41	2	3.5	0
24		870	20	30	20	1.45	2	2	0
25		870	20	30	20	1.36	2	2.5	0
26		870	20	30	20	2.26	2	3.5	0.3057
27		870	20	30	20	2.66	1	1	0
28		870	20	30	20	5.33	1	1	0
29		870	20	30	20	10.95	1	1	0
30		870	20	30	20	22.61	1	1	0
31	Heavy oil 2	5000	65	30	14	1.41	1	1	0
32		5000	65	30	14	1.38	1	1	0
33		5000	65	30	14	1.41	2	3.5	0
34		5000	65	30	14	2.62	1	1	0
35		5000	65	30	14	2.72	1	1	0
36		5000	65	30	14	5.35	1	1	0
37		5000	65	30	14	5.72	1	1	0
38		5000	65	30	14	9.07	1	1	0
39		5000	65	30	14	11.62	1	1	0

Water density (995 kg/m^3) and water viscosity 0.96 (cp) are the same for all cases.

The critical rate results are summarized in Figure 4, where the dimensionless critical rate is plotted vs. the dimensionless distance for all cases in Table 2. The solid line is the analytical solution from the hodograph method, and the points are computed by the numerical simulator. The analytical solution using the hodograph method is very similar to the results from the numerical model. Figure 4 also indicates that the dimensionless critical rate depends solely on the dimensionless reservoir distance, independent of rock and fluid properties, the shape of the water/oil relative permeability between end points, and the capillary pressure.

The percentage deviations of the hodograph solution compared to the numerical simulation results are presented in Figure 5. The percentage deviation is defined as percent error:

$$\text{Percent error} = \frac{\text{Result (analytical)} - \text{result (numerical)}}{\text{Result (numerical)}} \times 100$$

The result indicates that the new analytical solution differs by less than 10 percent from a solution obtained by the numerical simulation and the difference becomes smaller (less than 5

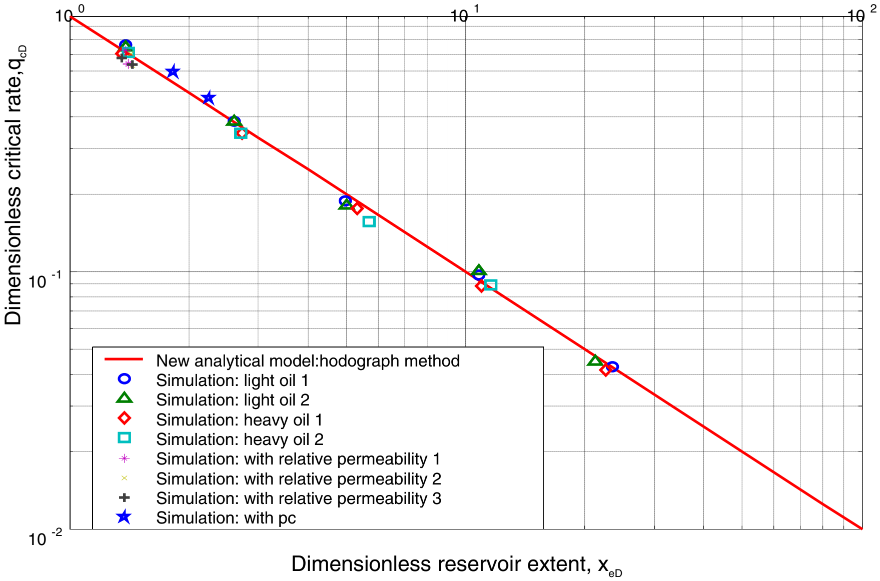

Fig. 4. Critical rate comparisons between analytical solution and numerical simulation

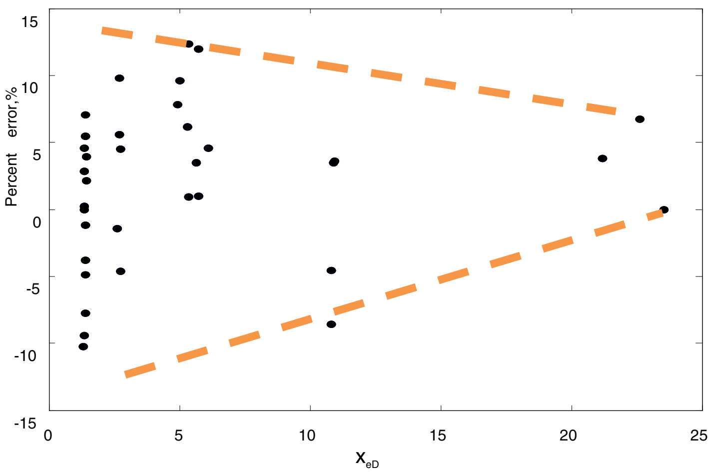

Fig. 5. The percentage deviations

percent) when the dimensionless drainage distance approaches realistic values ($x_{eD} > 5$). We can conclude that the new analytical solution matches the numerical simulation. The critical rate may be determined within 10 percent accuracy through this new analytical model.

A second validation was performed by comparing the new analytical results, derived in this work to the existing analytical solutions (Giger, 1989; Chaperon, 1986; McCarthy, 1993), and simulation results published by Dikken (1990). As shown in Figure 6, Giger and McCarthy's model are closely matched (less than 5 percent) with only a slight discrepancy at a small dimensionless reservoir distance when compared to the analytical solution of the new model. The main advantage of the new analytical solution is a removal of the physical inconsistencies in the other models. The new model has fewer restrictions and is more general than other models. The new analytical solution is within 5 percent of the critical rates estimated in Dikken's simulations. It also shows that Chaperon's model provides a higher critical rate than other models. The differences are caused by neglecting shrinkage of the oil zone caused by the crest, therefore ignoring the effect of the crest on the flow restrictions. Without considering the crest shape, Chaperon's model can lead to an overestimation of the critical rate by up to 70 percent.

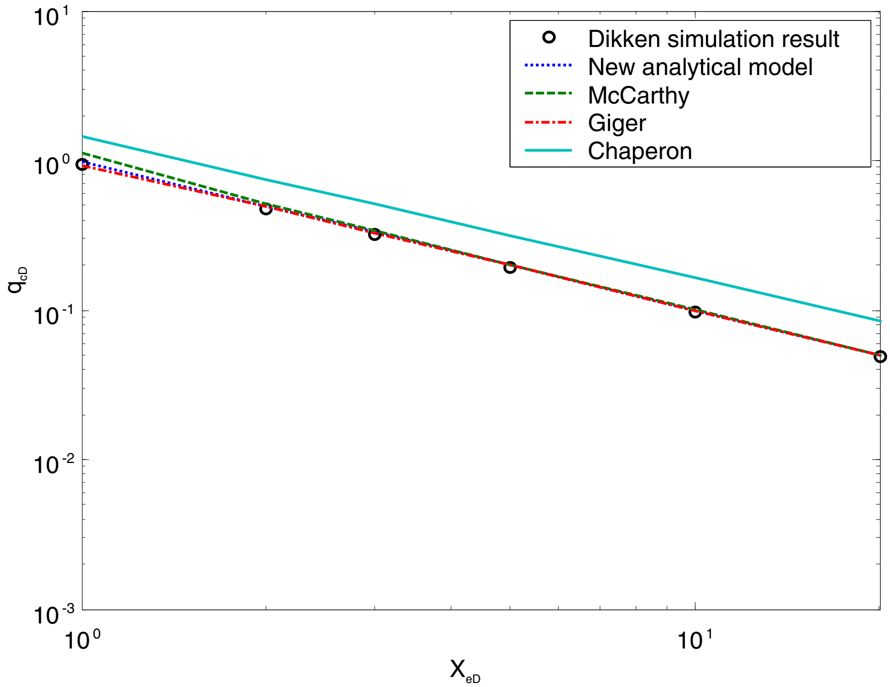

Fig. 6. Critical rate comparisons with existing models

4. Conclusions

- The new analytical model of critical rate based on the hodograph method is more accurate than the published solutions as it considers the presence of the crest shape. The theoretical model shows a good agreement with the numerical results, with a error less than 20 percent, indicating it effectiveness to predict critical rate in horizontal well.
- The dimensionless critical rate depends solely on the dimensionless reservoir distance, independent of rock and fluid properties, the shape of the water/oil relative permeability

between end points, and the capillary pressure. The relationship is linear with the (-1) slope on a log-log scale.
- Neglecting the influence of the crest shape on the flow restriction would lead to overestimate the critical rate up to 70 percent.

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APPENDIX A
DERIVATION OF CRITICAL RATE IN HORIZONTAL WELL USING HODOGRAPH METHOD

Bear and Dagan (1964) used this method to solve the water cresting problem in an infinitely large reservoir. Their detailed description of how they applied this hodograph method to the specific cases will be reviewed in this section.

Assumptions:
- Oil reservoir is homogeneous and isotropic.
- Oil is incompressible and shows a steady state flow condition.
- Sharp interface between oil and water exists.
- Friction loss along the wellbore and end point effect are negligible.

The physical plane is shown as Figure A-1.

Due to the symmetrical shape of the crest, it is sufficient to analyze the region FAGF, where A is the drain. The flow region is bounded by no flow boundary AF and free surface GF.

The mapping of the flow domain FAGF onto the hodograph plane is shown in Figure A-2.

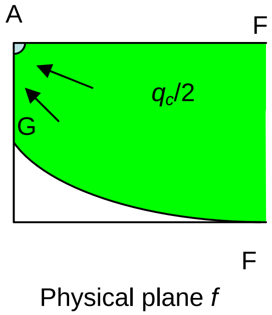

Fig. A-1. Physical plane of water cresting problem

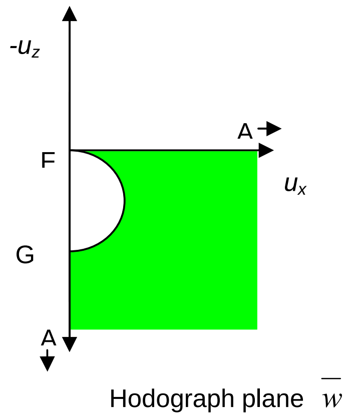

Fig. A-2. Hodograph plane for flow domain FAGF

Segment AF is a no flow boundary with vertical velocity $u_z = 0$, while segment FG, a critical free surface, is represented in the hodograph plane by a circle centered at (0, $K/2$) and with radius ($K/2$). The critical crest shape is associated with a maximum water free oil rate which is called critical rate per unit length q_c. Any rate above the critical rate will lead to water production in the well. Segment GA is a streamline which shows only vertical velocity toward to the well. A is a sink point, with infinite velocity in different directions.

The corresponding complex potential plane is shown in Figure A-3. The physical flow region is bounded by two streamlines, line AGF and line AF in ζ plane.

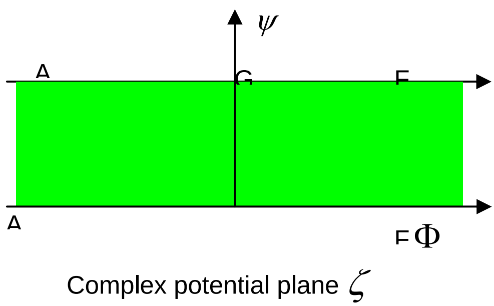

Fig. A-3. Corresponding complex potential plane

The next step is to find the relationship that maps the ζ and $\overline{w}$ planes onto an auxiliary B plane, shown in Figure A-4. This can be done by using the Schwarz-Christoffel transformation.

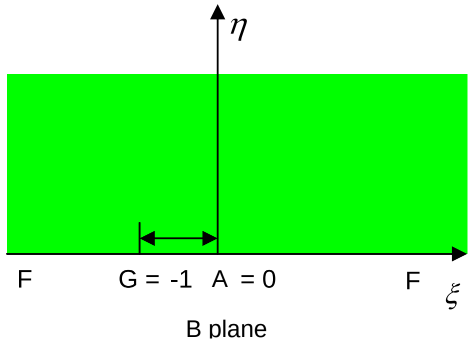

Fig. A-4. The auxiliary B plane

The ζ complex potential plane is mapped onto the B plane by using the Schwarz-Christoffel transformation:

$$\zeta = \frac{q_c}{2\pi}\ln B \tag{A-1}$$

The $\overline{w}$ plane is mapped onto the B plane by:

$$\overline{w} = \frac{K}{\frac{2}{\pi} arc\sinh(\sqrt{B})} \tag{A-2}$$

The relationship between the three planes can be expressed as:

$$\frac{df}{dB} = -\frac{d\zeta}{\overline{w}dB} = -\frac{q_c}{\pi^2 K}\left(\frac{arc\sinh\sqrt{B}}{B}\right)$$

$$\int_{FG} df = \int_{FG} -\frac{d\zeta}{\overline{w}dB}dB = \int_{FG} -\frac{q_c}{\pi^2 K}\left(\frac{arc\sinh\sqrt{B}}{B}\right)dB = x + iz \tag{A-3}$$

Integrating the Equation A-3 along segment FG in B plane yields the following parametric equations for the interface:

$$x = \frac{2q_c\mu_o}{\pi^2 k_o \Delta\rho g}\int_0^t dt\,\tanh t \tag{A-4}$$

$$h_c = -\frac{q_c\mu_o}{\pi k_o \Delta\rho g}\ln\cosh t \tag{A-5}$$

where t is a parameter and $0 < t < \infty$.

Equations A-4 and A-5 were presented by Bear and Dagan (1964) to describe the interface equation in a reservoir with infinite thickness and infinite lateral distance. Cone\crest height tends to be infinite based on the equations. Since the solution is only restricted to the infinite reservoir case, it is not applicable to the cases where the well is produced from a confined reservoir.

To calculate the critical rate and crest shape in a reservoir with oil zone thickness h_o and an outer boundary extent x_e as shown in Figure A-5, two constraint conditions must be introduced and combined with the parametric equations given by Bear and Dagan. The two constraint conditions are:

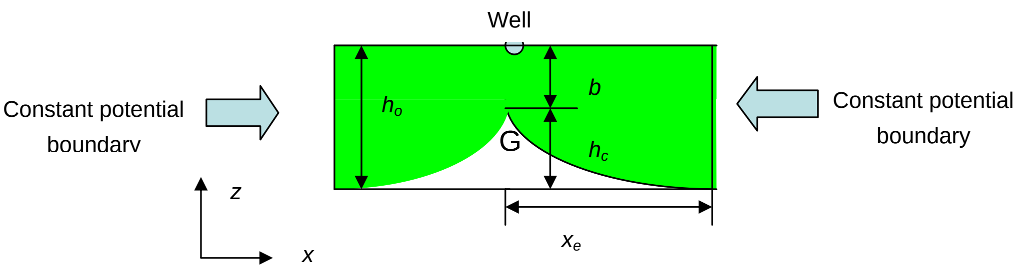

Fig. A-5. Cross-section schematic of water cresting in a horizontal well

$$h_c + b = h_o \tag{A-6}$$

$$x = x_e \tag{A-7}$$

where: h_c is the critical crest height; b is the distance between the apex of the crest to the well (Figure A-5).

To determine b, in the B plane, we integrate Equations A-8 and A-9

$$\int_{GA} df = \int_{GA} -\frac{d\zeta}{\overline{w}dB} dB = \int_{GA} -\frac{q_c}{\pi^2 K}\left(\frac{arc(\sinh)\sqrt{B}}{B}\right) dB \tag{A-8}$$

$$\int_{B=-1}^{B=0} -\frac{q_c \mu_o}{\pi^2 k_o (\rho_w - \rho_o) g}\left(\frac{arc(\sinh)\sqrt{B}}{B}\right) dB = x + ib \tag{A-9}$$

The integration gives,

$$\frac{bk_o(\rho_w - \rho_o)g}{\mu_o q_c} = \frac{1}{\pi}\ln 2 = 0.221 \tag{A-10}$$

Where b is expressed as :

$$b = \frac{q_c \mu_o}{k_o(\rho_w - \rho_o) g \pi} \ln 2 \tag{A-11}$$

Inserting Equations A-5 and A-11 into Equation A-6 gives:

$$\frac{q_c \mu_o}{k_o(\rho_w - \rho_o) g\pi} \ln 2 + \frac{q_c \mu_o \ln \cosh t}{k_o(\rho_w - \rho_o) g\pi} = h_o \tag{A-12}$$

At the outer boundary, reservoir extent is defined by x_e; inserting Equation A-4 into Equation A-7 yields:

$$\frac{2 \mu_o q_c}{k_o(\rho_w - \rho_o) g\pi^2} \int_0^t t \tanh t dt = x_e \tag{A-13}$$

Combining the above two equations, we obtain equations for calculating critical rate in a reservoir with thickness at h_o and bounded by outer distance at x_e:

$$\frac{q_c \mu_o}{k_o(\rho_w - \rho_o) g\pi} \ln 2 + \frac{q_c \mu_o \ln \cosh t}{k_o(\rho_w - \rho_o) g\pi} = h_o \tag{A-14}$$

$$\frac{2 q_c \mu_o}{k_o(\rho_w - \rho_o) g\pi^2} \int_0^t t \tanh t dt = x_e \tag{A-15}$$