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## Determination of transient fluid temperature using the inverse method

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**Abstract** This paper proposes an inverse method to obtain accurate measurements of the transient temperature of fluid. A method for unit step and linear rise of temperature is presented. For this purpose, the thermometer housing is modelled as a full cylindrical element (with no inner hole), divided into four control volumes. Using the control volume method, the heat balance equations can be written for each of the nodes for each of the control volumes. Thus, for a known temperature in the middle of the cylindrical element, the distribution of temperature in three nodes and heat flux at the outer surface were obtained. For a known value of the heat transfer coefficient the temperature of the fluid can be calculated using the boundary condition. Additionally, results of experimental research are presented. The research was carried out during the start-up of an experimental installation, which comprises: a steam generator unit, an installation for boiler feed water treatment, a tray-type deaerator, a blow down flash-vessel for heat recovery, a steam pressure reduction station, a boiler control system and a steam header made of martensitic high alloy P91 steel. Based on temperature measurements made in the steam header using the inverse method, accurate measurements of the transient temperature of the steam were obtained. The results of the calculations are compared with the real temperature of the steam, which can be determined for a known pressure and enthalpy.

**Keywords:** Temperature measurement; Transient conditions; Uncertainty analysis

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## Nomenclature

$Bi$	–	Biot number
$c(T)$	–	specific heat, J/(kgK)
$c_p$	–	specific heat at constant pressure, J/(kgK)
$d$	–	diameter of the thermometer, m
$f(t)$	–	temperature, °C
$ Fo$	–	Fourier number
$h$	–	heat transfer coefficient, W/(m <sup>2</sup> K)
$h_k$	–	heat transfer coefficient on the surface on which the steam is condensed, W/(m <sup>2</sup> K)
$h_s$	–	enthalpy of the steam, kJ/kg
$J_0, J_1$	–	Bessel functions of the first kind for integer order 0 and 1
$k(T)$	–	thermal conductivity, W/(mK)
$M$	–	number of time steps
$N$	–	number of measurement points
$Nu_d$	–	Nusselt number
$\dot{q}_v$	–	energy generation rate per unit volume (uniform within the body), W/m <sup>3</sup>
$\dot{Q}_i$	–	rate of heat, W
$p$	–	pressure, kPa
$r$	–	radius, m
$r_{out}$	–	radius of the outer surface, m
$Re$	–	Reynolds number
$Pr$	–	Prandtl number
$s_N$	–	mean square error, °C
$t$	–	time, s
$T$	–	temperature, °C
$T_0$	–	initial temperature, °C
$T_f$	–	fluid temperature, °C
$T_i$	–	temperature in the $i$ node, °C
$T_m$	–	measured temperature, °C
$T_P$	–	temperature inside the cell under consideration, °C
$v_T$	–	temperature change rate, K/s
$w$	–	fluid velocity, m/s
$X$	–	constant value

## Greek symbols

$\Delta r$	–	spatial step in radial direction, m
$\Delta t$	–	time step, s
$\Delta V$	–	volume of single cell, m <sup>3</sup>
$\varepsilon$	–	accuracy of calculation, K
$\kappa$	–	coefficient of thermal diffusivity, m <sup>2</sup> /s
$\mu$	–	dynamic viscosity, kg/(m·s)
$\nu$	–	kinematic viscosity, m <sup>2</sup> /s
$\rho(T)$	–	density, kg/m <sup>3</sup>
$\vartheta$	–	time, s

## 1 Introduction

Most studies on temperature measurements concentrate on steady-state measurements of the fluid temperature. Only the unit-step response of thermometers is considered to estimate the dynamic error of the temperature measurement. Little attention is paid to measurements of transient fluid temperature, despite the great practical significance of the problem.

It is very difficult to measure the transient temperature of steam or flue gases in thermal power stations. Massive housings and low heat transfer coefficients cause the measured temperature to differ significantly from the actual temperature of the fluid [1,2]. Some particularly heavy thermometers have time constants of three minutes or longer, which require about 15 minutes for a single measurement. On the other hand, some thermometer designs include more than one time constant in order to describe the transient response of a temperature sensor immersed in a thermowell. Measuring the temperature of a medium in a controlled process may require having two or three time constants that characterise the transient thermometer response.

The problem of dynamic errors in temperature measurements of superheated steam becomes particularly important when superheated steam temperature control systems use injection coolers (spray attenuators). Due to the large inertia of the thermometer, the measurement of the transient temperature of the fluid can be inaccurate, adversely affecting the automatic control of the superheated steam system. A similar problem is encountered in flue gas temperature measurements, because the thermometer time constant and time delay are large [3].

## 2 Inverse method to obtain accurate transient temperature of fluid

An inverse marching method determines the distribution of temperature inside the wall of a thermometer and the fluid temperature.

The heat equation for a cylindrical wall is [4]

$$c(T) \rho(T) \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ k(T) r \frac{\partial T}{\partial r} \right], \quad (1)$$

where  $0 \leq r \leq r_{out}$ .

The solution of the inverse problem is obtained by using the control volume method [5]

$$\Delta V c(T_P) \rho(T_P) \frac{dT_P}{dt} = \sum_{i=1}^4 \dot{Q}_i + \Delta V \dot{q}_v(T_P). \quad (2)$$

For this purpose, the thermometer housing is modelled as a full cylindrical element (with no inner hole), divided into four control volumes (Fig. 1). Using the control volume method, the heat balance equations can be written for each of the nodes for each of the control volumes.

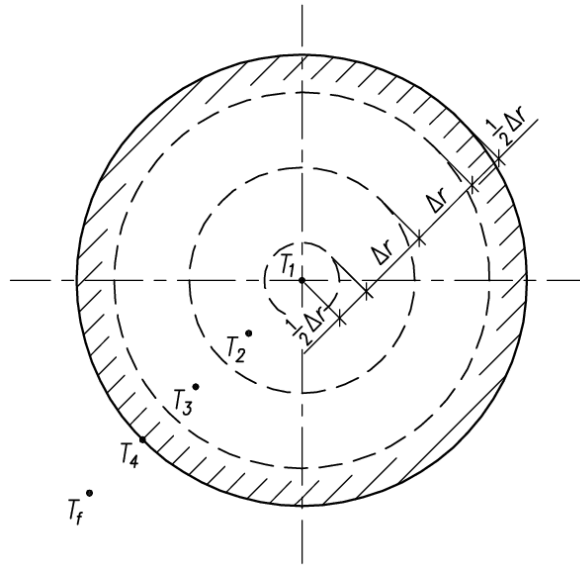


Figure 1. Dividing the cylindrical element into four control volumes.

The heat balance equations for each of the nodes of the control volume are

$$T_2 = T_1 + \frac{1}{2} \frac{(\Delta r)^2}{k(T_1) + k(T_2)} c(T_1) \rho(T_1) \frac{dT_1}{dt}, \quad (3)$$

$$T_3 = T_2 - \frac{1}{3} \frac{k(T_1) + k(T_2)}{k(T_3) + k(T_2)} (T_1 - T_2) + \frac{4}{3} (\Delta r)^2 \frac{c(T_2) \rho(T_2)}{k(T_3) + k(T_2)} \frac{dT_2}{dt}, \quad (4)$$

$$T_4 = T_3 - \frac{3}{5} \frac{k(T_2) + k(T_3)}{k(T_4) + k(T_3)} (T_2 - T_3) + \frac{8}{5} (\Delta r)^2 \frac{c(T_3) \rho(T_3)}{k(T_4) + k(T_3)} \frac{dT_3}{dt}, \quad (5)$$

$$T_f = T_4 - \frac{5}{12} \frac{k(T_3) + k(T_4)}{h\Delta r} (T_3 - T_4) + \frac{11}{24} \Delta r \frac{c(T_4) \rho(T_4)}{h} \frac{dT_4}{dt}, \quad (6)$$

where  $\Delta r = \frac{r_{out}}{3}$ . The temperature of fluid  $T_f$  can be obtained by substituting for  $T_1$ , the measured temperature by thermometer and subsequently substituting the obtained values of temperature in the following equations. Values of thermal conductivity  $k(T_i)$  for  $T_i$  (where  $i = 2, 3, 4$ ) are unknown. It is necessary to execute several iterations to obtain temperature  $T_i$ . It was assumed that in each of the nodes for the first iteration

$$k(T_i^{(0)}) = k(T_{i-1}). \quad (7)$$

The iteration continues until [4]:

$$\left| T_{i-1}^{(n+1)} - T_{i-1}^{(n)} \right| \leq \varepsilon, \quad (8)$$

where  $\varepsilon \approx 0.0001$  K.

The described method was used to determine transient fluid temperature (for unit step and linear rise). However, the temperature at the centre of the solid cylinder was calculated initially using an analytical method.

### 3 Analytical method used to generate data in order to validate inverse method

The heat conduction equation for a cylindrical wall (1) can be written in a different way after assuming temperature independent physical properties of the thermometer material

$$\frac{\partial \theta}{\partial t} = \kappa \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right), \quad (9)$$

where  $0 \leq r \leq r_{out}$ .

The solution of Eq. (9) will be obtained for the following boundary conditions

$$\left. \frac{\partial \theta}{\partial r} \right|_{r=0} = 0, \quad (10)$$

$$-k \left. \frac{\partial \theta}{\partial r} \right|_{r=r_{out}} = h \theta|_{r=r_{out}} \quad (11)$$

and initial condition:

$$\theta|_{r=0} = \theta_0, \quad (12)$$

where  $\theta = T(r, t) - T_f$  and  $\theta_0 = T_0 - T_f$ .

The temperature distribution inside the wall when the fluid temperature is the unit step function, is presented by [6]

$$u(r, t) = \frac{\theta}{\theta_0} = \frac{T(r, t) - T_f}{T_0 - T_f} = 2 \sum_{n=1}^{\infty} \frac{J_1(\mu_n) J_0(\mu_n R)}{\mu_n [J_0^2(\mu_n) + J_1^2(\mu_n)]} e^{-\mu_n^2 \text{Fo}} \quad (13)$$

where  $\text{Fo} = \frac{\kappa t}{r_{out}^2}$ ,  $R = \frac{r}{r_{out}}$ . The symbol  $\mu_n$  denotes roots of equation [6]:

$$\frac{J_0(\mu)}{J_1(\mu)} = \frac{\mu}{\text{Bi}}, \quad (14)$$

where  $\text{Bi} = \frac{hr_{out}}{k}$ .

For the linear rise of fluid temperature  $f(t)$ , the temperature distribution inside the wall is determined by using the Duhamel integral [7]

$$T(r, t) = T_0 + \int_0^t f(\vartheta) \frac{\partial u(r, t - \vartheta)}{\partial t} d\vartheta, \quad (15)$$

where  $u$  is step function. It is applied for uniform distribution of initial temperature,  $T_0$ , irrespective of position, inside the cylindrical element. The temperature field is determined based on function  $u(r, t)$  (calculated using Eq. (13)), which is temperature distribution in the cylinder for a unit stepwise temperature increase in fluid temperature

$$f(t) = \begin{cases} 0, & t < 0, \\ 1, & t > 0. \end{cases} \quad (16)$$

The function  $u(r, t)$  is also called as the influence function.

The real fluid temperature  $f(t)$  is replaced by a stepwise function (Fig. 2). Next, approximating the derivative in Eq. (15) by a finite difference quotient, the temperature distribution is as follows [7]

$$\begin{aligned} T(r, t) - T_0 = & f_1 [u(r, t_M - \vartheta_0) - u(r, t_M - \vartheta_1)] + \\ & + f_2 [u(r, t_M - \vartheta_1) - u(r, t_M - \vartheta_2)] + \\ & \vdots \\ & + f_M [u(r, t_M - \vartheta_{M-1}) - u(r, t_M - \vartheta_M)], \end{aligned} \quad (17)$$

where  $u(r, t_M - \vartheta_M) = u(r, 0) = 0$  and  $\vartheta_0 = 0$ .

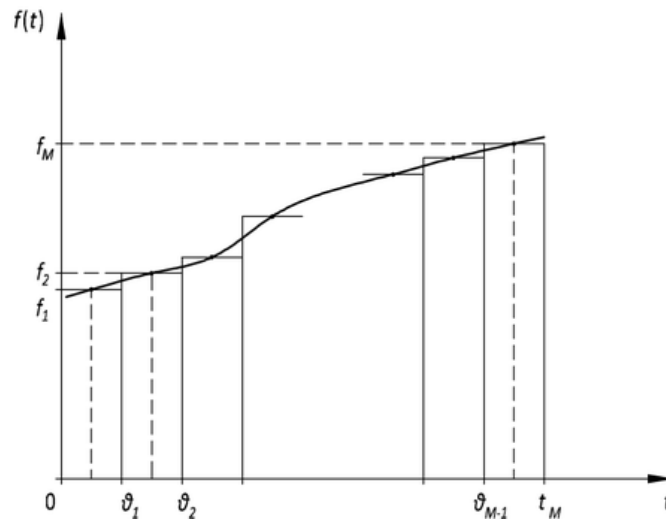


Figure 2. Approximation of fluid temperature.

#### 4 Computational validation of the inverse method

The calculations were made for a thermometer with an outer diameter of 7.0 mm, made of 1.4541 (austenitic steel), with the following physical properties:  $\rho = 7.9 \times 10^3 \text{ kg/m}^3$ ,  $k = 18 \text{ W/(m K)}$ ,  $\kappa = 4.55696 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $c = 500 \text{ J/(kg K)}$ . The heat transfer coefficient at the outer surface is  $h = 2000 \text{ W/(m}^2\text{K)}$ . The results of the inverse calculations for the step wise fluid temperature change from 20 to 100 °C and linear rise of fluid temperature:  $T_f = T_0 + v_T \times t$  from 0 to 170 °C with  $v_T = 0.33333 \text{ K/s}$  will be presented. The time step for the thermometer was determined by the following equation [4]:

$$\Delta t \geq 0.05 r_{out}^2 / \kappa \quad (18)$$

and its value is 0.2 s.

Numerical calculations were conducted using the FORTRAN programming language. Roots of the transcendental equation (14) were obtained using the procedure ZREAL from the IMSL library [10]. Results of calculations of the temperature distribution at the outer surfaces of the thermometer and inside the wall ( $r_1 = 2.33333 \text{ mm}$  and  $r_2 = 1.6667 \text{ mm}$ ) by an analytical exact method and by the inverse method, are presented in Fig. 3.

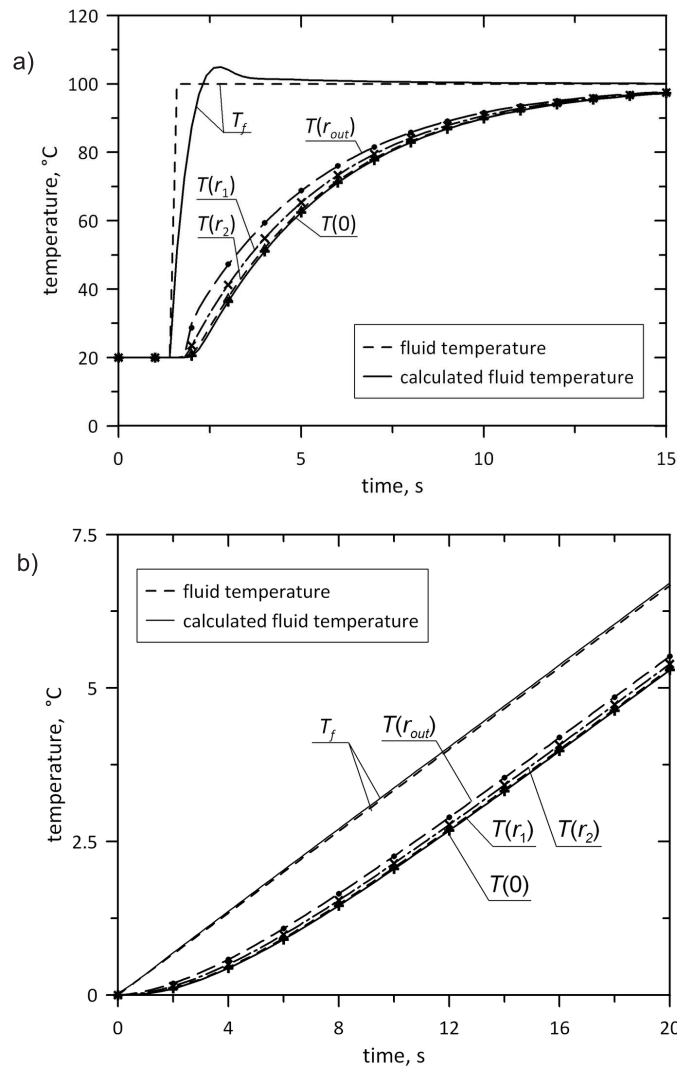


Figure 3. The results of calculations of fluid temperature and temperature distribution at the outer surface and inside the cylindrical element (analytical method – solid lines, inverse method – dashed line with symbols) with an outer diameter of 7.0 mm: a) step of temperature; b) linear rise of temperature.

A comparison of the results shown in Figs. 3a and 3b show that better compatibility between the fluid temperature determined from the solution of the inverse problem, and fluid the temperature adopted for the solution of



the direct problem (exact temperature) was achieved with a linear increase in temperature of the medium. It should be noted that the results for the stepwise fluid temperature change are also very satisfactory (Fig. 3a).

## 5 Experimental validation of the inverse method

The inverse method was validated. The research entailed measuring the temperature of steam during the start-up of an experimental installation, using a thermometer with an outer diameter of 7.0 mm (Fig. 4). The thermometer was placed in the steam header, which is made of martensitic high alloy P91 steel. The installation also comprises: a steam generator unit, an installation for boiler feed water treatment, a tray-type deaerator, a blow down flash-vessel for heat recovery, a steam pressure reduction station and a boiler control system (Fig. 5) [8].

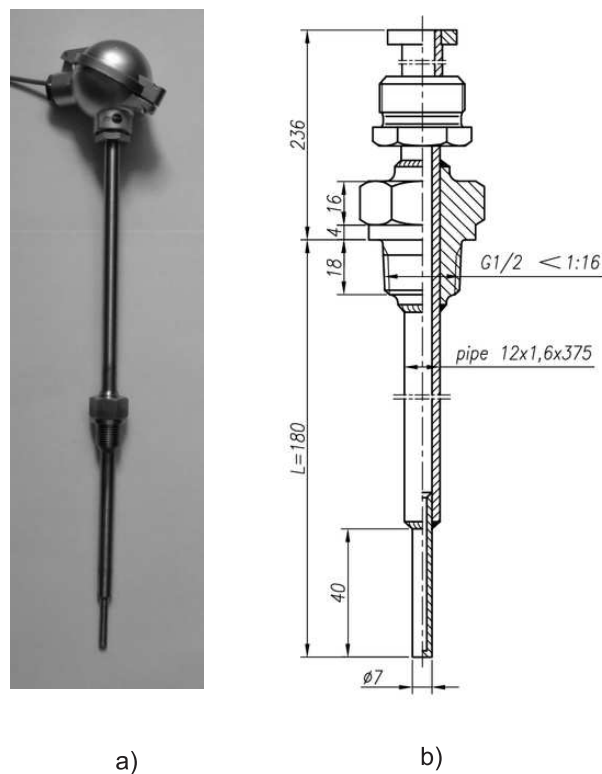


Figure 4. Thermometers with outer diameter 7.0 mm: a) general view, b) construction.

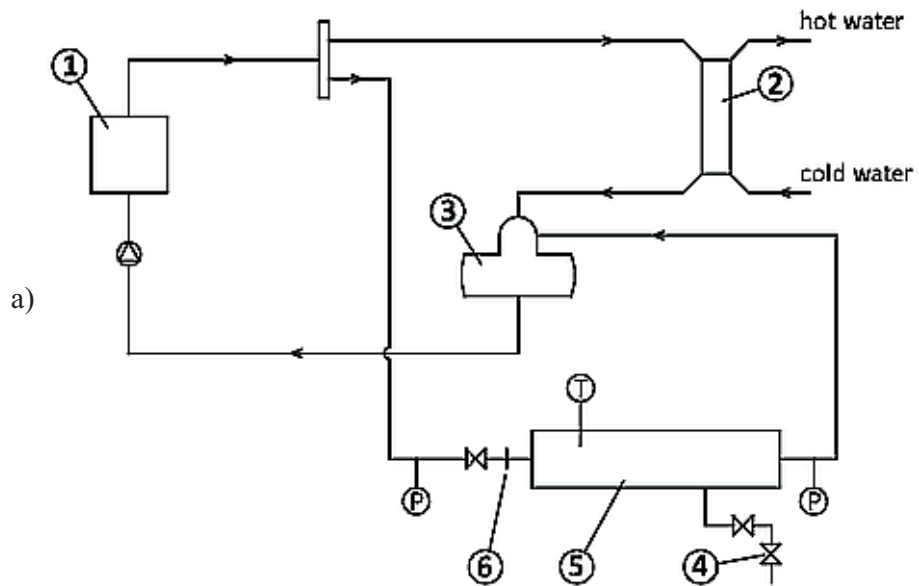


Figure 5. Experimental installation: a) main components of experimental facility: 1 – steam boiler, 2 – heat exchanger, 3 – deaerator, 4 – drain valve, 5 – steam header, 6 – orifice, P – pressure measuring device, T – thermometer; b) general view.

The measurements were recorded by a data acquisition system with a time step  $\Delta t = 1$  s. The calculations of temperature were based on temperature measurements made by a thermocouple in the massive housings, for the period of time from start-up of the boiler and after opening the valve before the steam header.

The boiler (1) produces saturated steam. After expansion of dry saturated steam at orifice (6), it becomes the superheated steam. Considering that the steam expansion at the orifice is an isenthalpic process and knowing the vapour pressure behind the orifice, the temperature  $T_f$  of the superheated steam can easily be determined as a function of pressure,  $p$ , and enthalpy,  $h_s$ , i.e.,  $T_f = T_f(p, h_s)$ .

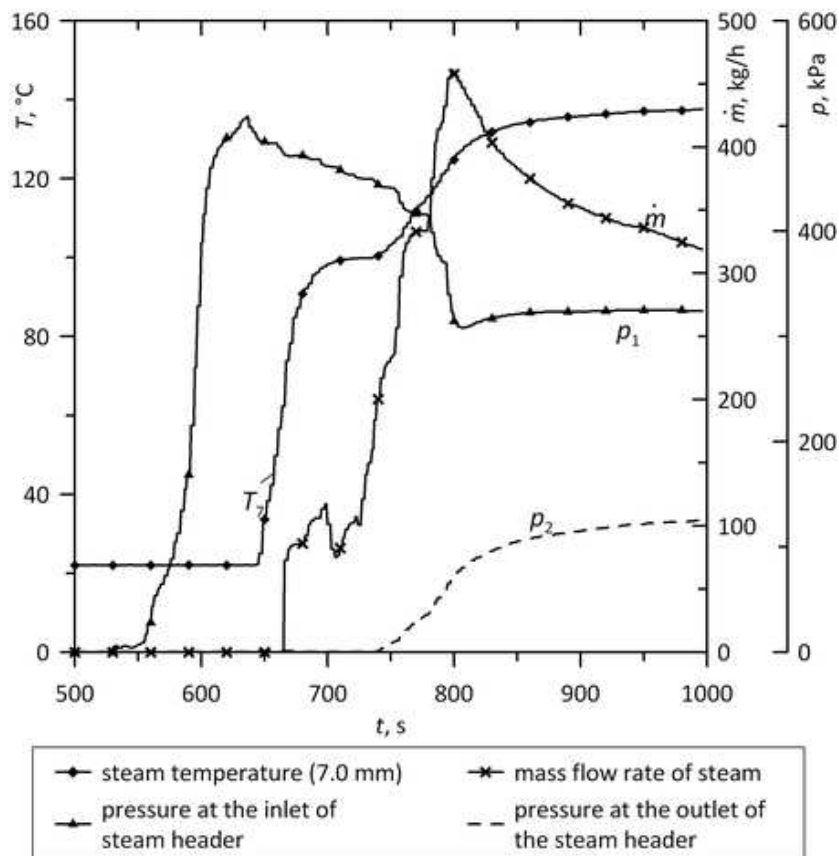


Figure 6. Measurements of temperature and pressure at the inlet and outlet of the steam header and mass flow rate of the steam during start-up of the steam boiler.

Figure 6 shows two temperature surges. The first of them occurred at  $t = 642$  s at a temperature equal to about  $100$  °C and the other at  $t = 739$  s at a temperature equal to about  $140$  °C. The opening of the valve at the inlet of the steam header and the drain valve (4) caused the first temperature jump; the pressure in the header was  $101.3$  kPa (atmospheric pressure). The second was caused by closing the drain valve; the pressure in the header rapidly rose.

Numerical calculations of the temperature were made using the FORTRAN programming language. The solution of the inverse problem was obtained based on the assumption that the physical properties of the housing material of the thermometer (1.4541 austenitic steel) are temperature dependent, and those physical properties are as follow

$$\rho(T) = 7900 + 0 \cdot T, \quad (19)$$

$$k(T) = 14.902547 + 0.01197859 T, \quad (20)$$

$$c(T) = 489.78959 + 0.21336286 T. \quad (21)$$

The cylindrical element was divided into four control volumes. The temperature in each node and the fluid was calculated using Eqs. (3)-(6). Because the values of thermal conductivity  $k(T_i)$  for  $T_i$  are unknown, it is necessary to make several iterations to obtain the temperature  $T_i$ .

It was assumed that in each of the nodes for the first iteration

$$k(T_i^{(0)}) = k(T_{i-1}). \quad (22)$$

The iteration continues until  $|T_{i-1}^{(n+1)} - T_{i-1}^{(n)}| \leq \varepsilon$ , where  $\varepsilon \approx 0.0001$  K. To partially eliminate the influence of random errors on the results of the calculation, a 9-point digital filter was used.

When the steam condensed on the thermometer, a constant heat transfer coefficient,  $h_k$ , was assumed. When the steam mass flow rapidly increased, the heat transfer coefficient was determined based on the Nusselt number, defined by the modified correlation of Churchill and Bernstein [9]

$$\text{Nu}_d = 0.3 + \frac{X \cdot \text{Re}_d^{1/2} \text{Pr}^{1/3}}{\left[1 + (0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\text{Re}_d}{282000}\right)^{1/2}\right], \quad (23)$$

where

$$\text{Nu}_d = \frac{h_k d}{k}, \quad \text{Re}_d = \frac{w d}{\nu}, \quad \text{Pr} = \frac{c_p \mu}{k}.$$

and  $X$  is the unknown constant value. Equation (22) is used when the fluid flows around the cylinder and [9]

$$\text{Re}_d \text{Pr} > 0.2, \quad 10\,000 < \text{Re}_d < 400\,000 .$$

The modification of Eq. (23) was based on introducing a constant  $X$ , instead of the constant equal to 0.62. In order to determine the value of the constant  $X$  and the heat transfer coefficient  $h_k$ , the least squares method was used. In both cases, the minimum value of the sum of the temperature difference squares was found by the method of interval searching. In a time interval  $500 \text{ s} \leq t < 739 \text{ s}$ , in which steam condensation takes place on the surface of the thermometer, the heat transfer coefficient was equal to a constant value of  $h_k = 550 \text{ W}/(\text{m}^2\text{K})$ . After opening the the drain valve at time  $t = 739 \text{ s}$  steam expansion occurs at the orifice and around the thermometer flows superheated steam. At this period ( $t \geq 739 \text{ s}$ ) the heat transfer coefficient at the thermometer surface was calculated using Eq. (23) with  $X = 1.13$ .

The time step was obtained using formula [4]

$$\Delta t \geq 0.05 \cdot r_{out}^2 / \kappa \quad (24)$$

and depends on the frequency of the download data from the acquisition system. The value of the time step  $\Delta t$ , used for the thermometer is 1.0 s.

The results of the calculations of the fluid temperature are shown in Fig. 7. The means square error of measurements was obtained using the following formula:

$$s_N = \sqrt{\frac{S}{N-1}}, \quad (25)$$

where

$$S = \sum_{i=1}^N [T_m(t_i) - T(t_i)]^2. \quad (26)$$

The result of calculation for the thermometer with an outer diameter of 7.0 mm is  $s_N = 11.77 \text{ }^\circ\text{C}$ .

The temperature of the saturated steam during the interval  $500 \leq t < 739 \text{ s}$  was determined from the measured pressure and for time  $t \geq 739 \text{ s}$ , the temperature of superheated steam was determined on the basis of enthalpy  $h_s$  and pressure,  $p$ . The steam temperature determined in this way was compared with the steam temperature obtained from the inverse solution (Fig. 7). An inspection of the results depicted in Fig. 7 indicates a good

agreement between the real steam temperature and calculated values obtained by the method proposed in the paper.

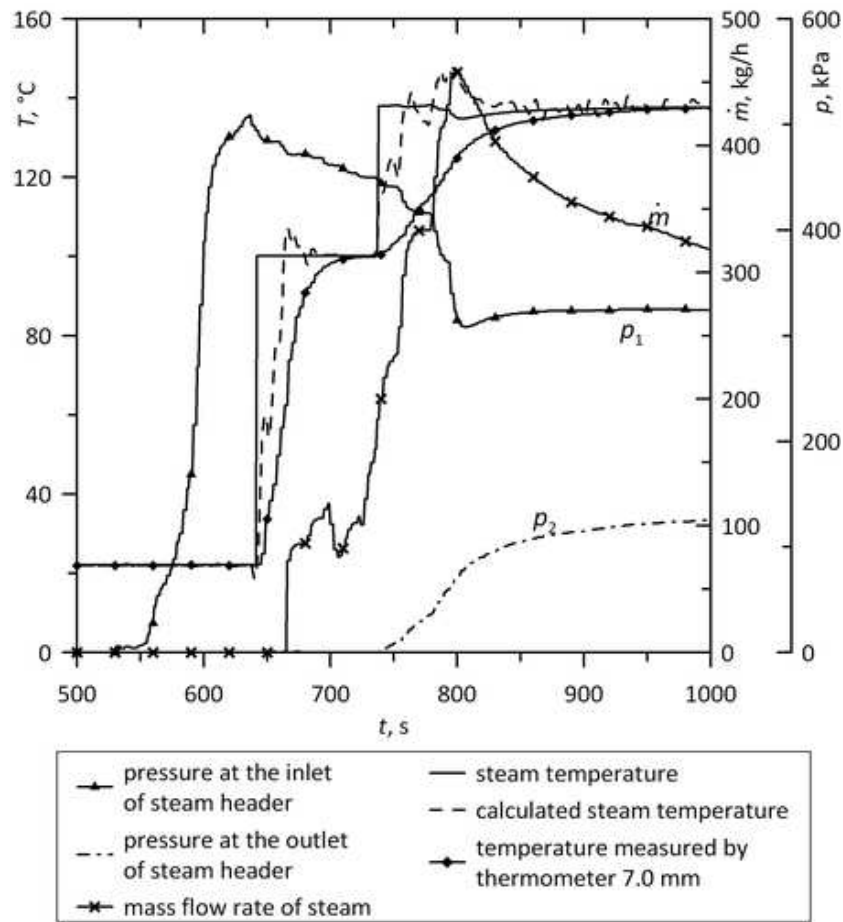


Figure 7. The correction of the measured temperature by the thermometer with an outer diameter of 7.0 mm,  $p_1$  – pressure at the inlet of steam header,  $p_2$  – pressure at the outlet of steam header,  $\dot{m}$  – mass flow rate of steam.

## 6 Conclusions

The paper presents an accurate inverse marching method to determine transient fluid temperature. Initially, the inverse method was validated computationally. Comparison of the generated fluid temperature with the cal-

culated fluid temperature gave good results. Subsequently, the presented method was validated using experimental data. The results obtained from the calculations also gave good results as indicated by the value of the mean square error. Large stability and accuracy in the determination of the actual fluid temperature, based on the time temperature changes indicated by the thermometer, can be achieved by using a 9 point digital filter.

The method can be used when measurements of temperature are conducted using industrial thermometers with massive housings and when the pressure of the fluid is high. The knowledge of the exact value of fluid temperature is very important for proper determination of thermal stresses. It is also very important in automatic control systems, for example in superheated steam temperature control systems. The presented method can be used for online applications. Another advantage of this method is the ease of its practical application.

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