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Method of track vertical stiffness estimation based on experiment

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Abstract

This paper presents a method of estimation of the track vertical stiffness changes based on the measured track vertical deflections during quasistatic railway vehicle motion along the track. Measurements of deflections were performed on the selected railway track section for applied static axle load of the wagon. The measured deflections were used to estimate track vertical stiffness variations. The courses of these variations were considered on the basis of two series of experiments (1st and 2nd stage) executed on the same track section at ten months long interval. In both stages of the experiment weather conditions were close to each other.

Keywords: railway track, rail vehicle, experiment, track vertical stiffness

1. Introduction

The problem presented here is a piece of the work realised by the authors of the paper. It is associated with the dynamics of a rail vehicle and track interactions. Essentially, the aim of the study is to answer the questions:

- How far the track vertical stiffness (in this case superstructure and ballast) affects the forces in the wheel-rail system?
- What is the mechanism of track geometric irregularities formation taking into account mechanical properties of ballast?

The determination of track vertical stiffness and also its variation along the track are relatively difficult and expensive. There are not many available references

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on the experimental methods and the results obtained. Moreover, published results of track vertical stiffness values are frequently divergent, because they refer to different design and state of track maintenance, e.g. [2, 5]. This circumstance was the motivation for us to carry out an appropriate experiment.

The key problem, earlier than experiment, is to fix on and arrange the proper track section being in operation. Due to technical and organizational requirements such railway track sections were accessible to the research team for a period of approximately 5 days by 8 hours each. The need of the experimental track section closures for the railway traffic means, that is impossible to meet the requirements of experiment and scheduled railway line operation. Hence, measurements were made on the secondary line, with a lower state of maintenance. After all, the track section nearly the Skarżysko Kamienna railway station was chosen for the experiment. However, it should be stressed that the track section selected is not nice-looking from a technical point of view. Nevertheless, as far as we know it is the first experimental approach in Poland, i.e. an effort to estimate changes in vertical deflection and stiffness of the track along some distance. Assumptions and scope of the experiment were developed jointly with the Rolling Stock Tests Laboratory of Railway Institute (formerly – Railway Scientific and Technical Centre) in Warsaw. That Laboratory also performed this experiment.

2. Description of the experiment

Measurements were carried out on the track number 401 linking Skarżysko Kamienna and Sandomierz. Experimental track section was located between 145.4165 and 145.3525km. The first stage of the experiment – was carried on in days 20÷25.11.2007, and the second one – in days 10÷13.09.2008. In the first stage, arranged test train (Fig. 1) consisted of locomotive series SM42, CNTK – measuring wagon and Kgns wagon of platform type, hereinafter named as test wagon (wb). In the second stage of the experiment, CNTK measuring wagon was removed from the test train and thereby vertical (Q) and lateral (Y) wheel-rail forces were not recorded. The railway track under tests was equipped with S60 rails, wooden sleepers and intermediate K – fastenings.

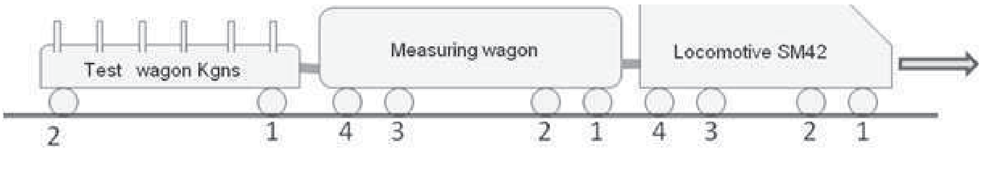


Fig. 1. Test train

For the period of measurements test train was passing slowly (up to 5 km/h) along the selected track section. During measurements rail vertical displacements above every second sleeper were recorded. There were performed three rounds of measurement. At every round the mass of tested car was fixed to obtain suitable axle load. There were applied the following axle loads of tested car: 100 kN/axle, 150 kN/axle and 200 kN/axle. Measurements of displacements for both rails were made for the above mentioned axle loads on the distance covered by the 111 sleepers (56 measured sleepers). During each test train pass – 10 rail deflection values were recorded successively above each of 56 sleepers. Four of them correspond to locomotive wheelsets, next four of them correspond to measuring wagon and the last two correspond to the tested wagon wheelsets (Fig. 1).

3. Measurement method of vertical deflections of rails

Non-contact method was applied for measuring vertical displacements of rails. Vertical deflections of both rails were recorded simultaneously on the same sleeper. To do this, two fast cameras having the high resolution (up to 0.05 mm) have been applied. They were placed on both sides of the track allowing to measure the deflections of two rails (left and right) above the same sleeper. On the rails were located markers equipped with the number of sleeper and scale (Fig. 2). Those allowed the reading of deflections (Fig. 3). Analysis of measured data consisted of

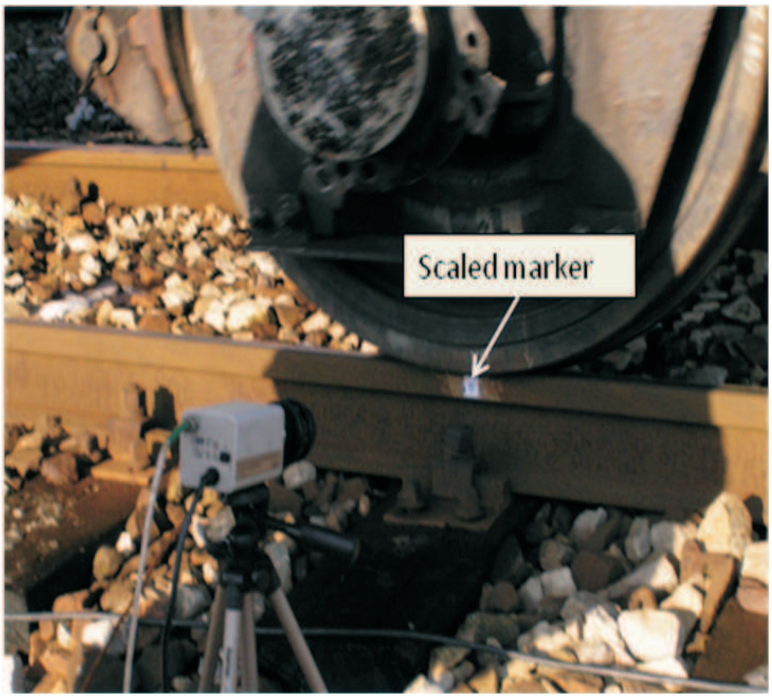


Fig. 2. View of rails with stick marker over the sleeper and the camera filming the passage

processing of movies during subsequent passes of the train. To do this, the selected sections of each video frames were recorded one after the other as a bitmap. In this way, the rail deflections courses were obtained for the sleeper vs. time. The interval between film frames should be understood as the time. This interval was 0.02 seconds in the case. Then, based on recorded changes in the positions of markers and fixed transformation coefficient η [mm/pixel], the bitmap sets were transformed to obtain the actual deflection of the rails. The results were recorded in the Excel worksheets.

In Fig. 3 are shown examples of two sets of bitmap for measuring points No. 5 and 10, illustrating the evolution of the deflection of a single rail above chosen sleeper due to passing train axles. Test wagon axle load was equal to 100 kN in this case. The results concern the first stage of the experiment.

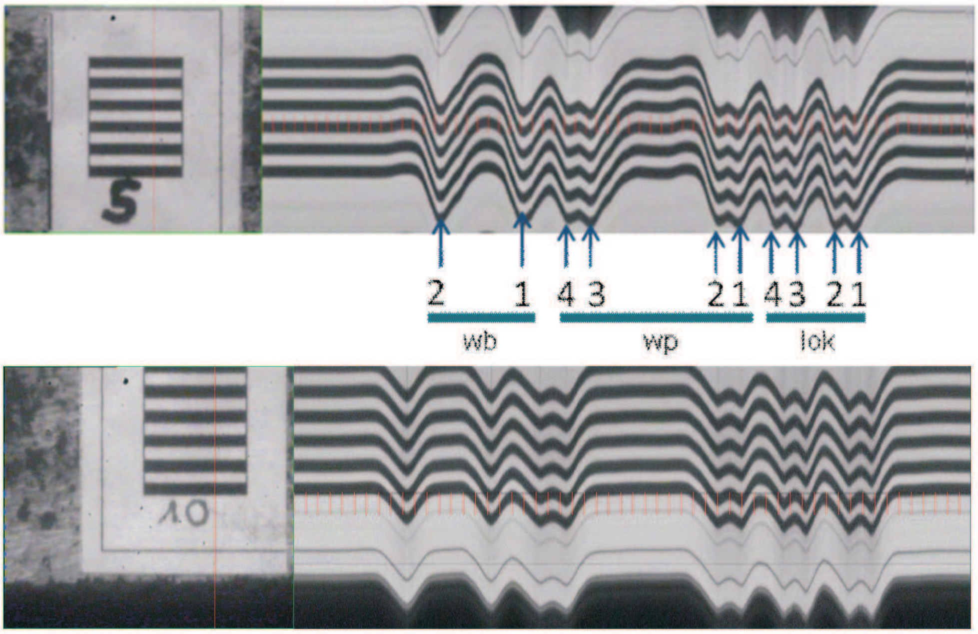


Fig. 3. 1st stage of the experiment. Illustration of vertical rail displacement (measured on the side of camera 1. location). Measurement points – No. 5 and 10

In Fig. 3 the deflection of the rail (for measurement points No. 5 and 10) is indicated by the numbers:

- for locomotive wheel – 1, 2, 3, 4;
- for wheels of measuring car – 1, 2, 3, 4;
- for wheels the biaxial test car – 1, 2.

For the other measuring points and for both rails, the numbering is the same.

4. Results of vertical deflections of rails

The rails deflections, caused by quasistatic train passage, were recorded over each sleeper for subsequent wheelsets of wagon. Wagon was loaded with 100, 150 and 200 kN per axle.

Examples of the results of measurements of vertical rail deflection (camera 1) during passage of the first axle of the wagon are shown in Fig. 4.

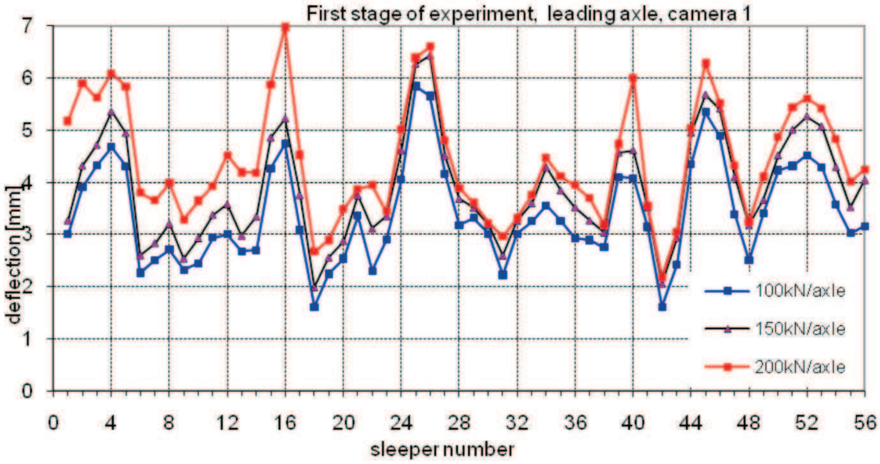


Fig. 4. Examples of changes of vertical rails deflections in the experimental section of railway track

In this Figure, sleeper numbers refer to those above, which the measurement points of rails deflections were located. In fact, on the experimental track section of 66.6 m length 111 sleepers were positioned (56 measured sleepers).

Displacements of the left and right rails on the same sleeper generally differ from each other, while the ranges of these changes for each axle load are comparable. The differences of displacements for the left and right rails may be caused by no symmetric mass distribution of the test car, the heterogeneity of ballast (in terms of its vertical stiffness) or different mutual adjacency of the superstructure to the ballast on the left and right side. It is easy to notice a difference in the values of the track deflections resulting from different static axle load. The estimators of mean values and standard deviations of the test track vertical deflection reduced to this track centre are included in Table 1.

Correlation coefficients were calculated between the deflections of the track centre line due to loads of the first and second axles of the test car. Very high correlation coefficient between displacements of the track under the first and second axle of the car takes place for the axle load of 100 and 150 kN. The lower value of this coefficient for axle load of 200 kN is possibly due to the irregular distribution of charge (additional mass) in the test car.

Table 1

1st stage of the experiment. Statistical parameters of vertical deflection of the track centre line

Statistical parameters [mm]	First axle	Second axle	Correlation coefficient	Axle load [kN]
μ	3.06	2.96	0.95	100
σ	0.18	0.34		
μ	3.55	3.54	0.99	150
σ	0.04	0.05		
μ	4.11	4.12	0.71	200
σ	0.89	0.93		

Notation: μ – estimator of the average deflection, σ – estimator of standard deviation.

In the second stage, measurements were made on 40 sleepers. There was selected the same starting point and the same registration points as in the first stage. In this case, a set of train consisted of locomotive and test wagon, and as mentioned above, wheel – rail forces *Q* and *Y* were not measured. Estimators of average values and standard deviations of vertical rail deflection reduced to the track centre line are shown in Table 2. Just as in the first stage of the experiment, the correlation coefficient was calculated between the deflections waveforms of the track centre line for the axle load of the test car successively: 100, 150 and 200 kN.

Table 2

2nd stage of the experiment. Statistical parameters of vertical deflection of the track centre line

Statistical parameters [mm]	First axle	Second axle	Correlation coefficient	Axle load [kN]
μ	3.29	3.27	0.99	100
σ	1.20	1.21		
μ	3.82	3.83	0.98	150
σ	1.38	1.40		
μ	4.31	4.22	0.99	200
σ	0.89	0.93		

5. Determination of vertical stiffness changes on the track measuring section

Track vertical stiffness calculation results are presented for the rear axle (at the end of the train) of the test car. Due to long distance between the front and rear axle (9 meters), the influence of the front axle load on the line deflection caused by the rear axle load is practically negligible. Thus, it was possible to calculate stiffness, adopting a model of a beam, with a single concentrated load, supported by the sleepers resting on the continuous foundation representing ballast. To prove this assumption, consider the track model, as the infinite continuous beam resting on elastic foundation, characterized by vertical ground stiffness coefficient $k[\mathrm{N/m^2}]$. The track is loaded with a vertical force Q, i.e. the pressure coming from the test car wheelset.

Beam displacement equation for a continuous elastic foundations under the influence of concentrated forces Q is [3, 4]:

$$EI\frac{\partial^4 y}{\partial x^4} + ky = q(x) \tag{1}$$

In this case, the load is not distributed (q = 0). The load in the form of concentrated force Q is included as the boundary condition.

The solution of equation (1) can be expressed in a form convenient to interpret [1] as specified below.

$$y(x) = \frac{Q}{2kL}u(x) \tag{2}$$

where:

$$L = \sqrt[4]{\frac{4EI}{k}} \tag{3}$$

L – is a characteristic length, depending on the bending stiffness EI of rails and the coefficient of the foundation (ballast) k.

$u(x)$ – normalized line of track centre line deflection (relative deflection).

The line $u(x)$ has the following form:

$$u(x) = -e^{-x/L}\left[\cos\frac{x}{L} + \sin\frac{x}{L}\right] \text{ for } x \geq 0, \tag{4a}$$

and

$$u(x) = -e^{x/L}\left[\cos\left(-\frac{x}{L}\right) + \sin\left(-\frac{x}{L}\right)\right] \text{ for } x \leq 0 \tag{4b}$$

Deflection line for the experimental parameters is shown in Fig. 5. Ground factor k can be estimated basing on the average equivalent stiffness of the measured section.

On the basis of formulas (4a) and (4b) it is possible to define the position of zero deflection (for $\xi_o = \pm(3/4)\,\xi g\pi L$), i.e. at $L = 1.22$ m, $\xi_o = 2.88$ m. Thus, under the influence of concentrated vertical force Q, the track will be in a downward deflection (negative part of the graph in Fig. 6) over a length of about $2\xi_o \cong 5.8$ m. As it is seen, the effect of the rear axle load of the car 9m away from the front one is irrelevant in this model.

Somewhat different concept of experiment was presented by Froehling [2]. That experiment was intended to estimate the vertical stiffness of the ballast and the rate of track settlement. Besides that, there was used a different measurement technique.

As mentioned earlier, in the first stage of the experiment, measurements were performed on 56 sleepers, whereas in the second one on 40 sleepers. Thus, the

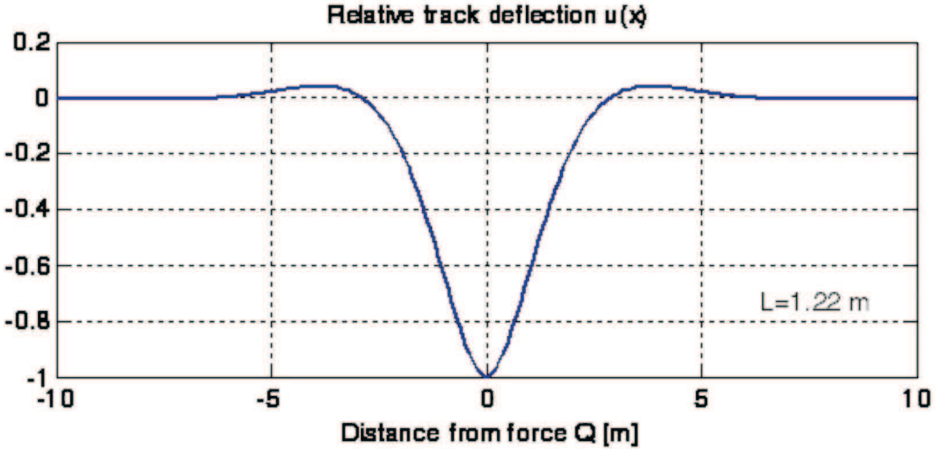
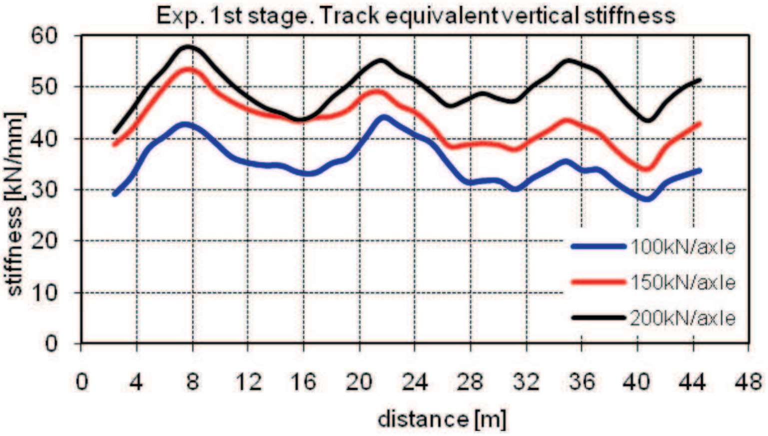


Fig. 5. Track centre line displacement (normalized)

results comparison of calculations of track stiffness changes for the first and second stages (Fig. 7) is limited to the section corresponding to 40 sleepers. Vertical stiffness of the track and its changes is interpreted as a variable coefficient along the track of an equivalent spring stiffness averaged over a length of $2\xi_o$ with a load Q = 100, 150 and 200 kN respectively. Track vertical stiffness understood in this way is called the equivalent stiffness. Based on the results of calculations (Fig. 6), it can be concluded that, for each measuring point equivalent stiffness depends nonlinearly on the axle load Q. If this relationship would be linear, then the stiffness curves in the graphs to be covered by. Together with the increase of force Q – equivalent stiffness increases (progressive characteristics). Considering the problem in spatial frequency domain, taking into account the waveform of track stiffness changes, we can say that their courses are alike to each other [5].

Those respond to both stages of the experiment. Estimators of average stiffness values for the stages slightly differ themselves, while the standard deviations are larger for the measurement of the 2^{nd} stage. This means that the operation of the track, which was the experimental section, in the period between measurements, contributed to the equivalent stiffness changes.

A characteristic feature of measurements, in all studied cases, is relatively large fluctuation of the stiffness appearing together with high gradients of change. This indicates the inadequate level of technical maintenance of the track, i.e., poor ballast and uneven adhesion the superstructure to ballast. Moreover, according to the calculations (Table 3), courses of track stiffness changes derived from measurements made during the first and second stage of experiment are relatively poorly correlated. Because there is repetition of the nature of the change of stiffness with different loads Q (in both stages of experiment), we assume that the probability of measurement errors is very small.



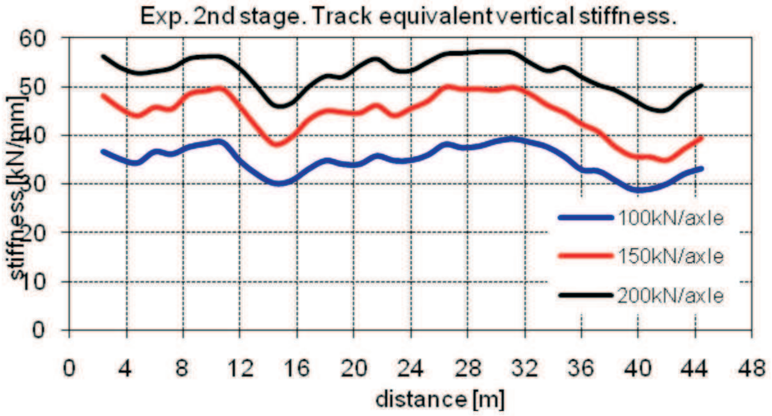


Fig. 6. Changes of equivalent vertical track stiffness determined on the basis of track deflections measurements in 1st and 2nd stage of the experiment

Table 3

Mean values and standard deviations of equivalent vertical track stiffness

1st stage of experiment			
Statistical parameters	100 kN/axle	150 kN/axle	200 kN/axle
Mean value μ_1 [kN/mm]	34.98	43.03	49.96
Std. deviation σ_1 [kN/mm]	1.62	0.09	3.05
2nd stage of experiment			
Mean value μ_2 [kN/mm]	34.76	44.16	52.61
Std. deviation σ_2 [kN/mm]	2.43	6.15	4.19
Correlation coefficient (1st/2nd stage)	0.58	0.66	0.46

6. Conclusions

Range of the vertical track stiffness changes for different axle loads on the tested track section is significant. It takes place for the first and second stage of the experiment. On the basis of calculations it can be concluded, that track equivalent vertical stiffness depends on vertical force *Q* in each measuring point. It is apparent from the courses of the stiffness change that characteristics of the track displacements depend nonlinearly on this force. Proposed method allows to estimate track equivalent vertical stiffness and its changes along the track. Nevertheless, when planning the next experiment, the method should be modified to obtain an enhanced representation of the results (e.g. increasing the length of measuring section, simultaneous recording of rail displacement in a number of points). Due to practical importance of those results and opportunity of making comparisons to other works (mostly foreign), it is advisable to perform such experiment on better maintained railway lines.

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