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INSURANCE AND SELF-INSURANCE – SUBSTITUTES OR COMPLEMENTS?

1. INTRODUCTION

Individuals facing a potential loss may undertake various efforts to protect themselves against risk. One of them is market insurance, but there are possible alternatives to it. Ehrlich and Becker (1972) were first to present and systematically analyze concepts of self-insurance and self-protection. Self-insurance is defined as an effort made towards a reduction in the size of a loss, whereas self-protection leads to reduction in the probability of a loss. Ehrlich and Becker showed that market insurance and self-insurance are substitutes, but market insurance and self-protection may be complements or substitutes, depending on the initial probability of the loss. That result was confirmed by Courbage (2001) in Yaari's Dual Theory of Choice setting. Over time, concept of self-protection has attracted many researchers appearing to be more complex and interesting phenomenon than self-insurance. The reason is that self-insurance reduces large losses in the bad state more effectively than smaller loss in the good state and therefore may be considered as a type of insurance. However, it is no longer true in more general model that takes into the account many states of the world.

It is easy to see that under decreasing absolute risk aversion (DARA) self-insurance (as well as insurance) is inferior. However, Lee (2010a) showed that if the model provides for many states of the world then with DARA insurance is inferior but self-insurance may by inferior or normal, depending on productivity of self-insurance. Therefore, in more general setting self-insurance cannot be considered as special type of insurance.

The effect of an increase in risk aversion on self-insurance is another important question. Dionne, Eeckhoudt (1985) and Bryis, Schlesinger (1990) proved that more risk-averse individuals invest more in self-insurance. Lee (2010b) again generalized the model to many states and presented conditions for more risk averse individuals to invest more or less in self-protection.

It is therefore interesting whether Ehrlich and Becker's classical result about substitutability of market insurance and self-insurance does hold in more general and realistic model with multiple states of the world. It has not been analyzed yet in the literature. This paper fills that gap to some extent. We present sufficient conditions for self-insurance and market insurance to be substitutes or complements, making use of

Diamond and Stiglitz „single crossing condition" and the notion of supermodularity. We also provide economic interpretation of that result. The key concept here is an effectivity of self-insurance, which reflects its technology.

The result presented in this paper has certain limitations. We consider here only the case when level of market insurance is exogeneous. It represents the situation when insurance is obligatory, forced by law. It usually depends on the country, in Poland there are a number of cases of mandatory insurance regulated by the Compulsory Insurance Act (2003). As in all EU countries, all vehicles must have third party liability insurance. It is mandatory for a car owner to take out insurance against injury and damage. Third-party liability is mandatory to any person who owns a farm. Also the insurance of farm buildings from fire and other accidents is compulsory. There are several more examples of compulsory insurance and in those cases insurance cannot be considered as decision variable. The problem is, when insurance is mandatory then there is no demand in the usual sense. For that reason terms „substitutes" or „complements" may seem to be inappropriate in that context. Nevertheless, we define and use them as a description of reaction of the demand for self-insurance generated by increase in price of market insurance.

2. THE MODEL

Consider a risk-averse individual who has initial wealth w_0 that is subject to possible loss. The size of a loss depends on the state of the world θ is denoted by $l(\theta)$. θ is a continuous random variable such that $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, with the density function $f(\theta)$. Without loss of generality we assume that a state with higher θ represents larger loss, that is $l'(\theta) > 0$. We denote full insurance cost by π, and an individual has bought an insurance coverage $\alpha l(\theta)$ for a premium $\alpha\pi$ where factor α is determined by law and $\alpha \in [0,1]$. Moreover, he may independently invest in self-insurance that also reduces the loss. In this model, effects of insurance and self-insurance are separated in order to capture interactions between them. The amount invested in self-insurance is e (it is decision variable in our model), and it leads to reduction in loss by $d(e, \theta)$. By the definition, an increase in self-insurance reduces the loss and it is reasonable and customary to assume that reduction happens at a decreasing rate. Therefore we have $d_e(e,\theta) = \partial d(e,\theta)/\partial e > 0$ and $d_{ee}(e,\theta) < 0$. It is also assumed that the same self-insurance activity cannot lead to higher reduction of the loss in the worse state, so we have $d_\theta(e,\theta) \leq 0$. It seems like technical assumption, but typical examples of self-insurance show that it is not restrictive. Violation of that condition might lead to the conclusion that it would be profitable to incur larger loss, which makes no sense.

The final wealth in the state θ is thus

$$w = w_0 - e - \alpha\pi - (1 - \alpha)[l(\theta) - d(e,\theta)]. \tag{1}$$

Let us denote $L(e, \theta) = l(\theta) - d(e, \theta)$. Hence

$$w = w_0 - e - \alpha\pi - (1 - \alpha)L(e, \theta). \tag{2}$$

Due to the above assumptions, we have

$$L_\theta(e, \theta) = l'(\theta) - d_\theta(e, \theta) > 0, \, L_e(e, \theta) = -d_e(e, \theta) < 0. \tag{3}$$

As a consequence of our assumptions we obtain that

$$w_\theta = -(1 - \alpha)L_\theta(e, \theta) \leq 0, \tag{4}$$

which reads that the worse state means smaller final wealth for the same level of investment in self-insurance, which is intuitive.

The individual's problem is to choose e to maximize expected utility of final wealth

$$Eu(w) = \int_{\underline{\theta}}^{\overline{\theta}} u\big(w(e, \theta)\big)f(\theta)d\theta,$$

where u denotes von Neumann-Morgenstern utility such that $u' > 0$, $u'' < 0$. The first-order condition – necessary for internal solution of the problem – is then

$$\frac{\partial Eu}{\partial e} = \int_{\underline{\theta}}^{\overline{\theta}} u'\big(w(e, \theta)\big)\big(-1 - (1 - \alpha)L_e(e, \theta)\big)f(\theta)d\theta = 0. \tag{5}$$

Obviously, for that to happen, the factor $w_e = -1 - (1 - \alpha)L_e(e, \theta)$ has to be positive for some values of θ and negative for other θs.

Observe that the second-order condition is satisfied. Indeed, after straightforward calculations we have

$$\frac{\partial^2 Eu}{\partial e^2} = \int_{\underline{\theta}}^{\overline{\theta}} \left[u''(w)\big(-1 - (1 - \alpha)L_e(e, \theta)\big)^2 + u'(w)(1 - \alpha)d_{ee} \right]f(\theta)d\theta < 0.$$

Due to our assumptions, the sign of the above expression is unambiguously negative. Hence the problem becomes concave and there exists its unique solution. Let us denote by e^* the optimal level of self-insurance, satisfying equation (5).

3. ANALYSIS OF THE MODEL

Our problem is to answer the question if insurance and self-insurance are substitutes or complements and to derive conditions sufficient to give the unambiguous answer. However, one must be careful about terms „substitutes" and „complements" in our context. In this paper we consider market insurance as mandatory, so demand for it does not make the usual sense. Especially use of the word „substitute" raises justified doubt. Nevertheless, price of the insurance is set always by insurer and an individual may adjust his self-insurance activity to an increase in price of the insurance. If demand for self-insurance increases in presence of increased prices of market insurance then we say that self-insurance is *substitute for* market insurance. If demand for self-insurance decreases then we say that self-insurance is *complement of* market insurance. It seems like classical definition, but it is not. When price of the insurance increases then „demand" for it remains the same as before. Therefore there is no substitution in traditional meaning. What we investigate here is the effect of increase in price of the market insurance on the demand for self-insurance. We use terms „substitutes" and „complements" in specific meaning, defined above. We do it for simplicity and because those terms are in present context as close as possible to their original sense.

There are three main, well-known types of absolute risk-aversion: decreasing, increasing and constant with regard to wealth of an individual, abbreviated to DARA, IARA and CARA respectively. However, DARA is considered a natural assumption and it is confirmed empirically. For example, under DARA, risky assets are normal goods, whereas with IARA it becomes inferior. DARA means that $A(w) = -\frac{u''(w)}{u'(w)}$, the Arrow-Pratt index of absolute risk-aversion is decreasing in wealth w (Pratt 1964), hence $A'(w) \leq 0$. IARA implies that $A'(w) \geq 0$.

Since DARA is intuitive and nonrestrictive and IARA case is symmetric, we will only cover the DARA case.

Proposition 1. Assume that individual exhibits decreasing absolute risk aversion (DARA).

(i) If the function $\left(-1 - (1-\alpha)L_e(e^*,\theta)\right)$ crosses singly the θ – axis changing its sign from plus to minus then $\frac{\partial e^*}{\partial \pi}$ is negative and self-insurance is complementary to market insurance.

(ii) If the function $\left(-1 - (1-\alpha)L_e(e^*,\theta)\right)$ crosses singly the θ – axis changing its sign from minus to plus then $\frac{\partial e^*}{\partial \pi}$ is positive and self-insurance is a substitute for market insurance.

Proof. By implicit function theorem, equation (5) may be written in general form:

$$F\big(\pi, e^*(\pi)\big) = 0. \tag{6}$$

It reflects the basic intuition that the demand for self-insurance depends somehow on the price of the market insurance. Our aim is to find out what happens with e^*, when the price of market insurance goes up. In other words, we are interested in the sign of $\frac{\partial e^*}{\partial \pi}$ and in what determines that sign.

By totally differentiating (6) we obtain

$$\frac{\partial e^*}{\partial \pi} = -\frac{\partial F/\partial \pi}{\partial F/\partial e^*}.$$

By the second-order condition, the sign of the denumerator of the above is negative, and therefore

$$\text{sign } \frac{\partial e^*}{\partial \pi} = \text{sign } \frac{\partial F}{\partial \pi}. \tag{7}$$

Consequently, we calculate:

$$\frac{\partial F}{\partial \pi} = \int_{\underline{\theta}}^{\overline{\theta}} u''\big(w(e^*,\theta)\big)(-\alpha)\big(-1 - (1-\alpha)L_e(e^*,\theta)\big)f(\theta)d\theta =$$

$$= \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left(-\frac{u''\big(w(e^*,\theta)\big)}{u'\big(w(e^*,\theta)\big)}\right) u'\big(w(e^*,\theta)\big)\big(-1 - (1-\alpha)L_e(e^*,\theta)\big)f(\theta)d\theta.$$

We recognize the expression $-\frac{u''\big(w(e^*,\theta)\big)}{u'\big(w(e^*,\theta)\big)}$ as an Arrow-Pratt index of absolute risk aversion, which will be denoted by $A\big(w(e^*,\theta)\big)$ from now on. Hence we may write

$$\frac{\partial F}{\partial \pi} = \alpha \int_{\underline{\theta}}^{\overline{\theta}} A\big(w(e^*,\theta)\big)u'\big(w(e^*,\theta)\big)\big(-1 - (1-\alpha)L_e(e^*,\theta)\big)f(\theta)d\theta. \tag{8}$$

In order to determine the sign of expression (8), we will make use of the single crossing condition, a method introduced to economics by Diamond, Stiglitz (1974). Basically, it says that if the factor $\big(-1 - (1-\alpha)L_e(e^*,\theta)\big)$ crosses singly the θ – axis and $A\big(w(e^*,\theta)\big)$ is monotonic in θ and has constant sign then it is possible to evaluate the sign of (8) unambiguously.

Firstly, observe that

$$\frac{\partial A\bigl(w(e,\theta)\bigr)}{\partial \theta} = A'\bigl(w(e,\theta)\bigr)w_\theta.$$

By (4), the sign of w_θ is negative, so the signs of $\frac{\partial A(w(e,\theta))}{\partial \theta}$ and $A'\bigl(w(e,\theta)\bigr)$ are opposite.

On the other hand, the sign of $A'\bigl(w(e,\theta)\bigr)$ is related to an individual risk perception of an individual. Our assumption is that the agent's preferences exhibit decreasing absolute risk aversion (DARA), hence $A'\bigl(w(e,\theta)\bigr) < 0$. Therefore the sign of $\frac{\partial A(w(e,\theta))}{\partial \theta}$ is positive and $A\bigl(w(e,\theta)\bigr)$ is increasing in θ.

Due to the risk-aversion, $A\bigl(w(e,\theta)\bigr) = -\frac{u''(w(e,\theta))}{u'(w(e,\theta))}$ is always positive. Now we are able to use method of Diamond and Stiglitz and the resut follows.   ∎

Unfortunately, the formulation of the above conditions itself generates certain problem. The function $L_e(e^*,\theta)$ is evaluated at the point e^*, which is unknown and it makes conditions (i) and (ii) virtually impossible to verify. Our next aim is to find verifiable sufficient conditions for insurance and self-insurance to be substitutes or complements.

By the definition, $L_e(e^*,\theta) = -d_e(e^*,\theta)$, hence $\bigl(-1-(1-\alpha)L_e(e^*,\theta)\bigr) = -1+(1-\alpha)d_e(e^*,\theta)$. Negative sign of the cross-derivative $d_{e\theta}$ means that the function $-1+(1-\alpha)d_e(e^*,\theta)$ is decreasing in θ, hence condition (i) follows. Analogous reasoning applies to (ii).

Obviously, the monotonicity in θ of $\bigl(-1-(1-\alpha)L_e(e^*,\theta)\bigr)$ guarantees the single-crossing conditions, hence we may formulate:

Proposition 2. Assume that individual exhibits decreasing absolute risk aversion (DARA).
(i) If $d_{e\theta} < 0$, then self-insurance is complementary to market insurance.
(ii) If $d_{e\theta} > 0$, then self-insurance is a substitute for market insurance.

Proposition 2 is then slightly weaker then Proposition 1. There is no apparent mathematical reason for function $-1+(1-\alpha)d_e(e^*,\theta)$ to be monotone in order to satisfy single-crossing condition. However, many examples of self-insurance suggest that it is usually the case. On the other hand, the condition $d_{e\theta} < 0$ is simple, verifiable and has clear economic interpretation.

Property $d_{e\theta} > 0$ is known as supermodularity of the function d, which in turn is equivalent to increasing differences notion (provided function d is twice continuously differentiable). It states that increases with regard to one variable are increasing in

second variable. In general, the sign of cross-derivative $d_{e\theta}$ is intrinsically related to self-insurance technology. It reflects how self-insurance deals with the losses in different states.

4. ECONOMIC INTERPRETATION OF THE RESULTS

Single crossing condition in Proposition 1 (i) means precisely that there exists $\bar{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that $w_e = -1 + (1 - \alpha)d_e(e^*, \theta) \geq 0$ for $\theta < \theta^*$ and $w_e = -1 + (1 - \alpha)$ $d_e(e^*, \theta) \leq 0$ for $\theta < \theta^*$. In states $\theta > \theta^*$ marginal increase in self-insurance costs more than its benefit; in that case we say that self-insurance is *ineffective*. Analogously, in states $\theta < \theta^*$ self-insurance is called *effective*. Hence single crossing condition (i) says that in good states (small θ) self-insurance is effective and in bad states (large θ) self-insurance is ineffective. For example, one can quench small fire by using fire extinguisher, but it does not help when the fire is severe. Also, bicycle helmet is effective in light accidents, but it does not help if the accident is severe.

It turns out that it is much harder to find real examples representing change from ineffectivity to effectivity of self-insurance as in (ii). If we consider hiring a lawyer as a form of self-insurance, then it may serve as an illustration for (ii). In minor cases investing in more expensive defense attorney is costly and results with small improvement. However, if losing the case means serious financial consequences then it is usually profitable to hire experienced, more expensive lawyer.

More generally, self-insurance aims often (not always) at small-scale problems and involves only small expenses. Therefore we may say that the technology of self-insurance usually has its limitations. If that is the case, then it fails at severe accidents. So it seems that (i) case is more frequent and realistic than (ii).

The economic interpretation of Proposition 1 is as follows. As before, single crossing condition (i) means that in good states self-insurance is effective and in bad states self-insurance is ineffective. In a way it is then opposite to insurance; increasing it in a good state does not benefit but increases the cost, and in the bad state it reduces the loss more than it costs. In other words, insurance is more effective in bad states than in good ones. Hence it is natural to think that self-insurance is then complementary to market insurance. On the other hand, condition (ii) works the other way around and makes self-insurance similar to market insurance. It may be easily perceived as a type of insurance. Therefore it is considered as a substitute for market insurance.

One may consider the problem from a different point of view. In the described situation, increasing price of the insurance (π) with constant insurance expenditures has the same effect as decreasing coverage of the insurance. It creates the situation of increasing underinsurance with its well-known adverse effects. The insurer then needs „more insurance". In the case (ii), self-insurance has the same feature as the

insurance. Therefore the insurer is willing to invest more in self-insurance. In the case (i) the situation is reversed. Self-insurance cannot be used as a replacement for lost part of insurance. In order to deal with increasing risk, the insurer decreases the expenditures on self-insurance.

5. CONCLUSION

With two states of the world, self-insurance and market insurance are substitutes. It turns out that this result does not extend to more general case with many states. In general setting the relation between self-insurance and market insurance becomes more complex. The key to understanding that relation is effectivity of self-insurance which reflects the technology of self-insurance. Under DARA, if in good (bad) states self-insurance is effective and in bad (good) states it is ineffective then self-insurance is complementary (substitute for) market insurance. The result presented here has its limitations: we have considered only the case of exogenous level of insurance, as it is in many cases forced by law. The general problem requires further research.

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UBEZPIECZENIE I SAMOUBEZPIECZENIE – DOBRA SUBSTYTUCYJNE CZY KOMPLEMENTARNE?

Streszczenie

Klasyczny wynik Ehrlicha i Beckera stwierdza, że samoubezpieczenie jest substytutem ubezpieczenia. Twierdzenie to zostało jednak uzyskane po analizie modelu, w którym występują tylko dwa stany świata. Niniejszy artykuł uogólnia ten model dopuszczając możliwość wystąpienia wielu stanów świata, prowadząc do wniosku, że teza twierdzenia Ehrlicha i Beckera przestaje obowiązywać w sposób bezwzględny. Artykuł poświęcony jest badaniu interakcji pomiędzy ceną obowiązkowego ubezpieczenia a popytem na samoubezpieczenie. Przedstawione zostały warunki dostateczne na to, aby samoubezpieczenie było substytutem (w specyficznie określonym sensie) lub dobrem komplementarnym względem ubezpieczenia. Zaprezentowana została także interpretacja ekonomiczna wyniku oraz jego założeń, gdzie podkreślono, że kluczową rolę dla zrozumienia badanego zjawiska odgrywa pojęcie efektywności samoubezpieczenia.

Słowa kluczowe: samoubezpieczenie, ubezpieczenie, dobra komplementarne i substytucyjne

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Abstract

Classical result by Ehrlich and Becker states that with two states of the world, market insurance and self-insurance are substitutes. However, it turns out that conclusion does not hold in the model with many states. This paper considers interactions between price of compulsory market insurance and demand for self-insurance. We present sufficient conditions for self-insurance to be complementary or substitute for market insurance. We provide economic interpretation of that result, highlighting the role of an efficiency of self-insurace as a key to understanding the phenomenon.

Keywords: self-insurance, insurance, substitution, complementarity