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Arch. Min. Sci., Vol. 59 (2014), No 4, p. 971–986

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2014-0067

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THE NATURE OF MINING-INDUCED HORIZONTAL DISPLACEMENT OF SURFACE ON THE EXAMPLE OF SEVERAL COAL MINES

OPIS ZJAWISKA PRZEMIESZCZEŃ POZIOMYCH POWIERZCHNI TERENU WYWOŁANYCH PODZIEMNĄ EKSPLOATACJĄ GÓRNICZĄ NA PRZYKŁADZIE KILKU KOPALNI PODZIEMNYCH

The paper presents the analysis of the phenomenon of horizontal displacement of surface induced by underground mining exploitation. In the initial part, the basic theories describing horizontal displacement are discussed, followed by three illustrative examples of underground exploitation in varied mining conditions. It is argued that center of gravity (COG) method presented in the paper, hypothesis of Awierszyn and model studies carried out in Strata Mechanics Research Institute of the Polish Academy of Sciences indicate the proportionality between vectors of horizontal displacement and the vector of surface slope. The differences practically relate to the value of proportionality coefficient *B*, whose estimated values in currently realized design projects for mining industry range between 0.23*r* to 0.42*r* for deep exploitations, whereas in the present article the values of 0.33*r* and 0.47*r* were obtained for two instances of shallow exploitation. Furthermore, observations on changes of horizontal displacement vectors with face advancement indicated the possibility of existence of COG zones above the mined-out field, which proved the conclusions of hitherto carried out research studies (Tajduś 2013).

Keywords: horizontal displacement, COG method, COG zone, Awierszyn hypothesis, rock and soil mechanics

Artykuł prezentuje analizę zjawiska przemieszczeń poziomych powierzchni terenu wywołanych podziemną eksploatacją górniczą. W pierwszej części przedstawia podstawowe teorie opisujące zjawisko przemieszczeń powierzchni, a następnie w dalszej kolejności prezentuje trzy przykłady eksploatacji podziemnych w różnych warunkach górniczych.

W kontekście przedstawionej w artykule metody punktu środka ciężkości, hipotezy Awierszyna i wyników badań modelowych IMG PAN w Krakowie stwierdzono, że wskazują one na proporcjonalność pomiędzy wektorami przemieszczenia poziomego a wektorem nachylenia powierzchni terenu. Różnice dotyczą w zasadzie wartości współczynnika proporcjonalności *B*, którego wartości w ramach prowadzonych aktualnie prac projektowych dla przemysłu górniczego leżą w granicach od 0.23*r* do 0.42*r* dla eksploatacji

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głębokich, natomiast w niniejszym artykule dla dwóch przykładów płytkich eksploatacji uzyskano wartości 0.33r oraz 0.47r. Dodatkowo obserwacje zmian kierunków wektorów przemieszczeń poziomych wraz z postępującym frontem wykazały możliwość istnienia strefy środka ciężkości nad wybranym polem. Potwierdziło to wnioski z dotychczas przeprowadzonych badań (Tajduś, 2013).

Słowa kluczowe: przemieszczenia poziome, metoda punktu środka ciężkości, strefa środka ciężkości, hipoteza Awierszyna, mechanika skał i gruntów

1. Introduction

For the last several years, Strata Mechanics Institute of the Polish Academy of Sciences (Tajduś et al., 2010, 2012; Tajduś, 2013) has been carrying out research study on the phenomenon of mining-enhanced horizontal displacement of surface and its prognostic model. Such dislocations result from underground mining exploitation causing disturbance of rock mass structure. The volume of the disturbance depends upon the following factors:

- number of mined-out seams,
- their thickness,
- dimensions of mined-out fields in each seam,
- methods of void liquidation,
- speed of mining,
- geological and hydro-geological structure of overburden,
- tectonics,
- strength and strain parameters of strata.

The studies of, *inter alia*, Keinhorst (1925), Bals (1931/32), Awierszyn (1947), Budryk (1953) or Popiołek et al. (1978), Majcherczyk et al. (2008) on the influence of underground exploitation on rock mass and terrain surface failed to lead to a uniform solution of the problem of proper description of distribution and values of horizontal displacement. There are two prevailing groups of theories describing horizontal displacement in literature: (a) **theories assuming that the surface points displace towards the center of gravity of mined-out element of the seam** and (b) **theories assuming that there is a relationship between the value of horizontal displacement and the value of slope**.

2. Theories describing dislocation of the point towards the center of gravity of mined-out element of seam

The theory was proposed by Keinhorst (1925), who presented analyses on calculating the values of horizontal displacements of points, caused by mining of underground deposits (Figs. 1 and 2). The theory argues that also "the center of gravity (mass)" of the exploitation affecting the points changes depending on the position of calculation points. However, there is a problem with estimating the position of the COG (center of gravity) point of exploitation influences for particular calculation points.

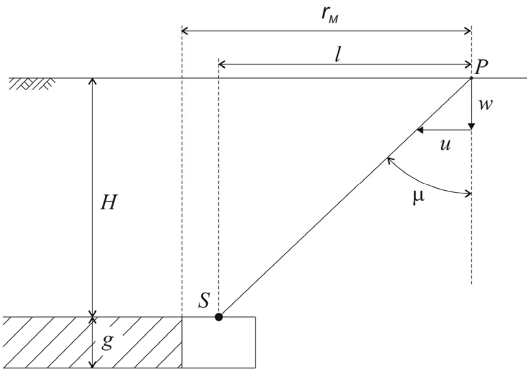


Fig. 1. Horizontal displacement of point *P* according to Keinhorst theory in elementary exploitation

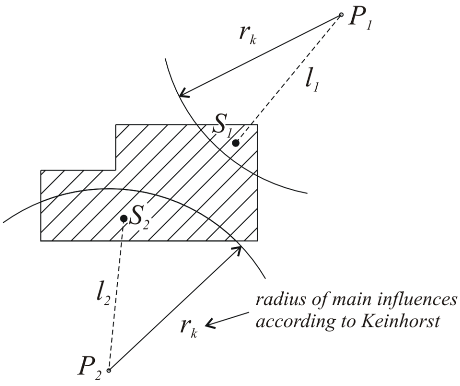


Fig. 2. Horizontal projection of COG according to Keinhorst theory in exploitation of field of any shape

$$u = \mathrm{tg}\mu \cdot w \tag{1}$$

where:

μ — angle between the straight line crossing the point at terrain surface and the straight line connecting it with the center of the part of exploitation affecting it (Fig. 1). Horizontal projection of the section connecting calculation point *P* with the point of center of gravity *S* was called "the radius of COG". The symbols used in Fig. 1 and Fig. 2 refer to the following:
l — length of COG radius,
H — depth of mining,
g — thickness of deposit mining,
r_M — radius of range of mining influence dependent from calculation method,
w — subsidence of point *P*,
u — horizontal displacement of point *P*,
S — COG point of exploitation affecting subsidence of point *P*.

It should be pointed out that the COG is certain for elementary exploitation (Fig. 1); however it is "uncertain" for the exploitation field of any shape (Fig. 2). "Uncertain" means in this case that, depending on the assumed calculation point, the position of COG of exploitation changed.

Further development of Keinhorst method is related to the development of calculation methods of point subsidence by means of graphic integration on the basis of networks of segments of equal influence. First of all, the fundamental geometric-integral method of Bals, as well as the later methods of Beyer and Sann, should be mentioned in this respect.

Later works of Bals (1931/32), as well as Lehmann, Neubert and Schafstein (1942), with the use of the assumption of Keinhorst's COG point, allowed for the calculation of the value of horizontal displacement of any point.

If we assume that the beginning of coordinate system (x,y) is located in the calculation point, we will obtain the formulae for displacement components u_{ix} and u_{iy} in the direction of coordinate system axis, caused by mining excavation of the area of the i^{th} segment, whose COG S_i has the coordinates x_i, y_i (Fig. 3).

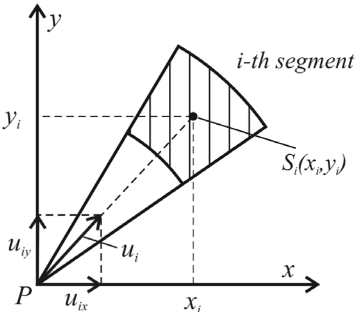


Fig. 3. Displacement components u_{ix} and u_{iy} in the direction of coordinate system axis, caused by seam excavation in the area of the i^{th} segment whose COG S_i has coordinates x_i, y_i

$$u_{ix} = \frac{u_i}{l_i} x_i, \qquad u_{iy} = \frac{u_i}{l_i} y_i$$
$$l_i = \sqrt{x_i^2 + y_i^2} \tag{2}$$
$$u_i = w_i \frac{l_i}{H}, \qquad w_i = \frac{a \cdot g}{N} = \text{const}$$

where:

l_i — radius of COG S_{i} of the area of the i^{th} segment,
u_i — horizontal displacement in the direction S_i caused by excavation of a single segment of the seam,
N — total number of meshes in the segment network.

The total horizontal displacement of the point was obtained by means of assuming the segments overlapping with the selected surface.

The total values of horizontal displacement was calculated according to the following formulae:

$$u_x = \sum_{i=1}^{i=n} u_{ix}, \qquad u_y = \sum_{i=1}^{i=n} u_{iy} \tag{3}$$

where: n — number of exploitation segments.

As the above shows, the calculations were extremely tedious and laborious.

German methods of calculating subsidence of Keinhorst, Bals, Sann, Beyer, as well as the so-called Ruhrkohle method, all using COG theory, differ from one another only with the values of the assumed angles of range of exploitation influences and with the assumed functions of influence of elementary seam volume on the point in the area of exploitation impact. Using Keinhorst method, the formulae for maximum value of horizontal displacement for the so-called infinite semi plane in a general form were obtained:

$$u_{\max} = c_m \cdot a \cdot g \cdot \cot \gamma_m \tag{4}$$

where:

c_m — the constant depending on calculation method (the values of coefficients c_m are provided in Table 1),

a — the so-called exploitation coefficient,
g — exploitation thickness of the seam,
γ_m — angle of range of main influences, depending on calculation method.

TABLE 1

Values of coefficients c_m for German calculation methods of Keinhorst, Bals, Sann, Beyer and the so-called Ruhrkohle method

Method	c_m
Keinhorst	0.152
Bals	0.138
Beyer	0.148
Sann	0.106
Ruhrkohle	0.131

The maximum values of horizontal displacement calculated according to the general formula (1) are significantly smaller (approx. 5÷7 times) than the values observed in the site. For this reason, in Ruhrkohle method the solution according to Awierszyn hypothesis was applied in order to calculate the value of horizontal displacement.

3. Theories assuming correlation between horizontal displacement value and slope value

On the basis of observation of mining-inducted subsidence troughs, Awierszyn (1947) arrived at a conclusion that the volume of horizontal displacement $u(x)$ was proportional to the profile slope $T(x)$ of the subsidence trough.

$$u(x) = -B \cdot T(x) = -B\frac{dw(x)}{dx} \tag{5}$$

where:

$u(x)$ — horizontal displacement in the direction of axis x,
B — displacement coefficient (or horizontal strain coefficient),
$T(x)$ — slope of elementary trough in the direction of axis x.

Further research studies on proportionality of slope to horizontal displacement were also carried out by, among others, Budryk (1953), Popiołek et al. (1978) and Hegemann (2003), and were discussed in detail in Tajduś, Misa and Sroka (2010). In the latter publication, a special attention was given to the studies realized in Strata Mechanics Research Institute of the Polish Academy of Sciences on the so-called elementary dumping hopper (joint dumping hopper) (Krzysztoń, 1965), where the linear relation was determined, described in the following formula (6):

$$\frac{u(x)}{w(x)} = \alpha \cdot x \tag{6}$$

where: x — horizontal distance from the dumping hopper.

The conclusion was that the distribution of horizontal displacement for the elementary dumping hopper can be described by means of the function:

$$u(x) = \alpha \cdot x \cdot w(x) \tag{7}$$

If we assume that the elementary trough is described with the formula:

$$w(x) = w_{\max} \cdot \exp\left(-\pi \frac{x^2}{r^2}\right) \tag{8}$$

we will obtain the distribution of horizontal displacement in the elementary trough:

$$u(x) = \alpha \cdot x \cdot w_{\max} \cdot \exp\left(-\pi \frac{x^2}{r^2}\right) \tag{9}$$

where: $w_{\max}$ — maximum subsidence value in elementary trough.

If we compare the formula based on the study of Krzysztoń with the formula based on Awierszyn hypothesis, we will see that the relations essentially differ with the assumed coefficients (Tajduś et al., 2010). We will obtain the following formula from the comparison:

$$\alpha = B \cdot \frac{2\pi}{r^2} \tag{10}$$

and

$$B = \frac{\alpha}{2\pi} \cdot r^2 = \frac{\alpha}{2\pi} \cdot H^2 \cdot \cot^2 \beta \tag{11}$$

Prof. Sroka carried out the calculation of value of parameter B by means of comparing the plain assumptions of Keinhorst hypothesis with Knothe theory:

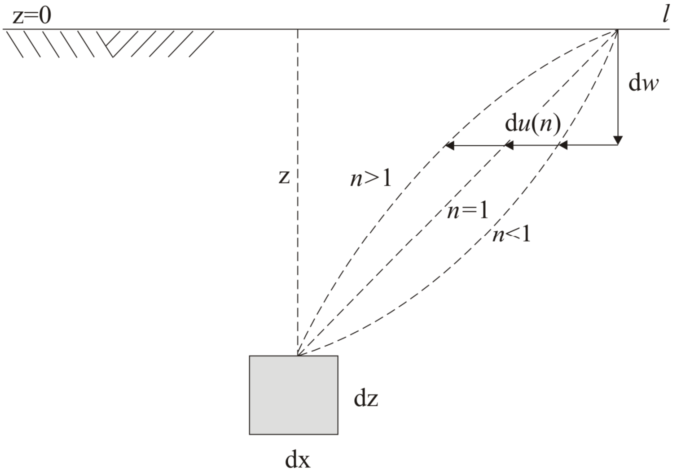


Fig. 4. Distribution of displacement vectors for elementary exploitation

If we assume a parabolic course of displacement trace (Fig. 4), from Keinhorst hypothesis we will obtain the following formula for elementary excavation (Sroka & Schober, 1982):

$$du = n\frac{l}{z} \cdot dw \tag{12}$$

In the case of Knothe theory, the distribution of subsidence in elementary trough is described by means of Gauss function, which leads to the formula:

$$du = -D \cdot dw' \tag{13}$$

where: D — the constant depending on depth and properties of the medium.

The formula is qualitatively identical with Awierszyn assumption. It can be seen that, for the above analyses, the value of coefficient D is determined as follows:

$$D = \frac{n}{2\pi} H \cdot \cot^2 \beta \tag{14}$$

and qualitatively it corresponds with the assumption presented by Litwiniszyn (*cf.* Knothe & Sroka, 2010).

In sum, it should be pointed out that both the theory of COG point of Keinhorst and the hypothesis of Awierszyn indicate the proportionality between the vector of horizontal displacement and the vector of subsidence trough slope.

Currently, both in Poland and Germany, geometric-integral theory (e.g. Knothe method, Ruhrkohle method) are applied for displacement description with Awierszyn hypothesis used for horizontal displacement (Sroka, 1995). Displacement coefficient B became subject to numerous analyses, in which many researchers tried to adapt it to local mining and geological conditions (Tajduś, 2013).

4. Examples of horizontal displacement distribution in areas of underground exploitation

Three sample exploitations in Germany were selected for the sake of the analysis. Two cases were taken from the archive databases: the exploitations took place at small depths and surface measurements were taken with traditional methods. The third example refers to the exploitation carried out in the late 1990s and the measurement was taken with the GPS method.

Example 1.

The German ore mine executed exploitation at the depth of 160 m÷180 m with the thickness of g = 1.80 m (Lehmann et al., 1956), whereas simultaneously on the terrain surface, geodesic measurements were taken along line 1, perpendicular to the front advancement, and line 2, parallel to the front advancement (Fig. 5).

The exploitation led to the occurrence of subsidence trough in the terrain surface. Fig. 6 presents the measured vertical displacements (w), horizontal displacements (u) and the relation between the values (u/w) for the line No. 1, as well as the function of relations between horizontal displacement and slope.

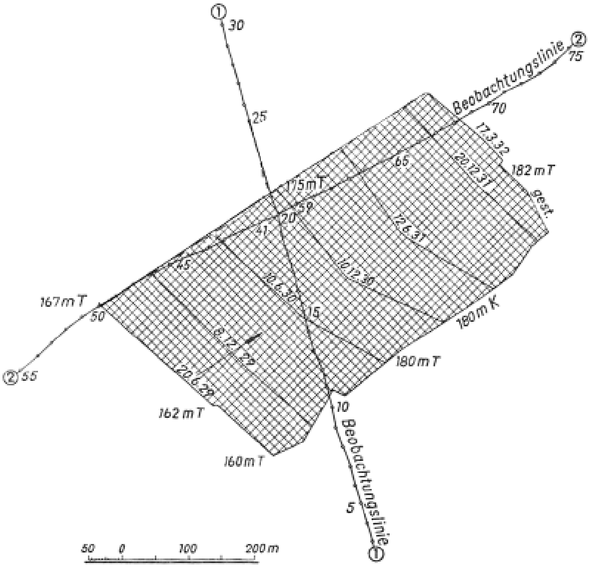
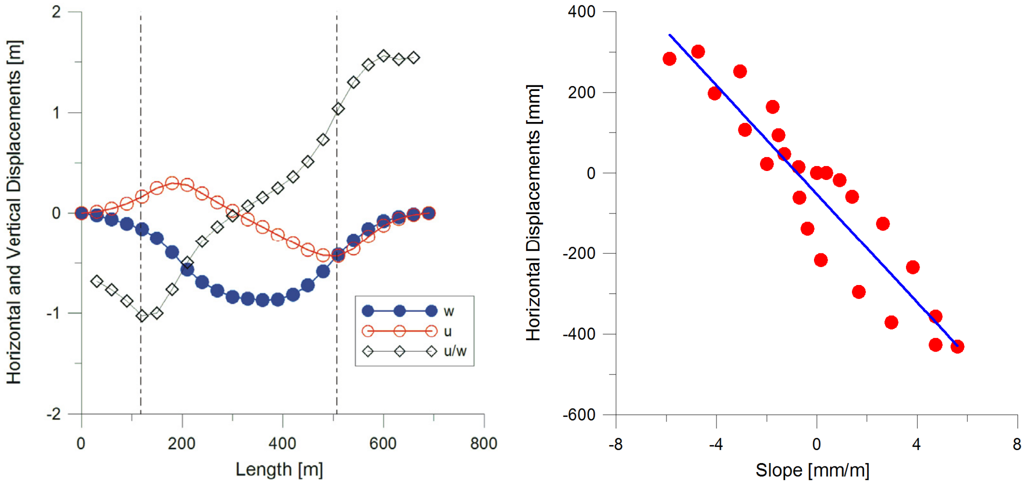


Fig. 5. Exploitation scheme of southern area of sample German ore mine (Lehmann et al., 1956)



a)

b)

Fig. 6. Measurements for line No. 1:
a) of vertical and horizontal displacement and their relation (with marked longwall panel edges),
b) the diagram of linear regression matching for the total values of *B* parameter

The relation of the values of vertical displacement to horizontal displacement is in the range $0 < \frac{|u|}{w} \leq 1.57$. The values of relation above the unity was obtained for the points beyond the contour of exploitation and their maximum is in the point approx. 70 m distant from the exploitation edge ($d_{\max}$). If we assume that the value of maximum horizontal displacement depends upon the method of excavation, longwall panel length and run, depth of exploitation, quality of rock mass, properties of soil strata, speed of mining and disturbance inside rock mass, we will be able, on the basis of gathered data, to determine the relation of standardized exploitation dimensions L and S (L – standardized wall length; S – standardized longwall panel run) to the standardized length of occurrence of maximum horizontal displacement $M_u(S,L)$, which is:

$$\begin{cases} S = \dfrac{s}{r} \rightarrow M_u(S) = 16.9 \cdot S \\ L = \dfrac{l}{r} \rightarrow M_u(L) = 31.1 \cdot L \end{cases}$$

where:

l — length of longwall,
s — longwall panel run.

The value $M_u(S,L)$ was given in standardized units.

The following formula describing the course of linear regression was obtained for the particular relation of function between horizontal displacement and slope:

$$U(\alpha) = -67.25 \cdot T - 51.90 \quad \text{with matching} \quad R^2 = 0.87.$$

The above formula of matching regression line to measurements results differs from the applied Awierszyn hypothesis by the value of absolute term. In order to approximate the value to the solution presented by Awierszyn, it was assumed that the regression line crosses the center of coordinate system. Then, the following formula was obtained:

$$U(\alpha) = -67.30 \cdot T \quad \text{with matching} \quad R^2 = 0.81.$$

It can be also noticed that the curve $|u|/w$ determined for the measurement points situated directly above the exploited plot is approximately linear and it can be described by means of the following formula:

$$D = 204 \cdot \frac{|u|}{w} + 327\,^{1} \quad \text{with matching} \quad R^2 = 0.98.$$

where: D — distance from the first point [m].

On the basis of the calculations it was proven that for the above-mentioned example the coefficient B can be presented in the form $B = 0.47r$ (with matching $\tan\beta = 1.2$) or $B = 0.40H$.

Example 2.

A similar analysis was carried out for the second example of exploitation in a German mine (Lehmann et al. 1956).

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[1] The formula refers to the points situated directly above the exploited wall, i.e. for the points: from No. 10 to 23 for the Example 1 (Fig. 5), and for the No. 59-65 for the Example 2 (Fig. 7).

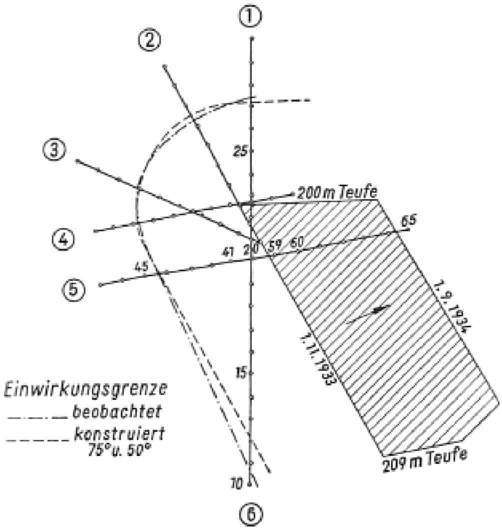
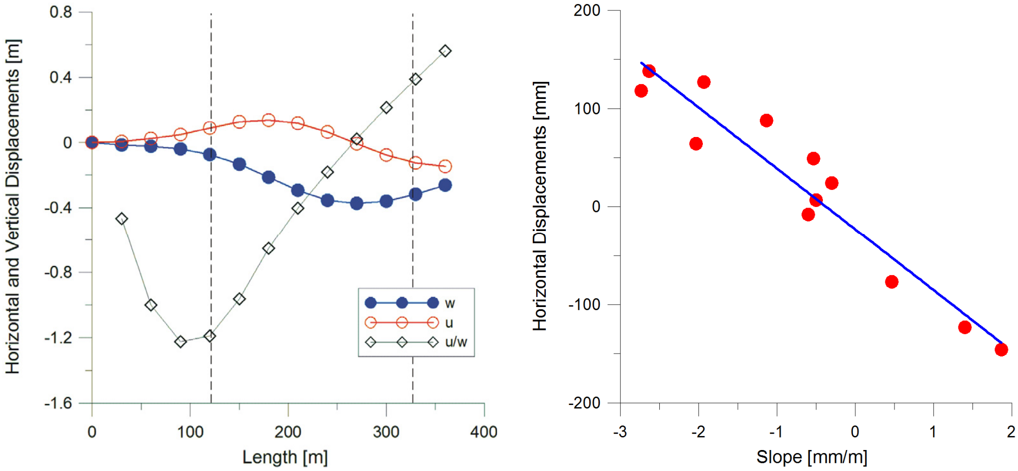


Fig. 7. Scheme of longwall exploitation in the sample German mine (Lehmann et al., 1956)

The mine carried out a longwall exploitation (Fig. 7) at the depth ranging from 200 m to 210 m, with the height of exploitation gate of $g = 1.45$ m. On the surface, geodesic measurements were carried out with the lines 1, 2, 3, 4, 5, 6. In further discussion, the author used the results of measurements for the line No. 5. Figure 8 presents the charts of measured vertical displace-



a)

b)

Fig. 8. Measurement diagrams for line No. 5:
a) for vertical and horizontal displacement and their relation (with marked longwall panel edges),
b) the diagram of linear regression matching for the total values of *B* parameter

ments (w), horizontal displacements (u) and the relation of both the values (u/w), as well as the function of relations between the measured horizontal displacements and slope.

For the relation between horizontal displacement and slope, the following formula describing the course of linear regression was obtained:

$$U(\alpha) = -62.03 \cdot T - 23.05 \quad \text{with matching} \quad R^2 = 0.92.$$

Whilst assuming that the regression line crosses in the center of coordinate system, the following was obtained:

$$U(\alpha) = -55.40 \cdot T \quad \text{with matching} \quad R^2 = 0.86.$$

It can be also seen that the curve $|u|/w$ determined for the measurement points situated directly above the exploited plot is approximately linear (Fig. 8a) and it can be described by means of the following formula:

$$D = 149.25 \cdot \frac{|u|}{w} - 271 \; \text{[2]} \quad \text{with matching } R^2 = 0.99.$$

Displacement coefficient B can be presented in the form $B = 0.33r$ (with matching $\tan\beta = 1.2$) or $B = 0.28H$.

The comparison of the course of curves u and w for the examples presented above slope that at a certain distance from the exploitation edge, horizontal displacement begins to assume higher values than the subsidence (40 m for the Example 1 and 50 m for the Example 2). This fact remains in accordance with the formulae of Knothe theory and for the so-called elementary trough, where this characteristic distance amounts to $x/r = 0.4$ from the dumping hopper.

Example 3.

On 13th March 1995, Friedrich Heinrich/Rheinland Mine commenced its exploitation in the seam Mathilde with the wall No. 204 with the length of 255 m and the longwall panel run 1,260 m (Korittke et al. 1996). The average thickness of the seam was 1.40 m, whereas its depth ranged $H = 760 \div 900$ m. The exploitation of the longwall was terminated on 15th October 1995. On the terrain surface, geodesic measurements were carried out with the use of GPS system, which is characterized by the precision of (S_x and S_y) = 3÷5 mm for horizontal directions and $S_z = 5 \div 8$ mm for vertical displacement (Korittke et al., 1996), where S means standard deviation.

The first measurement was taken on 14th March 1995, followed by regular measurements carried out until 30th January 1996 (final measurement numbered as 27).

Mining situation in the area of exploitation of the longwall panel 204, with a contemporary situation of the front of the wall 204 corresponding to the particular measurements of terrain surface, is presented in Fig. 9. The figure additionally shows the situation of the abandoned workings of the longwall 203 of the same seam, whose exploitation was led in the period from 20th June 1994 to 28th December 1994.

On the basis of the measurements it was concluded that the exploitation of the panel 204 resulted in the occurrence of the trough in the terrain surface with the point of maximum subsidence of $w_{\max} = 700$ mm, whose center is moved towards the mined-out workings of the longwall panel 203.

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[2] The formula refers to the points situated directly above the exploited wall, i.e. for the points: from No. 10 to 23 for the Example 1 (Fig. 5), and for the No. 59-65 for the Example 2 (Fig. 7).

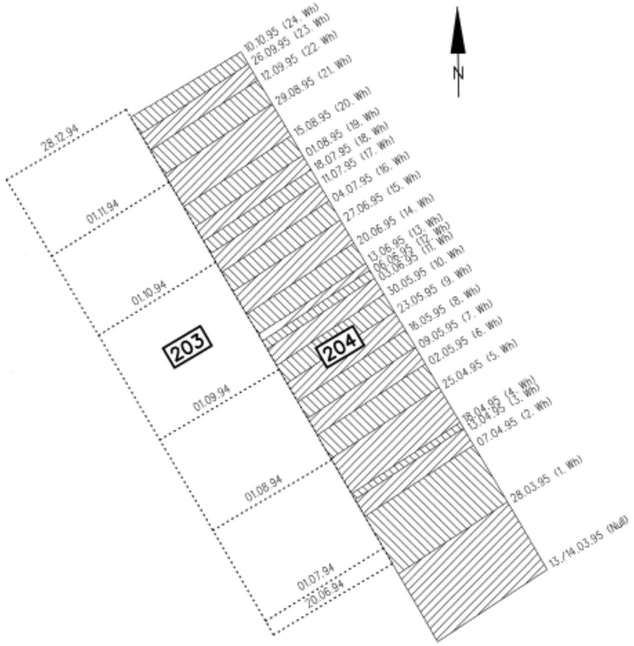


Fig. 9. Mining situation of the longwall panel 204 with the situation of exploitation front corresponding to the time of measurement and neighboring abandoned workings of the wall 203

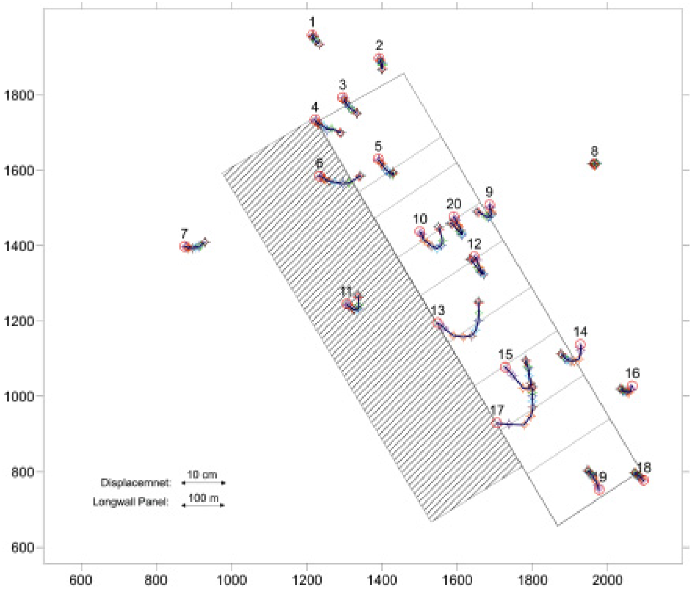
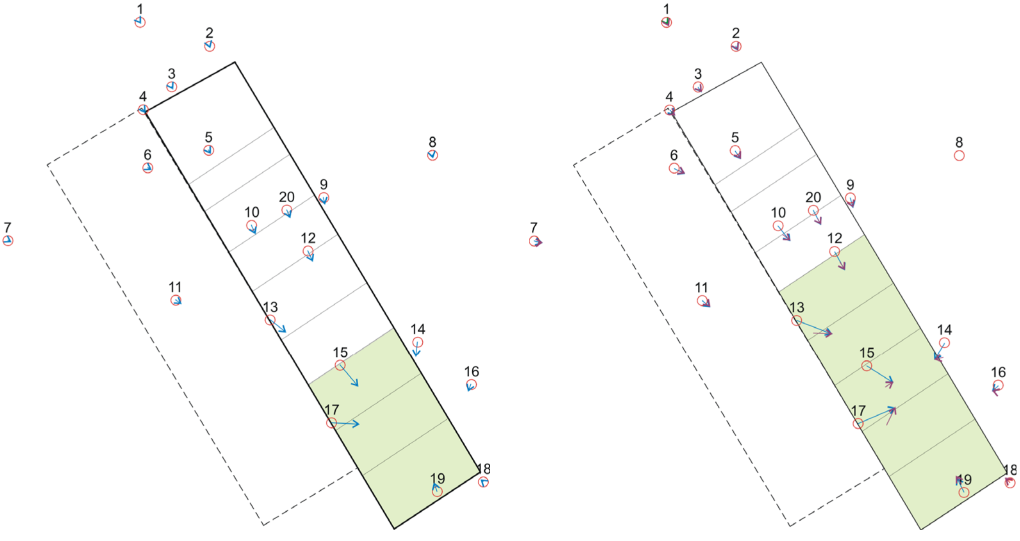


Fig.10. Mining situation of the wall 204 with the situation of the exploitation front corresponding to the time of measurement

In order to illustrate the changes of horizontal displacement vectors alongside with the advancement of exploitation front, the analysis was carried out for four selected phases of front advancement of the wall 204 (Figs. 11a, 11b and Figs. 12a, 12b). The vectors created by the initial points and the points corresponding to the current front advancement were marked with the blue color (darkened field). The purple color marks the vectors created by the points corresponding to the difference between: measurement 5 and 12 (Fig. 11b), measurement 18 and 12 (Fig. 12a) and measurement 27 and 18 (Fig. 12b).

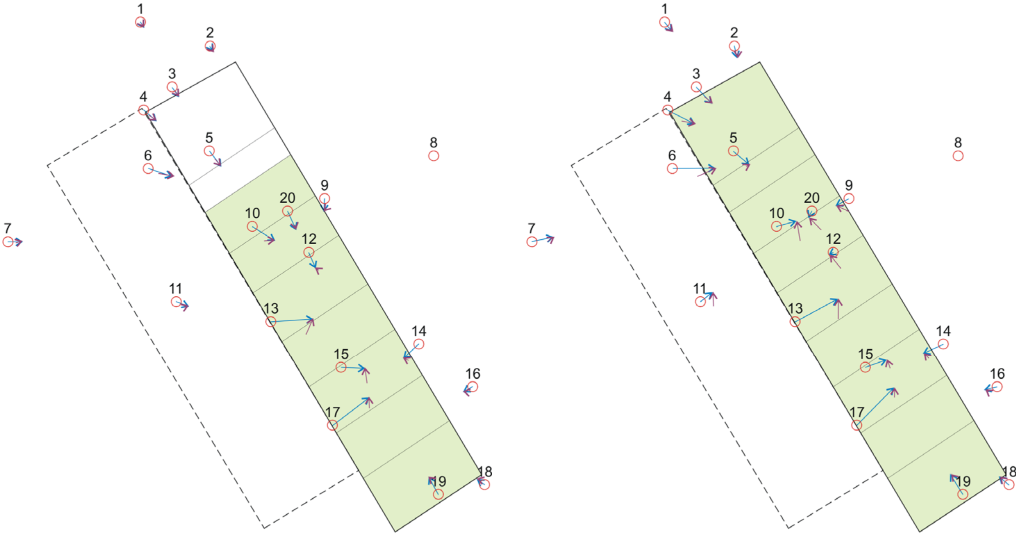


a) b)

Fig.11. Horizontal displacement vectors measured in the terrain surface for various phases of exploitation a) measurement No. 5 corresponding to the front advancement of 440 m, b) measurement No. 12 corresponding to the front advancement of 715 m

Figures 11 and 12 indicate that alongside with the exploitation front advancement, both the value of horizontal displacement vector and the direction of its activity change. However, there is a certain delay in the point's reaction to the front advancement. For instance, the measurement point No. 5 in the northern part of exploitation field experiences the first significant turn of displacement vector in the moment when the front stops in the longwall panel 1,115 m, i.e. for the measurement No. 21. This measurement corresponds to the situation, when the front exceeds the location of point No. 5 in the surface by 34 m. Then, the vector will turned by 18° in relation to the horizontal axis. However, for the front advancement corresponding to the measurement No. 22, i.e. 93 m beyond the location of point No.5, the vector will be turned by 83°.

A similar correlation can be observed in the case of point No. 9, where for the situation of exploitation front corresponding to the measurement No. 15, the vector will be turned in relation to the horizontal axis only by 22°. The measurement corresponds to the situation of exploitation front at the distance of 25 m after the measurement point. For the front advance-



a) b)

Fig.12. Horizontal displacement vectors measured in the terrain surface for various phases of exploitation a) measurement No. 18 corresponding to the front advancement of 995 m, b) measurement No. 27 corresponding to the period of 107 days after exploitation termination

ment 115 m northbound beyond the point No. 9, the vector was turned by 88° in relation to the horizontal axis. On the basis of the studies, it can be noticed that the angle of vector turning of horizontal displacement is determined by the speed of exploitation advancement, size of mined-out field, situation of exploitation front and properties of overburden, which often minimize and delay the reaction of surface points.

In order to additionally illustrate the activity of the so-called exploitation COG zone, the scheme of distribution of displacement vectors' directions was prepared. It was observed that for the situation, when the exploitation is in the panel 440 m (measurement No. 5), the directions of measurement point vectors numbered as 14, 15, 16, 17, 18, 19 cross above the field of exploitation corresponding to measurement No. 2 (panel 290 m) (Fig. 13). The measurement points 1, 3, 4, 5, 9, 10, 12, 13, 20 cross above the field determined for the measurement No. 5. Several measurement points were skipped in the observations due to their large distance from exploitation field and immediate vicinity of the mined-out workings of the wall 203, which would interfere and disturb the measured results.

The observations suggest that the points situated directly above the mined-out field and in a close distance tend to indicate some delays in the change of direction of horizontal displacement vector in relation to front advancement. The delay is caused by continuous movements of rock mass above the earlier mined-out portion of the wall 204 and, to a certain degree, it can also result from the disturbance of rock mass structure (fracture, voids etc.). The model assuming the constant advancement of longwall exploitation, worked out on the basis of Knothe theory, was presented by Sroka (1978). The remaining measurement points situated in the further area of the wall 204 only react to the exploitation within the range of a certain radius of the main influences.

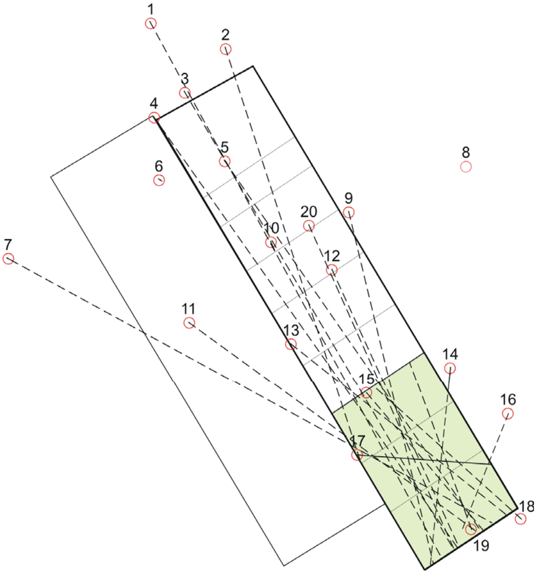


Fig. 13. Projections of horizontal displacement vectors' directions for measurement 5

5. Conclusions

The paper discussed the development and currently applied methods of determining the values of horizontal displacement of surface points enhanced by underground mining exploitation. In relation to the COG method, Awierszyn hypothesis and results of model studies of Strata Mechanics Research Institute of the Polish Academy of Sciences in Cracow, it was argued that they indicate the proportionality between the horizontal displacement vector and the vector of surface slope. In fact, the differences refer to the value of proportionality coefficient B, whose values in currently realized design works for mining industry range between $0.23r$ and $0.42r$ for deep exploitations. However, in the present publication the values of $0.33r$ and $0.47r$ were obtained for two examples of shallow mining exploitation. In addition, the observations of direction changes of horizontal displacement vectors alongside with the front advancement proved the possibility of existence of COG zone above the mined-out field, which clearly confirmed the conclusions from the hitherto carried out research projects.

The project was financed from the means of National Science Centre granted on the grounds of decision No. DEC – 2011/01/D/ST8/07280

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Received: 28 April 2014