www.czasopisma.pan.pl PAN www.journals.pan.pl
POLSKA AKADEMIA NAUK

DE GRUYTER
OPEN

Arch. Min. Sci., Vol. 60 (2015), No 3, p. 791–806

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.1515/amsc-2015-0052

JACEK MUCHA*, MONIKA WASILEWSKA-BŁASZCZYK*

INTERPOLATION AND SAMPLING ERRORS OF THE ASH AND SULPHUR CONTENTS IN SELECTED POLISH BITUMINOUS COAL DEPOSIT (UPPER SILESIAN COAL BASIN – USCB)

BŁĘDY INTERPOLACJI I OPRÓBOWANIA ZAWARTOŚCI POPIOŁU I SIARKI W WYTYPOWANYCH POLSKICH ZŁOŻACH WĘGLA KAMIENNEGO (GÓRNOŚLĄSKIE ZAGŁĘBIE WĘGLOWE)

The basic sources of information on the parameters characterizing the quality of coal (i.e. its ash and sulphur contents) in the deposits of The Upper Silesian Coal Basin (Poland) are drill core sampling (the first stage of exploration) and channel sampling in mine workings (the second stage of exploration). Boreholes are irregularly spaced but provide relatively uniform coverage over an entire deposit area. Channel samples are taken regularly in mine workings, but only in the developed parts of the deposit.

The present study considers selected seams of two mines. The methodology used is based on detailed geostatistical analysis, point kriging procedure and P. Gy's theory of sampling. Its purpose is:

- defining and comparing geostatistical models for variability of the ash and sulphur contents for data originating from boreholes and mine workings,
- predicting by means of point kriging the values of the parameters and errors of interpolation using data from boreholes at grid points where underground mine workings were later channel-sampled,
- assessing the accuracy of interpolation by comparison of predicted values of parameters with real values (found by channel sampling),
- evaluating the variances of total secondary sampling error (error of preparation of assay samples) and analytical error introduced by assaying of sulphur and ash,
- assessing the contribution of sampling and analytical errors (global estimation error) to the interpolation errors.

The authors found that the interpolation errors for ash or sulphur content are very large, with mean relative values of 35%-60%, mainly caused by the considerable natural variability, a significant role of random component of variability, and heterogeneity of spatial distribution of these characteristics. The sampling and analytical errors play a negligible role. Their values are smaller than 11% of interpolation error values. Presenting estimates of the spatial distribution of ash and sulphur contents in coal seams by means of contour maps is unreasonable if they are based on drill core sampling.

Keywords: geostatistics, contour maps, sampling errors, interpolation accuracy

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, FACULTY OF GEOLOGY, GEOPHYSICS AND ENVIRONMENTAL PROTECTION, DEPARTMENT OF ECONOMIC AND MINING GEOLOGY, AL. MICKIEWICZ 30, 30-059 KRAKÓW, POLAND

Prawo geologiczne i górnicze a w szczególności Rozporządzenie Ministra Środowiska w sprawie dokumentacji geologicznej złoża kopaliny z dnia 22 grudnia 2011 r. nakłada na dokumentatora złoża obowiązek ilustrowania zmienności jakości kopaliny za pomocą map. W odniesieniu do złóż węgla kamiennego rozmieszczenie wartości parametrów charakteryzujących jego jakość najczęściej wizualizuje się za pomocą map izoliniowych zawartości siarki, zawartości popiołu lub wartości opałowej konstruowanych w oparciu o wyniki ich interpolacji w regularnej sieci interpolacyjnej znacznie zagęszczonej w stosunku do sieci rozpoznania złoża. Podstawowymi źródłami informacji o wartościach tych parametrów w warunkach złóż Górnośląskiego Zagłębia Węglowego są wyniki opróbowania rdzeni z otworów wiertniczych (we wczesnych fazach rozpoznania złoża) i wyniki opróbowania bruzdowego pokładów w wyrobiskach górniczych (w zaawansowanych fazach rozpoznania złoża). Otwory wiertnicze są wykonywane w nieregularnej sieci ale z reguły zapewniają jednolite pokrycie złoża. Próby bruzdowe w wyrobiskach górniczych pobierane są na ogół ze stałym rozstawem ale charakteryzują jedynie części złóż rozciętych wyrobiskami.

W artykule przestawiono wyniki badań nad wielkością błędów interpolacji zawartości siarki i popiołu oraz na ich tle przedstawiono oszacowania błędów opróbowania wykonane dla dwóch pokładów kopalni Murcki (330 i 334/2) oraz jednego pokładu kopalni Brzeszcze (405/1) (Fig. 1 i 2). Metodyczną podstawę badań stanowiła statystyczna i geostatystyczna analiza zmienności wyników opróbowań, geostatystyczna procedura interpolacji tych parametrów za pomocą zwyczajnego krigingu punktowego oraz dla określenia błędów przygotowania próbki do analizy chemicznej elementy teorii opróbowania Gy (Mucha i Wasilewska, 2006).

Cele badań były wielokierunkowe i obejmowały:

- określenie i porównanie geostatystycznych modeli zmienności zawartości siarki i popiołu dla danych pochodzących z z opróbowania rdzeni wiertniczych i wyrobisk górniczych,
- prognozowanie za pomocą krigingu zwyczajnego na podstawie danych otworowych wartości parametrów w punktach złoża opróbowanych w późniejszej fazie rozpoznania w wyrobiskach górniczych,
- ocenę dokładności interpolacji przez porównanie wartości parametrów prognozowanych na podstawie danych otworowych z danymi z wyrobisk górniczych, które stanowiły zbiór referencyjny,
- ocenę wariancji błędów przygotowania próbek do analizy chemicznej w oparciu o formułę Gy (wzór (1)) oraz błędów oznaczeń zawartości siarki i popiołu na podstawie oznaczeń kontrolnych,
- ocenę udziału błędów opróbowania i oznaczeń zawartości siarki i popiołu w błędach interpolacji.

Wyniki analizy statystycznej nie pozwalają z jednym wyjątkiem (zawartość siarki w pokładzie 405/1 kopalni Brzeszcze) na odrzucenie hipotezy o identyczności rozkładów prawdopodobieństwa parametrów dla danych z otworów wiertniczych i wyrobisk górniczych z ryzykiem błędu mniejszym od 5% (Fig. 3). Wyniki geostatystycznego modelowania zmienności są niejednoznaczne i pokazują w dwóch przypadkach (zawartość siarki w pokładzie 405/1 kopalni Brzeszcze i zawartości popiołu w pokładzie 334/2) drastyczne różnice w postaci modeli dla danych otworowych i górniczych. (Fig. 4). W pozostałych przypadkach modele dla obu rodzajów danych wykazują zadowalającą zgodność.

Interpolacja wartości parametrów na podstawie danych otworowych za pomocą procedury krigingu punktowego charakteryzuje się niską dokładnością ze średnimi względnymi (absolutnymi) błędami rzędu 35%-60% (Tab. 1). Przyczyn niskiej dokładności interpolacji należy upatrywać przede wszystkim w dość dużej zmienności zawartości siarki i popiołu (ze współczynnikami zmienności od 38% do 77%) (Fig. 3) oraz w znaczącym udziałem losowego składnika ich zmienności reprezentowanego na modelach semiwariogramów przez silnie zaznaczoną wartość parametru modeli C_0 (nugget effect) (Fig. 4). Znajduje to potwierdzenie w mapach izoliniowych sporządzonych metodą krigingu dla zawartości siarki (Fig. 5) i zawartości popiołu w węglu (Fig. 6) dla pokładu 330 w kopalni Murcki gdzie rezultaty interpolacji dokonanej metodą krigingu punktowego są najdokładniejsze.

Przebiegi izolinii dla danych otworowych i z wyrobisk górniczych wykazują zauważalne różnice, podobnie jak i prognozowane błędy interpolacji. Wysokie, rzeczywiste średnie błędy interpolacji zawartości siarki (35%) i zawartości popiołu (47%) dowodzą, że możliwości wiarygodnej predykcji wartości tych parametrów w punktach złoża na podstawie danych otworowych są dalece niewystarczające dla planowania eksploatacji uśredniającej.

Błędy opróbowania określone w oparciu o teoretyczną formułę Gy (wzór 1, Fig. 7) oraz błędy oznaczeń zawartości siarki i popiołu ocenione na podstawie oznaczeń kontrolnych maja marginalne znaczenie. Wariancja tych błędów stanowi zaledwie od 3% do 11% średniego kwadratowego błędu interpolacji (Fig. 8). Tak więc nie mają one zauważalnego wpływu na dokładność interpolacji. Wysokie wartości

błędów interpolacji zawartości siarki i popiołu czynią bezzasadnym ilustrowanie rozmieszczenia wartości tych parametrów w pokładach za pomocą map izoliniowych skonstruowanych na podstawie rzadko rozmieszczonych danych otworowych.

Słowa kluczowe: geostatystyka, mapy izoliniowe, błędy opróbowania, dokładność interpolacji

1. Introduction

The most common way of presenting the spatial distribution of various quality characteristics for coal seams in the Upper Silesian Coal Basin (USCB) in Poland includes contour maps. The maps are based upon interpolation of the parameters determined in the nodes of a square grid. In initial stages of exploration, contour maps of deposits are prepared on the basis of drilling results. They allow prediction of such coal quality characteristics as sulphur and ash contents in the parts of deposits being developed. The reliability of these maps is controlled by the accuracy (or, conversely, by the errors) of interpolation, and depends mainly upon the natural variability of the deposit and the density of the borehole grid, but is also affected by errors of sampling and assaying. Usually, no numerical estimate of the reliability of interpolation is presented.

2. Aim and scope of investigations

Investigations were carried out to establish the reliability of contour maps showing the amounts of sulphur and ash in selected coal seams of the USCB. They particularly deal with:

- assessment of accuracy of the interpolation made on the basis of drill core sampling,
- verification of the variances of interpolation errors predicted using the kriging method by comparing them with the variances of real interpolation errors,
- quantitative, theoretical evaluation of the variance of errors made during the comminution and preparation for assaying of samples taken from coal seams (total secondary sampling error) and analytical (assaying) error,
- assessment of the contribution of sampling and analytical errors to interpolation errors.

3. Materials

The investigations were based on the results of assaying for total sulphur (S_t^r) and ash (A^r) in samples collected from drill cores and underground workings in coal seams No's 330 and 334/2 in the Murcki Mine as well as assaying for total sulphur (S_t^r) in samples collected from underground development and exploitation workings in coal seam No. 405/1 in the Brzeszcze Mine (Fig. 1). It must be emphasized that pyrite constitutes main source of sulphur in coal seams.

These seams and mines were selected because there is sufficient amount of data for reliable statistical and geostatistical operations to be made. In Polish mines of bituminous coal, systematic seam sampling for assessing coal quality parameters is a rare practice and the sampling grid is usually irregular. Fig. 2 presents the locations of samples collected from drill cores and mine workings, whose sulphur and ash assays were used by the authors.

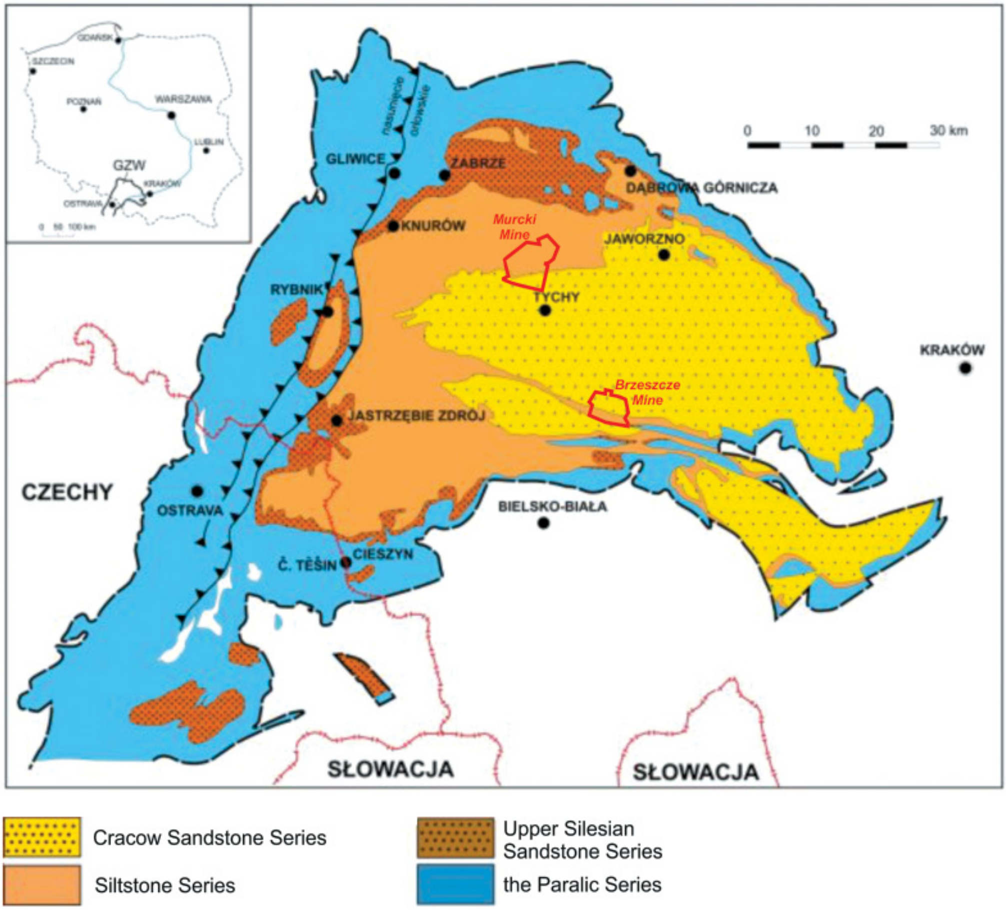

Fig. 1. Localization of the Murcki and Brzeszcze mines (in red) within the Upper Silesian Coal Basin (according to PIG-PIB Warszawa), http://geoportal.pgi.gov.pl/css/zrozum_z/images/jkc/Rys_JKCw1.jpg)

4. Methods

The sets of the numerical data obtained were divided into two groups. The first group is data that served as a basis for interpolation (results of sampling of drill cores in the Murcki Mine and of development workings in the Brzeszcze Mine), while the other group is data used in assessment of the accuracy of interpolation (results of sampling of exploitation workings in both mines). The two groups of data were statistically analysed, including preparation of histograms and calculation of basic variability measures, i.e. arithmetic mean, variance and coefficient of variation (Fig. 3). The authors have compared empirical distributions using the Kolmogorov-Smirnov test and compared medians of distributions by applying the Mann-Whitney test (Fig. 3).

The structure of variability of the sulphur and ash contents was studied using Matheron geostatistics (1963) by means of sample semivariograms that express the dependence between

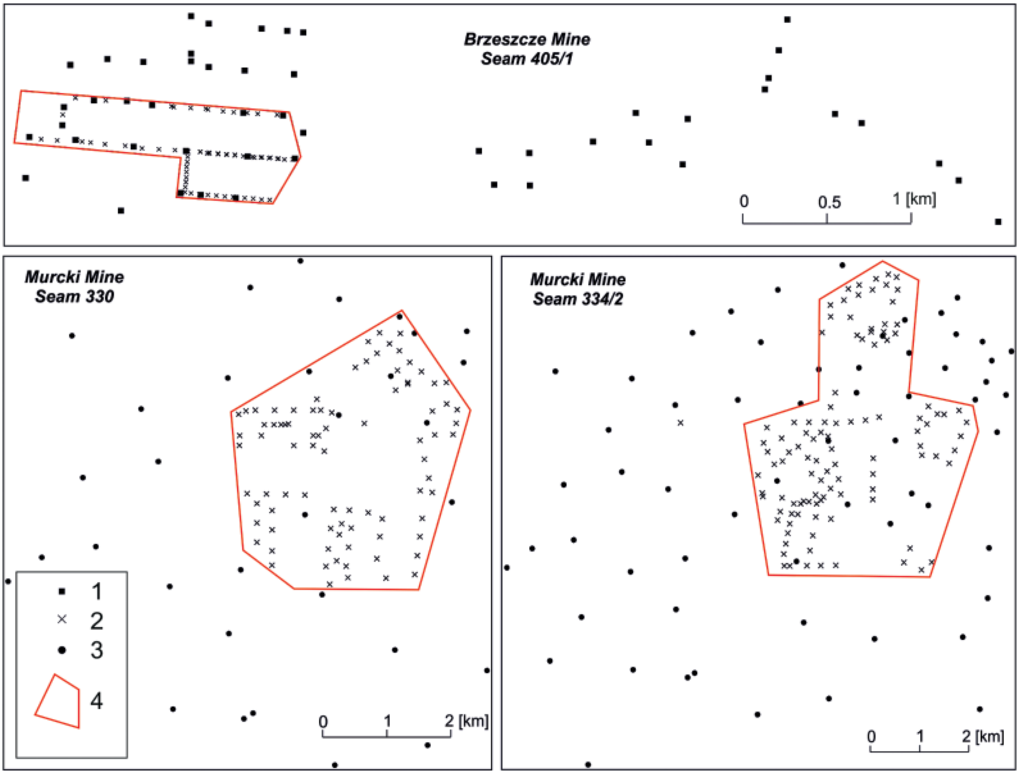

Fig. 2. Locations of the boreholes and underground sampling points.
1 – sampling points in development workings (Brzeszcze Mine), 2 – sampling points in exploitation workings (Murcki and Brzeszcze mines), 3 – boreholes sampled, 4 – area of mining exploration

the mean squared differences of the parameter values and the average distance between sampling points. The omnidirectional semivariograms calculated for each of the seams, separately for the data obtained from drill core sampling and those from underground sampling, along with theoretical fitting models are presented in Fig. 4. The reliability of the models was checked by cross-validation (Armstrong, 1998).

The values of the deposit parameters analysed were interpolated using the geostatistical point kriging method (Isaaks, Srivastava 1989). Its advantage stems from the fact that it estimates the value at a given point as a weighted average and provides an estimate of the magnitude of the interpolation error, considering in both cases the model of the semivariogram assumed, the positions of the sampling points for which the value of the data is known, and the distances of those points from the point at which the value is estimated. The sulphur and ash contents were interpolated for each of the seam sampling points in underground exploitation workings using data from the eight closest boreholes (Murcki Mine) or the eight nearest points of sampling in development workings (Brzeszcze Mine). The differences between the values interpolated and the values later measured in the workings were used as a measure of interpolation accuracy. Assuming a high accuracy of sampling and assaying, these differences may be regarded as real interpolation errors. The average sizes of the real interpolation errors were compared with estimates of the reliability predicted using point kriging. The accuracy of interpolations is displayed on contour maps of the data analysed. These contour maps also show predicted and real errors of interpolation (Fig. 5, Fig. 6).

The magnitude of the variance of total secondary sampling error was estimated using the theoretical formula of Gy (Gy 1983; Petersen, Minkkinen, Esbensen 2005), considering the flow sheet for analytical sample preparation (Fig. 7). The preparation of the assay portion involves two stages of sample size reduction with simultaneous reduction of the mass from 25 kg to 1 g (ash assaying) or 0.25 g (sulphur assaying). The relative variance of the fundamental error (FE) that appears during sample mass reduction was evaluated with a simplified formula (Mucha, Szuwarzyński, 2004).

$$\sigma^2\left[FE\right]=\sum_{i=1}^{N}\left(\frac{1}{Q_i}-\frac{1}{Q_{i-1}}\right)\cdot m\cdot f\cdot g\cdot l_i\cdot d_i^3 \tag{1}$$

where:

- N — number of stages of size reduction ($N = 2$),
- Q_i — mass of the sample at the "i-th" stage of size reduction [g] ($Q_0 = 25{,}000$g, $Q_1 = 150$g, $Q_2 = 1$ g for ash or 0.25g for sulphur),
- m — mineral constitution factor [g/cm^3], for sulphur: $m = 241.5$ g/cm^3 (mean pyrite content = 2% i.e. mean sulphur content ≈ 1%, specific densities of pyrite = 5.0 Mg/m^3 and coal = 1.4 Mg/m^3) and for ash: m = 15.6 g/cm^3 (mean ash content = 15%, specific density of ash = 3.0 Mg/m^3),
- f — grain shape factor ($f = 0.5$),
- g — granulometric distribution factor ($g = 0.5$),
- d_i — diameter of the largest grains of the material at the "i-th" stage of size reduction [cm] ($d_1 = 0.6$ cm, $d_2 = 0.02$ cm),
- l_i — factor of liberation of the mineral of interest; $l_i = (\frac{d_l}{d_i})^{1.5}$ (François-Bongarçon, Gy 2000), here it was used more conservative formula – $l_i = \sqrt{\frac{d_l}{d_i}}$ for $d_l \le d_i$, where d_l – liberation size – for pyrite = 0.002 cm hence for sulphur $l_1 = 0.058$ and $l_2 = 0.316$; the liberation size for ash is not known, therefore the safe value $l = 1$ was assumed.

The maximum relative variance of the total sampling errors [TSE], comprising selection and preparation errors, was obtained as a doubled variance of fundamental error [FE] what provides a safe estimate (Whateley & Scott, 2006). The variance of the chemical assaying error [AE] was assumed to be the same as for control assays for sulphur and ash carried out on two coal samples (internal standards) in 29 chemical laboratories of different coal mines within the USCB.

Variance of the global estimation error including combined total sampling and assaying errors was evaluated from the relation: $\sigma^2[GE] = \sigma^2[TSE] + \sigma^2[AE]$.

The variance of the total secondary sampling error and assaying error (global estimation error, according to Gy 1983, Whateley & Scott, 2006), relative to the mean squared error of interpolation are shown in Fig. 8.

5. Results

5.1. Statistical comparison of the data sets from drilling and mining explorations

The coefficients of variation of sulphur content range from 40% to 61%. Those of ash content cover the wider range of 38% to 77%. Both ranges indicate generally high variability of the parameters analysed. The distributions of sulphur and ash contents show strong positive asymmetry (Fig. 3). Standardized coefficients of asymmetry and of kurtosis for these distribu-

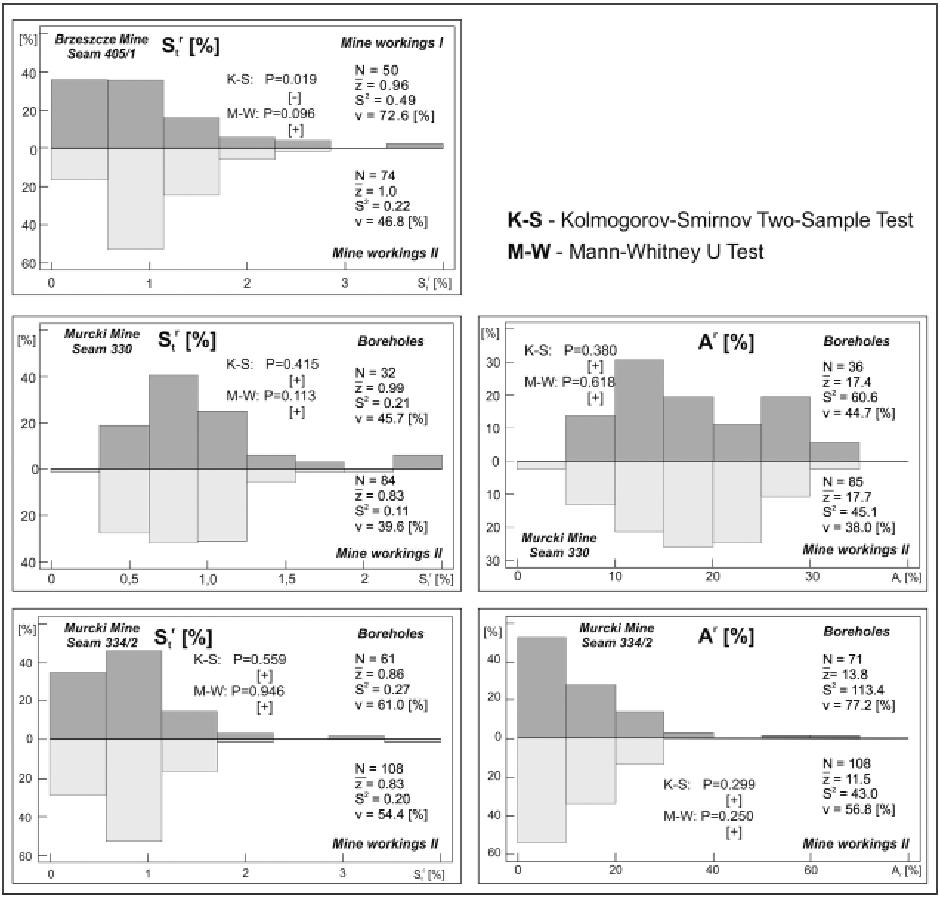

Fig. 3. Histograms of ash and sulphur contents for the data sets used in interpolation (upper section) and in estimation of interpolation accuracy (lower section).
N – number of data, z – arithmetic mean, S^2 – variance, v – coefficient of variation, K-S – Kolmogorov-Smirnov test to compare the distribution of the two samples, M-W – Mann-Whitney test to compare the medians of the two samples, [+] – there is not a statistically significant difference between the two distributions or the medians at the 95% confidence level, [-] there is a statistically significant difference between the two distributions or the medians at the 95% confidence level, I – development workings, II – exploitation workings

tions, except for the ash content in the seam No. 330 of the Murcki Mine, are higher than 2. These values indicate significant departures of the empirical distributions from normality. This means that the conditions for applying Snedecor's F-test for comparison of variances and Student's t-test for comparison of mean values have not been fulfilled for these data sets.

Therefore, the estimation of statistical similarity of both types of data sets have been made by comparing the empirical distributions of the parameters analysed (Kolmogorov-Smirnov test) and their medians (Mann-Whitney test) (Fig. 3). These tests do not provide a basis for rejecting (at the 95% confidence level) the hypothesis that the distributions or medians are identical for the sets of sulphur and ash contents in drill cores and underground workings. Statistically significant differences in distributions of the parameters analysed were only noted for the sulphur content of samples taken from underground development and exploitation workings in the Brzeszcze Mine.

5.2. Geostatistical models of parameter variability

Omnidirectional (isotropic) sample semivariograms were approximated using spherical models and, in single cases, the Gauss model and the linear model (Journel & Huijbregts, 1978) (Fig. 4). Cross-validation indicates that the models of semivariograms were properly selected (mean standardized differences between predicted and real values of parameters range from -0.08 to 0.06, while their standard deviations range from 0.98 to 1.05). The fitted semivariograms show a substantial variation in the random component (termed "nugget effect" by Royle 1991) from 21% to 75% for the sulphur content and from 12% to 82% for the ash content. Geostatistical models fitted using the data from exploration drillings (wider range of calculations) and from underground workings (shorter range of calculations) are consistent (i.e. they indicate similar semivariogram at intermediate ranges) for the sulphur content in seams 330 and 334/2 in the Murcki Mine and for the ash content in seam 330 of the same mine. In the remaining two cases they reveal a different structure of variability of both sulphur and ash for the two ranges of observations, possibly caused by differences between types and areas of exploration (Fig. 4).

5.3. Accuracy of interpolation of the sulphur and ash contents

Assessment of interpolation precision was based on the data obtained from sampling of underground workings (reference set). It reveals generally poor reliability of interpolation, expressed as a poor ability to predict the results of future channel sampling. The accuracy of interpolation has been quantitatively characterized by some measures shown in Table 1. On the basis of those measures, the following conclusions have been drawn.

- The differences between the predictions of geostatistical models fitted using data from drilling exploration and future measurements in underground workings are not consistent with the standard errors predicted by those geostatistical models. The values of cross-validation statistics ($\overline{\Delta}/s_{\Delta}$) deviate significantly from the ideal value ($\overline{\Delta} = 0/s_{\Delta} = 1$). This was particularly marked for ash content, but the deviation was sufficiently large that predictions of sulphur content must be regarded as untrustworthy.
- Interpolated values of sulphur and ash are often quite biased, with systematic errors ($\overline{\varepsilon}_R$), reaching 18% (see Table 1) in the cases analysed here. Mean relative errors, i.e. absolute ($\overline{\varepsilon}_{AR}$) and squared ($\overline{\Delta}_{IR}$), of the parameter estimation reach high values from 35% to 60% for sampling in underground workings.

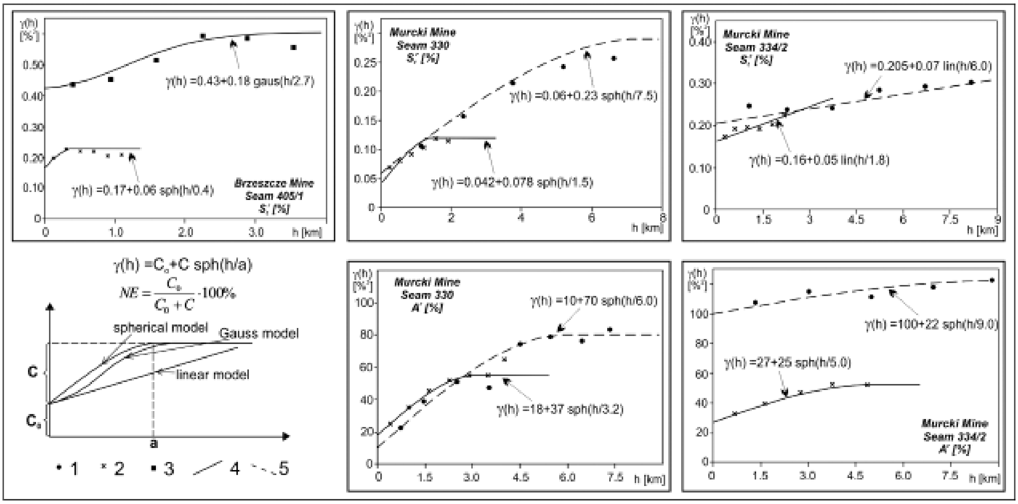

Fig. 4. Semivariograms and geostatistical models of the parameters analysed for the data from drilling and mining exploration (notation of model formulae according to Deutsch and Journel 1992). 1 – semivariogram for the data from drilling exploration, 2 – semivariogram for the data from mining exploration (exploitation workings), 3 – semivariogram for the data from mining exploration (development workings), 4 – semivariogram model for the data from mining exploration, 5 – semivariogram model for the data from drilling exploration, C_0 – variance of the local variability (nugget variance), C – variance of the spatial variability, a – the range of the semivariogram, NE – nugget effect

- Estimates of the variance of interpolation errors predicted by point kriging procedure ($\bar{\sigma}_{KR}$) are inaccurate by amounts ranging from 34 to 124%. They can drastically differ from the average variance of real interpolation errors.
- Coefficients of linear correlation r between the data and the predictions made using point kriging range from -0.13 to 0.41, which is regarded as surprisingly small.
- Assuming the normal distribution of errors, the absolute differences between the data and the values predicted using the kriging method should be smaller than the standard error predicted by kriging 68% of the time, and smaller than twice the kriging standard error 95% of the time. However, the numbers in Table 1 ($N\varepsilon$) reveal, except for sulphur in the seam No. 330 of the Murcki Mine, that the differences calculated are not in accordance with this expectation.

TABLE 1

Summary of the quality of interpolation for total sulphur content (S_t^r) and ash (A^r) contents by point kriging

Mine seam No. parameter	**Cross-validation $\bar{\Delta}/s_\Delta$ (eq. 2-4)**	**$\bar{\varepsilon}_R$ [%] (eq. 5)**	**$\bar{\varepsilon}_{AR}$ [%] [Lq-Uq] (eq. 6)**	**$\bar{\Delta}_{IR}$ [%] (eq. 7)**	**$\bar{\sigma}_{KR}$ [%] [Lq-Uq]**	**$r(z-z_K^*)$**	**$N\varepsilon(\%) < 1\sigma_K$ $N\varepsilon(\%) < 2\sigma_K$**
1	2	3	4	5	6	7	8
Brzeszcze, 405/1 S_t^r	0.25/0.71	18.0	58.5 [19.3-83.6]	51.9	84.6 [49.6-110.0]	-0.13	86.7/100

Murcki, 330 S_t^r	-0.05/0.94	-2.4	35.3 [11.8-45.0]	37.6	47.1 [32.4-54.2]	0.32	69.0/96.4
Murcki, 334/2 S_t^r	-0.29/0.96	-16.9	49.0 [17.1-55.7]	58.9	77.7 [48.1-91.1]	-0.01	74.1/98.1
Murcki, 330 A^r	0.50/1.27	14.0	47.1 [12.5-47.4]	38.1	33.9 [21.7-38.5]	0.41	50.6/87.1
Murcki, 334/2 A^r	0.02/0.63	1.6	59.7 [20.5-82.6]	58.4	124.2 [70.0-166.9]	0.12	88.0/100

$\overline{\Delta}/s_\Delta$ – measures of the cross-validation procedure: mean of the standardized estimation errors/standard deviation of the standardized estimation errors, $\overline{\varepsilon}_R$ – mean relative interpolation error, $\overline{\varepsilon}_{AR}$ – mean relative absolute interpolation error, $\overline{\Delta}_{IR}$ – mean squared relative interpolation error, $\overline{\sigma}_{KR}$ – mean relative kriging standard error (predicted interpolation error), $r(z - z_K^*)$ – coefficient of linear correlation between the predicted (z_K^*) and the observed (z) values of the parameter at sampling points, $N\varepsilon(\%) < 1\sigma_K$, $N\varepsilon(\%) < 2\sigma_K$ – percentage of interpolation errors lower than: kriging standard error / twice kriging standard error

The statistical parameters (Tab. 1) were calculated using following equations:

$$\Delta_i = \frac{z_{iK}^* - z_i}{\sigma_{iK}} \tag{2}$$

$$\overline{\Delta} = \frac{1}{N}\sum_{i=1}^{N}\Delta_i \tag{3}$$

$$s_\Delta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\Delta_i - \overline{\Delta})^2} \tag{4}$$

$$\overline{\varepsilon}_R = \frac{\frac{1}{N}\sum_{i=1}^{N}(z_{iK}^* - z_i)}{\overline{z}} \times 100\% \tag{5}$$

$$\overline{\varepsilon}_{AR} = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|z_{iK}^* - z_i\right|}{z_i} \times 100\% \tag{6}$$

$$\overline{\Delta}_{IR} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(z_{iK}^* - z_i\right)^2}}{\overline{z}} \times 100\% \tag{7}$$

where:

z_{iK}^* — the predicted with point kriging values of the parameter at sampling point "*i*",
z_i — the observed values of the parameter at sampling point "*i*".

The investigations of interpolation accuracy described above have also been visualized using contour maps of the sulphur content (Fig. 5) and the ash content (Fig. 6) for seam No. 330 of the Murcki Mine, for which the results of interpolation are the most accurate.

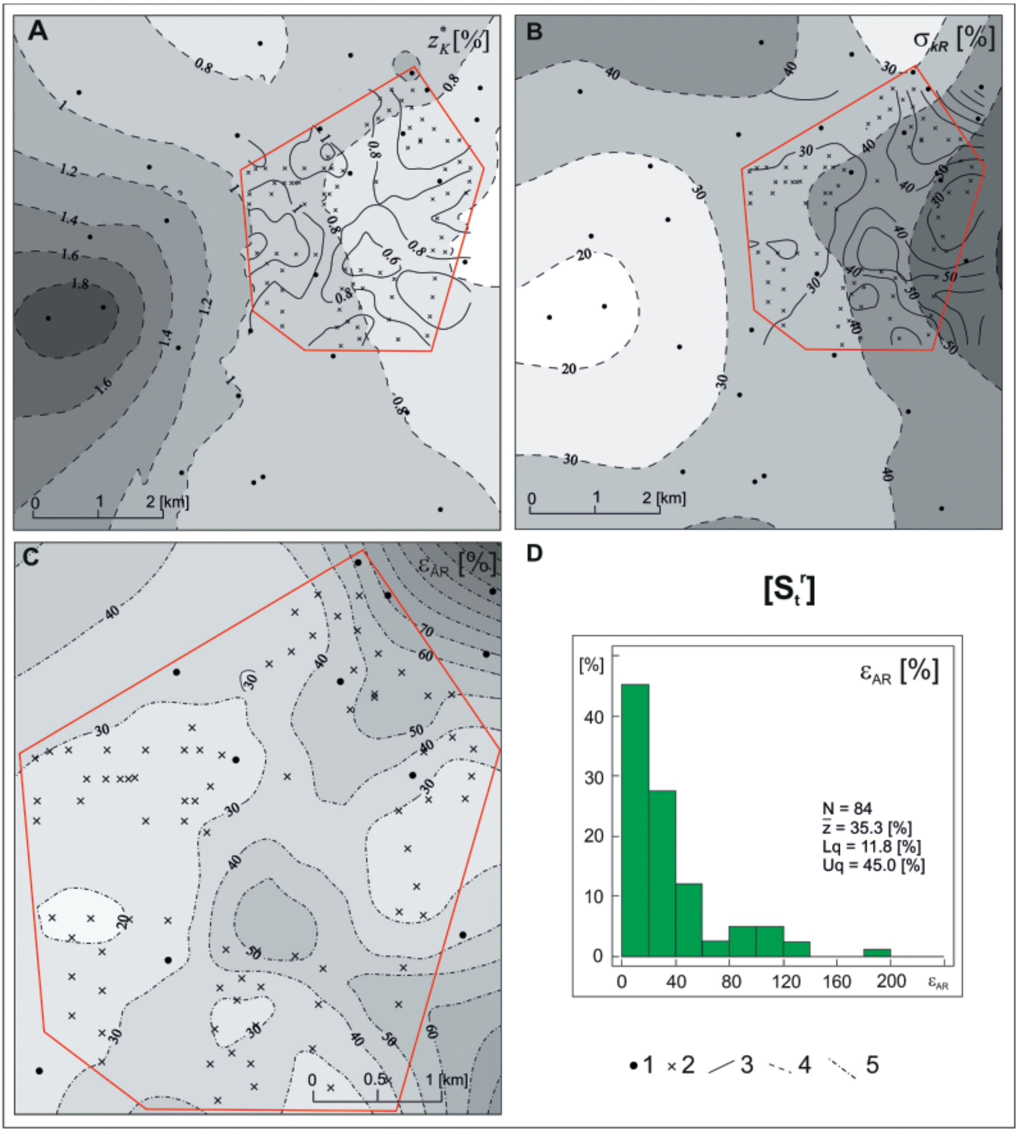

Fig. 5. Kriging contour maps: A – sulphur content (z_K^*), B – predicted relative interpolation errors ($\bar{\sigma}_{KR}$), C – real relative absolute interpolation errors ($\bar{\varepsilon}_{AR}$); D – histogram of relative interpolation errors ($\bar{\varepsilon}_{AR}$); seam No. 330, Murcki Mine.
1 – boreholes, 2 – underground sampling points, 3 – contours of sulphur contents and relative kriging errors based on results of channel sampling in mine workings, 4 – contours of sulphur contents and relative kriging errors based on drilling data, 5 – contours of absolute relative interpolation errors, N – number of data, $\bar{z}$ – mean value of a parameter, Lq – lower quartile, Uq – upper quartile

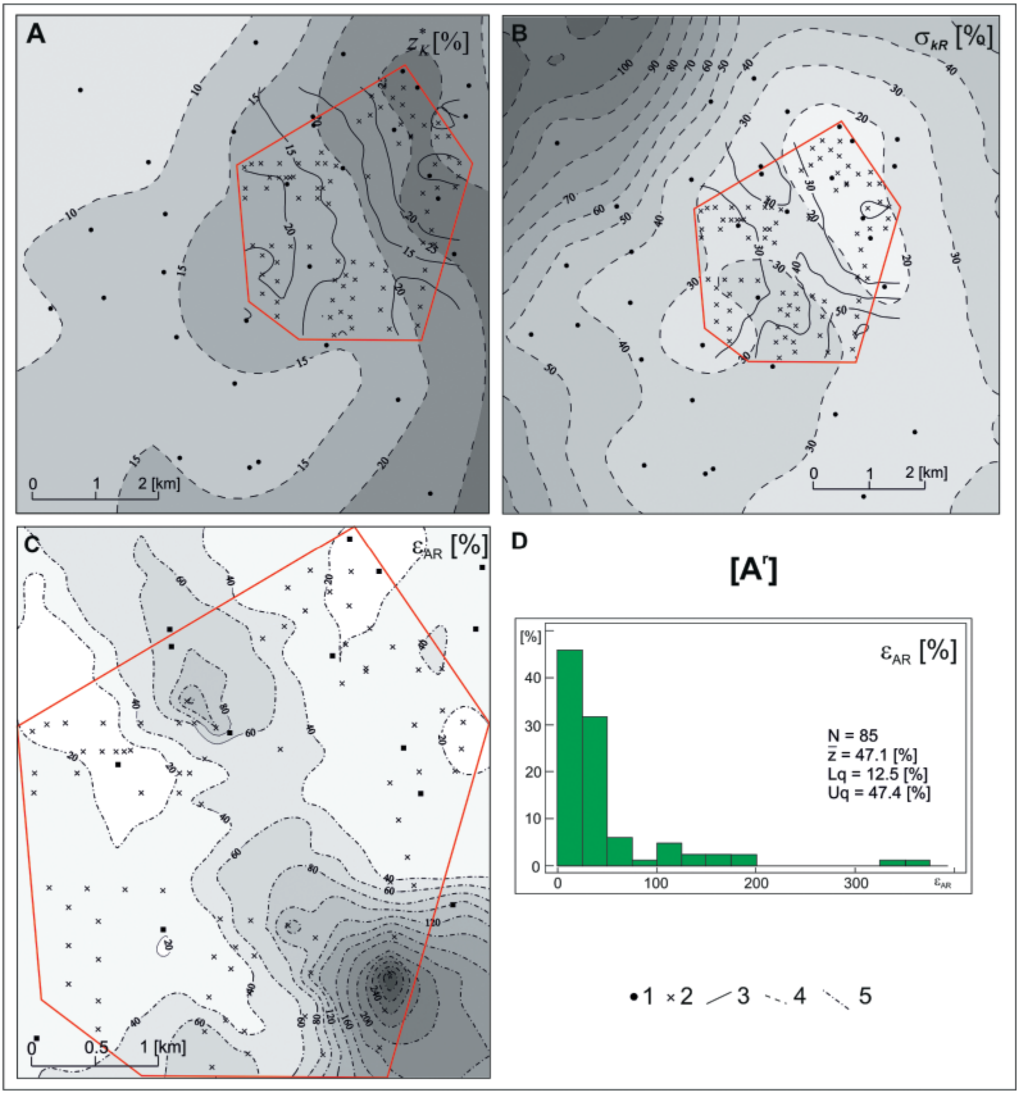

Fig. 6. Kriging contour maps: A – ash content (z_K^*), B – predicted relative interpolation errors ($\overline{\sigma}_{KR}$), C – real relative absolute interpolation errors ($\overline{\varepsilon}_{AR}$); D – histogram of relative absolute interpolation errors ($\overline{\varepsilon}_{AR}$); seam No. 330, Murcki Mine.

1 – boreholes, 2 – underground sampling points, 3 – contours of ash contents and relative kriging errors based onr results of channel sampling in mine workings, 4 – contours of ash contents and relative kriging errors based on drilling data, 5 – contours of absolute relative interpolation errors, *N* – number of data, $\overline{z}$ – mean value of a parameter, *Lq* – lower quartile, *Uq* – upper quartile

The contour maps based on point kriging show spatial distributions rather different for the data obtained from drilling and the data from underground workings (Fig. 5A and 6A). Comparison of kriging using the data from boreholes and the data from underground workings (Fig. 5B and 6B) shows substantial discrepancy between the locations of contour lines.

Accuracy of interpolation has been presented as a contour map of relative absolute interpolation errors within the area of mining exploration (Fig. 5C and 6C) and histograms of these errors (Fig. 5D and 6D). Large interpolation errors for the sulphur content and extremely large errors for the ash content suggest that point prediction of these quantities is insufficiently accurate to be useful in mine planning.

5.4. Total sampling and assaying errors

The relative total sampling errors [*TSE*] evaluated from P.Gy´s formula are identical for both characteristics of quality of bituminous coal and amount to 10.6% (Fig. 7). The relative standard errors of assaying [*AE*] estimated from control assays on two coal samples used as internal standards are distinctly lower, amounting to 1.9% for the ash content and 6.1% for the sulphur content.

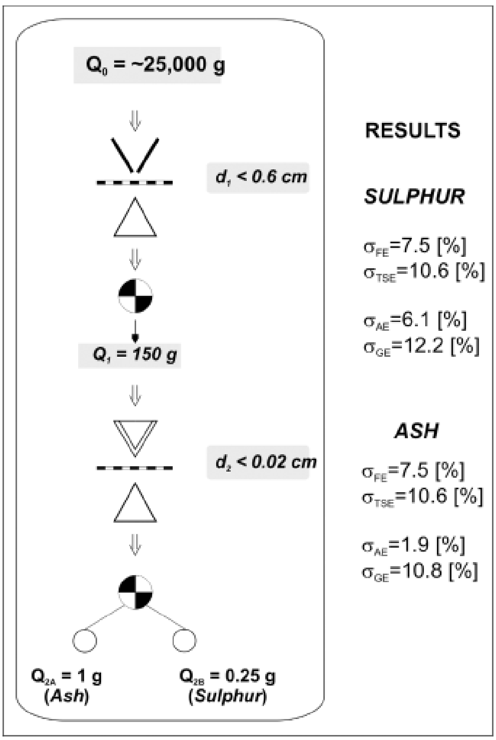

Fig. 7. Flow sheet of preparation of analytical samples.
σ_{FE}, σ_{TSE} – fundamental and total sampling errors, σ_{AE} – analytical error, σ_{GE} – global estimation error, Q_i – mass of the sample at the i-th stage of comminution, d_i – maximum diameter of grains at the i-th stage of comminution

The global estimation errors [*GE*] can be evaluated as 10.8% for ash content and 12.2% for sulphur content. The assessed accuracy of sampling and analytical procedures can be recognized as acceptable in mining practice.

5.5. Impact of total sampling and analytical errors on the accuracy of interpolation

The total variance of sampling and analytical errors can be added to the variance of parameters in the small scale of observation (microvariability) to determine the value of the nugget variance. In equations of semivariogram models, the nugget variance is the free term C_0 (Fig. 4). For the data analysed, the variances of sampling and analytical errors contribute in 3-35% to the nugget variances established for the data from drilling exploration. Therefore, they should not measurably affect interpolation errors. This conclusion is corroborated by the diagrams in Fig. 8, showing mean contributions of the combined variance of total sampling and analytical errors (global estimation error) to the mean squared interpolation errors. Their proportionate contribution is 3 to 11%. Hence total sampling and analytical errors can be treated as having only a marginal impact on prediction errors.

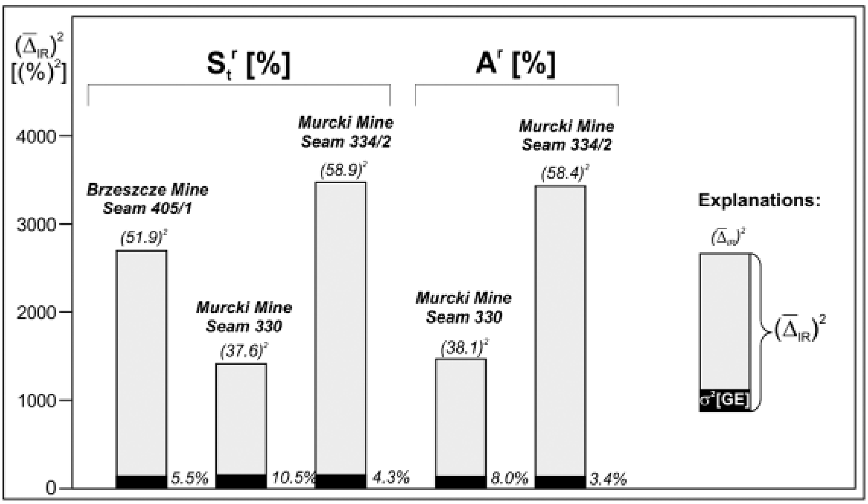

Fig. 8. Contribution (%) of variance of global estimation error to the mean squared error of interpolation for sulphur (S_t^r) and ash (A^r) contents.
$(\overline{\Delta}_{IR})^2$ – mean squared error of interpolation, $\sigma^2[GE]$ – variance of global estimation error

6. Summary and conclusions

The reliability of the predicted spatial distributions of the sulphur and ash contents represented as the contour maps for the selected seams of the Upper Silesian Coal Basin (Poland) is very low. Prediction of these quantities using point kriging is generally of low accuracy, particularly for the ash content. This prediction is burdened with a large mean relative absolute interpolation error, ranging from 35 to 60%. Quite often, the interpolated values are biased. Additionally, the predicted variance of interpolation errors can substantially differ from the observed variance of interpolation errors. It must be stressed that the authors have obtained similar results using nonlinear geostatistical methods such as the indicator and probability kriging.

The variance of sample preparation errors (i.e. errors of comminution and preparation for assaying of samples collected in underground workings or from drill cores) and the variance of errors of assaying are not major problems for the accuracy of interpolation. The variances of these types of errors do not exceed 11% of the variance of interpolation errors. This means that improving the accuracy of preparation of analytical samples and of assaying would not significantly improve the quality of interpolation.

The low interpolation accuracy can be explained, first of all, by enough high variability of the sulphur and ash contents in coal seams and considerable contribution of the random component of variability. An additional factor lowering the accuracy of interpolation may include heterogeneous spatial distribution of the sulphur and ash contents in the seams, statistically resulting in significant differences of empirical distributions or medians between various parts of the seams. Considering the high variability of sulphur and ash in other seams and mines of the USCB, it seems very likely that the authors conclusion of low interpolation accuracy can be projected onto the whole area of the USCB.

Summarising, the contour maps of the spatial distribution of sulphur and ash contents in coal seams predicted in the early stages of exploration on the basis of drilling data have no predictive value at all. These maps do not provide the sound and reliable prediction of the sulphur and ash values in the parts of a deposit that will be mined in the future.

The research project was financed from the AGH University of Science and Technology grant No. 11.11.140.320.

References

Armstrong M., 1998. *Basic Linear Geostatistics.* Springer-Verlag, Berlin Heidelberg New York, 115-116.

Deutsch C.V., Journel A.G., 1992. *GSLIB. Geostatistical Software Library and User's Guide.* Oxford University Press, New York, 23 p.

François-Bongarçon D., Gy P., 2000. *The most common error in applying "Gy's formula" in the theory of mineral sampling, and the history of the liberation factor*. Monograph 2000 "Towards 2000", AusIMM.

Gy P., 1983. *Les erreurs d'echantillonage.* Analysis, 11(9), p. 413-440.

Isaaks E.H., Srivastava R.M., 1989. *An Introduction to Applied Geostatistics.* Oxford University Press, New York, p. 142, 290-296.

Journel A.C., Huijbregts Ch.J., 1978. *Mining Geostatistics.* Academic Press, London, 164-165.

Matheron G., 1963. *Principles of Geostatistics.* Econ. Geol., 58, 1246-1266.

Mucha J., Szuwarzyński M., 2004. *Sampling errors and their influence on accuracy of zinc and lead content evaluation in ore from the Trzebionka mine (Silesian-Cracow Zn-Pb ore district, Poland)*. Chemometrics and Intelligent Laboratory Systems. Special Issue: 50 years of Pierre Gy's Theory of Sampling. Proceedings WCSB1, Eds: K.H. Esbensen, P. Minkkinen, Elsevier, vol. 74, 1, 165-170.

Mucha J., Wasilewska M., 2006. *Teoria opróbowania Gy i przykłady jej zastosowań w geologii górniczej w Polsce*. Przegląd Górniczy, nr 12, 33-38.

Petersen L., Minkkinen P., Esbensen K.H., 2005. *Representative sampling for reliable data analysis: Theory of Sampling*. Chemometrics and Intelligent Laboratory Systems, Vol. 77, 261-277.

Royle A.G., 1991. *Kriging.* Leeds University Mining Association, Leeds, 39-47.

Whateley M.K.G., Scott B.C., 2006. *Evaluation Techniques*. [In:] Introduction To Mineral Exploration. Eds.: Moon C.J., Whateley M.K.G, Evans A.M., Blackwell Publishing, 199-252.

Received: 05 May 2014