THE ARCHIVES OF TRANSPORT

VOL. XXII NO 2 2010

10.2478/v10174-010-0016-z

A method to achieve comparable thermal states of car brakes during braking on the road and on a high-speed roll-stand

Andrzej Wolff*

Received October 2010

Abstract

The temperature of a brake friction surface influences significantly the braking effectiveness. The paper describes a heat transfer process in car brakes. Using a developed program of finite element method, the temperature distributions in brake rotors (disc and drum brake) of a light truck have been calculated. As a preliminary consistency criterion of the brake thermal state in road and roll-stand braking conditions, a balance of the energy cumulated in the brake rotor has been taken into account. As the most reliable consistency criterion an equality of average temperatures of the friction surface has been assumed. The presented method allows to achieve on a roll-stand the analogical thermal states of automotive brakes, which are observed during braking in road conditions. Basing on this method, it is possible to calculate the braking time and force for a high-speed roll-stand. In contrast to the previous papers of the author, new calculation results have been presented.

Keywords: automotive brakes, heat transfer, thermal state of brakes, diagnostic tests

1. Introduction

Thermal processes play important role when braking effectiveness is considered. It should be noticed that typical friction material characteristics are strongly influenced by the temperature. It usually results in reduction of braking effectiveness. That is the reason of carrying out road tests for cold and heated brakes [14].

* Warsaw University of Technology, Faculty of Transport, ul. Koszykowa 75, 00-662 Warsaw, Poland, e-mail: wolff@it.pw.edu.pl

Periodical car tests take place on stands at service stations due to weather condition independence, lower costs and higher repeatability of results.

Basing on a developed heat transfer model [19], the author has applied a method for the roll-stand test to achieve the analogical thermal states of automotive brakes corresponding with that occurring during the road braking [10, 11].

2. Mathematical model of heat transfer in brakes

In the paper the heat transfer in the rotor of brake is considered. In lots of scientific articles different modelling approaches for the thermal analysis of car brakes can be found. The models ranging from one–dimensional [12,13], two–dimensional [2,10,11,17,19,20], to complex three–dimensional [3,20,21] are used.

The two–dimensional, axi-symmetrical model of transient heat transfer has been chosen by the author of this paper. It should be noticed that the geometry of a car brake rotor is axi-symmetrical. However the boundary conditions on the friction surface are not axi-symmetrical. It's because the brake linings contact with the rotor on a certain part of the brake circumference. Furthermore the pressure distribution between pad and rotor is not uniform. Nevertheless the rotational movement between mentioned parts of the brake should be remarked. In this case the assumption of axi-symmetric heat flux on the friction surface means only time - averaged boundary conditions over a rotor revolution. It should cause relatively slight error, especially at high angular velocity, when the heat flux is very intensive. The two - dimensional approach is mostly used by the scientists.

The mathematical model of unsteady heat transfer in automotive brake presents the partial differential equation of second order, which in case of two-dimensional, axi-symmetrical model can be written in the following form:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(r\lambda\frac{\partial T}{\partial z}\right)\right] \tag{1}$$

where: ρ – density, $\boldsymbol{c_p}$ – specific heat, λ – coefficient of heat conduction, $\boldsymbol{T}$ – temperature, ***r, z*** – coordinates, ***t*** – time

On the friction surface of a brake rotor Γ_q (Figs. 1a and 1c), where the heat flux ***q*** is generated, the boundary condition of second type has the form [16,18,19]:

$$-\lambda\frac{\partial T}{\partial n} = \dot{q}'\,(r, z, t) \tag{2}$$

where: λ – coefficient of heat conduction, ***T*** – temperature of friction surface, ***n*** – normal to the boundary surface, ***q'*** – heat flux generated on friction surface, ***r, z*** – coordinates, ***t*** – time

The heat flux distribution between brake rotor and linings is described by the following equation:

$$\dot{q}' = \xi\,\dot{q} \tag{3}$$

where: ξ – coefficient of heat flux distribution between brake rotor and linings, $\boldsymbol{q}$ – heat flux generated in the brake, $\boldsymbol{q'}$ – heat flux transferred from the friction surface to brake rotor.

The heat flux density $\boldsymbol{q}$ is derived by equating the decrease in kinetic energy to heat dissipated by the brake [19]. It depends on friction coefficient μ, pressure $\boldsymbol{p}$ between the rotor and brake lining and slip velocity $\boldsymbol{v}$.

$$\dot{q} = \mu\, p\, v \tag{4}$$

where: μ – friction coefficient, $\boldsymbol{p}$ – pressure between the rotor and brake lining, $\boldsymbol{v}$ – slip velocity of friction elements

On free surfaces of a brake rotor Γ_k (Figs. 1b and 1c), we have the boundary conditions of third type, which describe the heat transfer with ambient air [16,18,19]:

$$-\lambda \frac{\partial T}{\partial n} = \alpha\,(T\,(r,z,t) - T_\infty) \tag{5}$$

where: λ – coefficient of heat conduction, α – heat transfer coefficient, $\boldsymbol{T}$ – free surface temperature of brake rotor, $\boldsymbol{T_\infty}$ – ambient air temperature, $\boldsymbol{n}$ – normal to the boundary surface, $\boldsymbol{r, z}$ – coordinates, $\boldsymbol{t}$ – time

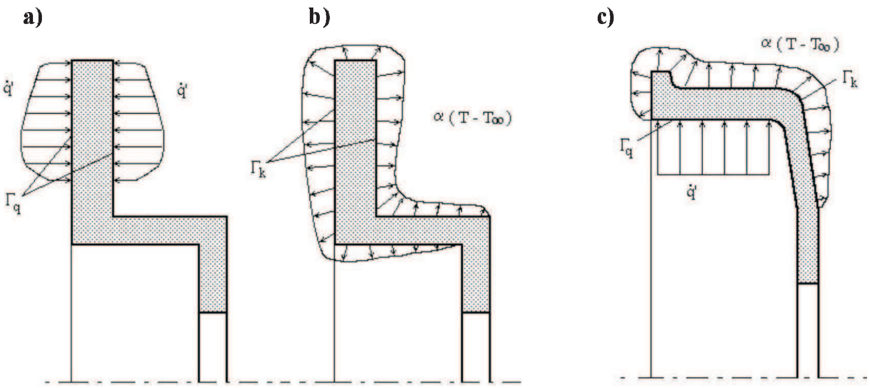


Fig. 1. Two-dimensional, axi-symmetrical model of unsteady heat transfer in brake rotors: a) heat flux q' generated on the friction surface Γ_q of disc brake, b) heat transferred to ambient air from free surfaces Γ_k of disc brake, c) boundary conditions of heat transfer in brake drum

The heat transfer coefficient α is the sum of the convective heat transfer coefficient α_{conv} and radiative heat transfer coefficient α_{rad}. The convective heat transfer coefficient α_{conv} depends on several parameters of ambient air in thermal boundary layer like: viscosity, density, thermal conductivity and specific heat [16,18]. These parameters are functions of air temperature and pressure (in less extent). But first of all the coefficient α_{conv} depends on air flow conditions (laminar or turbulent) and its velocity [5,8,9,15,19].

Convective heat transfer between the air and the brake rotor surfaces is the primary mean of heat rejection. Radiation heat transfer is less important especially for the low surface temperatures. However it was also taken into account in the analysis – the averaged radiation heat transfer coefficients for the predicted temperature range were calculated. The evaluation of the convective heat transfer coefficient α_{conv} is one of the most difficult tasks in the thermal analysis of a brake system. It varies over the brake surface and is a complex function of many factors such as geometry, air flow conditions, wheel angular velocity, surface temperature etc. Convective heat transfer coefficients for the various geometric segments of the brake rotor were estimated in this paper by relationships available in the heat transfer literature – functions of car speed and average surface temperature [5,8,9,15].

It need to be noticed that in order to obtain more accurate values of these coefficients experimental temperature measurements ought to be made and the identification procedure could be carried out. Such a procedure has been presented by the author of this paper in [19]. The values of the mentioned parameters are especially of great importance for the thermal analysis of long lasting continued braking or repeated brake applications.

The initial condition of the analysed heat transfer problem can be written as follows [16,18,19]:

$$T(r, z, t_0) = T_0 \tag{6}$$

where: $\boldsymbol{r}$, $\boldsymbol{z}$ – coordinates, $\boldsymbol{t}_0$ – initial time, $\boldsymbol{T}_0$ – initial temperature of brake rotor.

3. Numerical method

The transient heat conduction problem was solved using the finite element method. Formulation was based on the Galerkin method, a subclass of the method of weighted residuals. The relevant equations of finite element method for the problem are presented in [19]. For the space discretization of the problem domain the axisymmetrical, 8 – nodes, isoparametric elements with the curved or straight edges were applied [6,22]. The resulting set of differential equations was numerically integrated using the Crank – Nicolson recursion scheme [6,19], this method is second – order accurate and unconditionally stable.

4. Simulation results for short duration braking on the road

Vehicle model parameters corresponded with fully loaded light truck *Polonez-Truck* (2380 kg total mass). Details are shown in Table 1.

The vehicle is equipped in front disc brakes and rear drum brakes. The brake disc is made of cast iron, but the brake drum consists of two parts: one – cast iron ring and the second – cast aluminium drum.

At first the calculations were carried out for a vehicle braking rapidly with the constant average deceleration of 4.4 m/s^2 from initial speed 70 km/h to 0 km/h – according to [14]. In this case the total braking period is 4.42 s.

In Fig.2 the assumed surfaces Γ_k of brake rotors (i.e. disc and drum brake) concerning heat transfer to ambient air are shown. Furthermore in Fig. 3 the heat flux generated in the front disc brake as a function of time is presented. It was calculated as a part of kinetic energy of a braking car, which was dissipated in the brake. Heat transfer calculations were carried out for the front disc brakes and rear drum brakes of the analyzed car. Inspection of Fig. 3a and Fig. 3b shows that more heat is generated in rear drum brakes than in front disc brakes.

Table 1

Technical data of analysed vehicle *Polonez-Truck*

<table>
<tr><td colspan="2">Aerodynamic drag coefficient [-]</td><td colspan="3">0.46</td></tr>
<tr><td colspan="2">Front area of the car [m²]</td><td colspan="3">2.385</td></tr>
<tr><td colspan="2">Total mass of the car [kg]</td><td colspan="3">2380</td></tr>
<tr><td colspan="2">Braking forces distribution coefficient (front/rear) [-]</td><td colspan="3">0.736</td></tr>
<tr><td colspan="2">Tyre dynamic radius [m]</td><td colspan="3">0.306</td></tr>
<tr><td>Friction area of brakes:</td><td colspan="3">Brake rotor</td><td>Brake linings</td></tr>
<tr><td>Front brake (disc) [m²]</td><td colspan="3">0.02184</td><td>0.003157</td></tr>
<tr><td>Rear brake (drum) [m²]</td><td colspan="3">0.03848</td><td>0.024910</td></tr>
<tr><td>Material parameters of front brake:</td><td colspan="3">Cast iron disc</td><td>Brake linings</td></tr>
<tr><td>Thermal conductivity [W/mK]</td><td colspan="3">46</td><td>0.78</td></tr>
<tr><td>Density [kg/m³]</td><td colspan="3">7100</td><td>2400</td></tr>
<tr><td>Specific heat [J/kgK]</td><td colspan="3">500</td><td>980</td></tr>
<tr><td rowspan="2">Material parameters of rear brake:</td><td colspan="3">Bimetallic drum:</td><td rowspan="2">Brake linings</td></tr>
<tr><td colspan="2">Cast iron ring</td><td>Cast aluminium</td></tr>
<tr><td>Thermal conductivity [W/mK]</td><td colspan="2">46</td><td>160</td><td>0.78</td></tr>
<tr><td>Density [kg/m³]</td><td colspan="2">7100</td><td>2650</td><td>2400</td></tr>
<tr><td>Specific heat [J/kgK]</td><td colspan="2">500</td><td>883</td><td>980</td></tr>
</table>

An earlier developed two-dimensional, axi-symmetrical model of transient heat conduction for the brake was applied [19]. The relevant boundary conditions that describe the heat generated in the brake and the heat transferred to ambient air were used. The problem was solved by use of the finite element method [1,3,4,6,7,19,22].

In Fig. 4 the temperature response of the friction surface during braking time is shown. As one can see the temperature rise is quick and high. The highest value of temperature is nearly 144°C (for the front disc brake) and 132°C (for the rear drum brake). The thermal capacity of drum brakes is simply higher than for disc brakes. At the end of braking period on the road the temperature of friction surface

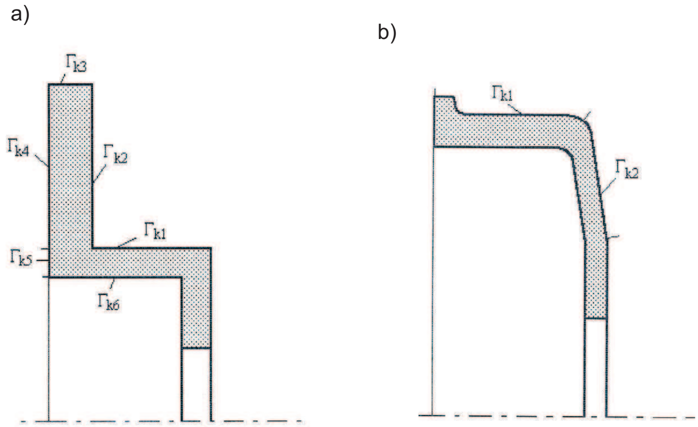


Fig. 2. Assumed surfaces of brake rotor concerning heat transfer to ambient air: a) disc brake (Γ_{ki}, i=1,. . . ,6), b) drum brake (Γ_{ki}, i=1,2)

(for disc and drum brakes) decreases a little bit (from 144°C to 138°C for the disc brake and from 132°C to 99°C for the drum brake) due to the decrease of the heat flux generated in the brake (see Fig. 3).

5. Consistency criterions of brake thermal states after braking on the road and on the roll-stand

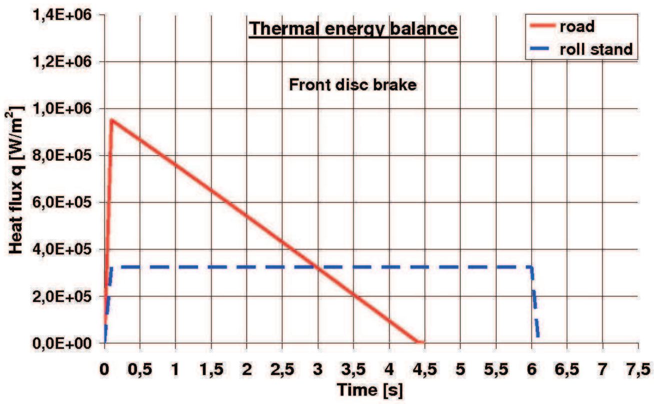
It is advantageous to use stands of the constant test velocity of rolls if the simplicity of their design is considered. For the constant brake force the braking time $\boldsymbol{t_b}$ is the main factor that decides on the amount of the dissipated energy.

As a preliminary consistency criterion of the brake thermal state in road braking conditions and on the roll-stand a balance of the energy cumulated in the brake rotor was assumed (*criterion I*). In that way the braking time $\boldsymbol{t_b}$ on the roll-stand for the assumed values of the rolls test velocity $\mathbf{v}_T$ and the brake force $\boldsymbol{F_b}$ can be determined (Fig. 3):

$$\int_0^{t_b^{road}} [\dot{q}'_{road} - \alpha_{road}(T - T_\infty)]\,dt = \int_0^{t_b^{roll}} [\dot{q}'_{roll} - \alpha_{roll}(T_{av} - T_\infty)]\,dt \tag{7}$$

where: $\boldsymbol{q'}$ – heat flux transferred from the friction surface to brake rotor, α - heat transfer coefficient to ambient air, $\boldsymbol{T}$ – free surface temperature of brake rotor, $\boldsymbol{T_{av}}$ – average value of temperature, $\boldsymbol{T}$, $\boldsymbol{T_\infty}$ – ambient air temperature, $\boldsymbol{t_b}$ – braking time.

In the previous papers of the author [10, 11] the braking time $\boldsymbol{t_b}$ on the roll-stand as a function of the brake force $\boldsymbol{F_b}$ for stands of different test velocities of rolls was a)



b)

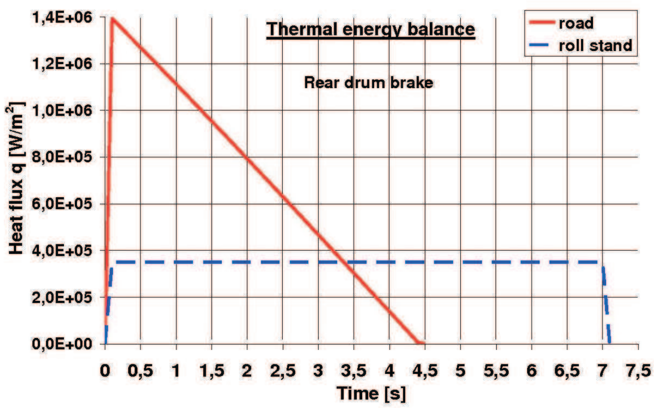


Fig. 3. Balance of thermal energy generated in car brakes during braking period. Braking on the road: initial velocity $\mathbf{v}_0$ = 70 km/h, average deceleration $\boldsymbol{a}_b$=4,45 m/s^2, braking time. $\boldsymbol{t}_b$=4,42 s. Braking on the *Clayton* roll-stand: test velocity $\mathbf{v}_T$ = 62 km/h, brake force $\boldsymbol{F}_b$=1000 N. Braking time on the roll-stand: **a)** $\boldsymbol{t}_b$=6 s for the front disc brake, **b)** $\boldsymbol{t}_b$=7 s for the rear drum brake

presented. It should be noticed that the most interesting and valuable tests can be made on high-speed roll-stands like *Clayton* stand ($\mathbf{v}_T$ = 17.22 m/s = 62 km/h).

As the most reliable consistency criterion of the brake thermal state in road braking conditions and on the roll-stand an equality of average temperatures of the friction surface was assumed (*criterion II*). It results from the fact that the friction coefficient between the brake rotor and linings is dependent on the temperature of the friction surface. Finally it influences significantly the braking effectiveness. The satisfying of the *criterion II* makes possible to carry out on a roll-stand the tests of braking effectiveness for heated brakes (Fig. 4).

The braking time $\boldsymbol{t}_b$ is predetermined from the *criterion I* of the energy balance. The accurate value of this parameter is calculated in an iterative way basing on the *criterion II* of the equality of average temperatures of the friction surface. For this purpose the heat transfer process for the brake is simulated and average temperature of the friction surface is calculated.

a)

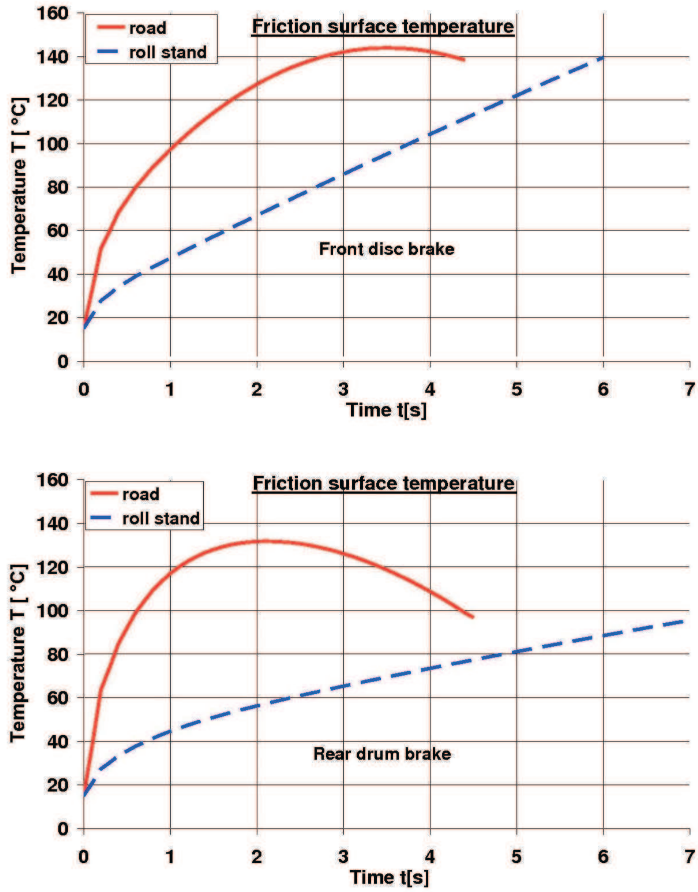


Fig. 4. Temperatures of the friction surfaces of car brakes versus time. Braking on the road: initial velocity $\mathbf{v}_0$ = 70 km/h, average deceleration $\boldsymbol{a}_b$=4,45 m/s^2, braking time $\boldsymbol{t}_b$=4,42 s. Braking on the *Clayton* roll-stand: test velocity $\mathbf{v}_T$ = 62 km/h, brake force $\boldsymbol{F}_b$=1000 N. Braking time on the roll-stand: **a)** $\boldsymbol{t}_b$=6 s (for the front disc brake), **b)** $\boldsymbol{t}_b$=7 s (for the rear drum brake)

The temperature field in the brake can be examined in convenient way, when temperature isolines are observed. Fig. 5a shows isothermal lines in the cross-section of the front disc brake at the end of the car braking on the road. The maximum temperature of the disc equals 144°C. The average temperature of the friction surfaces reaches the value of 118.6°C. Fig. 5b presents the temperature isolines at the axial cross-section of the brake disc at the end of braking time on the high-speed

Clayton roll-stand for the brake force $\boldsymbol{F}_b = 1000$ N. The highest temperature reaches the value of 142.9°C. The average temperature of friction surfaces is 119.1°C. In addition the applied finite element mesh of the front brake disc is shown in Fig. 5c.

Furthermore isothermal lines in the cross-section of the rear drum brake at the end of braking on the road are presented in Fig 6a. The maximum temperature of the drum friction surface equals 96.8°C and the average temperature of the friction surfaces (at the end of braking) reaches the value of 88.6°C. Similarly the temperature field of the brake drum at the end of braking ($\boldsymbol{t}_b$ = 7 s) on the *Clayton* roll-stand ($\mathbf{v}_T$ = 62 km/h, $\boldsymbol{F}_b = 1000$ N) is shown in Fig. 6b. The highest temperature reaches the value of 95.6°C. The average temperature of friction surfaces is 88.6°C (the same as after braking on the road). In addition the applied finite element mesh of the brake drum is presented in Fig. 6c.

The comparison of the simulation results for braking the car on the road and on the *Clayton* roll-stand (Figs. 5a, 5b and 6a, 6b) indicates the qualitative and quantitative consistency of the temperature profile in the material of disc and drum brakes. The maximum temperatures are almost equal (144°C and 142.9°C for the disc brake / 96.8°C and 95.6°C for the drum brake). The braking times are also comparable: 4.42 s (on the road) and 6 s for the disc brake and 7 s for the drum brake (on the roll-stand). In addition the temperatures of the disc hub (Figs. 5a and 5b) and of the drum hub (Figs. 6a and 6b) are almost the same at the end of braking on the road and on the roll-stand. But in case of braking on the roll-stand, a higher temperature gradient near the friction surface (of the disc and drum brake) can be observed.

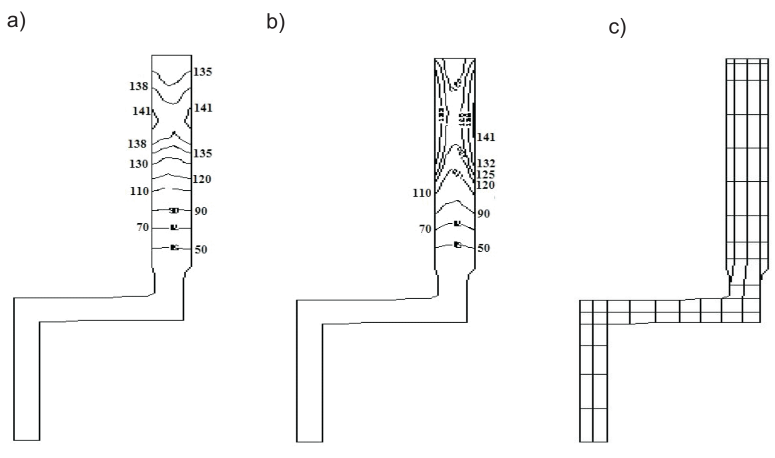


Fig. 5. Isothermal lines [°C] in the cross-sections of front disc brake at the end of braking: **a)** on road from $\mathbf{v}_0$ = 70 km/h, **b)** on the *Clayton* roll-stand ($\boldsymbol{F}_b$ = 1000 N, $\mathbf{v}_T$ = 62 km/h, $\boldsymbol{t}_b$ = 6 s), **c)** applied finite element mesh of brake disc

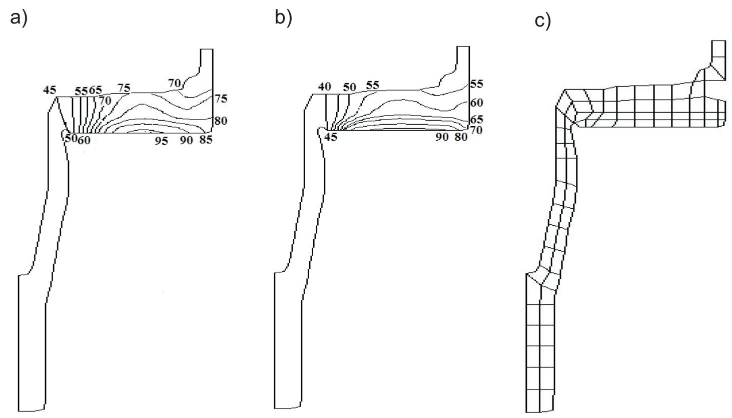


Fig. 6. Isothermal lines [°C] in the cross-sections of rear drum brake at the end of braking: **a)** on road from $\mathbf{v}_0$ = 70 km/h, **b)** on the "Clayton" roll-stand ($\boldsymbol{F}_b$ = 1000 N, $\mathbf{v}_T$ = 62 km/h, $\boldsymbol{t}_b$ = 7 s), **c)** applied finite element mesh of brake drum

6. Simulation results for repeated brake applications on the road and braking on the roll-stand

It is known that maximum increase of temperature during repeated brake applications [14] can be much higher than during a single stop. In this case the heat energy dissipated in every brake application cumulates in the rotor material.

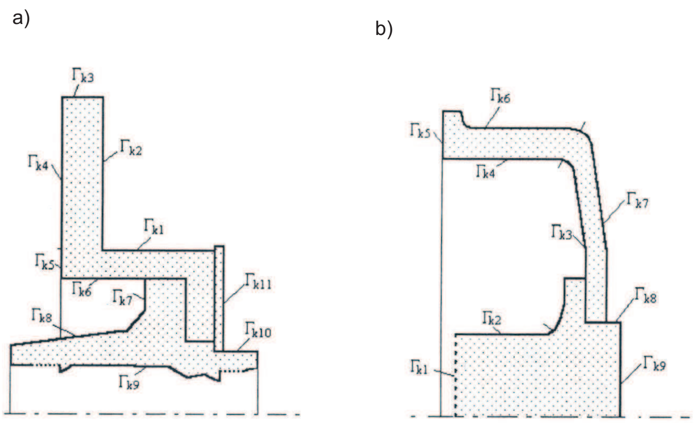


Fig. 7. Assumed surfaces of brake concerning heat transfer to ambient air in case of repeated brake applications: a) disc brake (Γ_{ki}, i=1,. . . ,11), b) drum brake (Γ_{ki}, i=1,. . . ,9)

In case of such a high thermal load of car brakes, the thermal capacity of additional parts connected to the brake rotor (the hub of disc brake and a part of axle for drum brake) should be taken into account. In addition the assumed surfaces

$\boldsymbol{\Gamma_k}$ of brake rotors (i.e. disc and drum brake) concerning heat transfer to ambient air are shown in Fig. 7.

The braking schedule [14] consisted of simulated 2.8 m/s^2 deceleration from 100 km/h to 50 km/h (the braking period 5 s), followed immediately by a 0.278 m/s^2 acceleration back to 100 km/h (the acceleration period 50 s). Computations of temperatures of front disc brake and rear drum brake were made for as many as 15 successive cycles.

The periodic fluctuations in temperature for friction surface of disc brake attained during the whole braking process on the road are shown in Fig. 8a. The maximum temperature of friction surface reaches approximately 600°C. The similar friction temperature of front disc brake can be observed for roll-stand braking (Fig. 8a) after $\boldsymbol{t_b}$ = 40 s using brake force $\boldsymbol{F_b}$ = 1000 N on the high-speed *Clayton* roll-stand.

In addition temperature fluctuations concerning the friction surface of rear drum brake during 15 repeated brake applications on the road are presented in Fig. 8b. The maximum temperature of friction surface reaches approximately 400°C, which is about 200°C lower than for the front disc brake (see Fig. 8a). Although more heat is dissipated in rear drum brakes than in front disc brakes, the thermal capacity of drum brakes is much higher than for disc brakes. The friction temperature of 400°C for the drum brake can be achieved on the roll-stand (Fig. 8b) after braking time $\boldsymbol{t_b}$ = 65 s ($\mathbf{v}_T$ = 62 km/h, $\boldsymbol{F_b}$ = 1000 N).

The temperature isolines in the cross-section of the front brake disc after 15 successive cycles of repeated braking (t = 775 s) are illustrated in Fig. 9a. For the comparison in Fig. 9b isothermal lines in the cross-section of the brake disc at the end of braking on the roll-stand are shown. This is an analogical result as the result presented in Figs. 5a and 5b, but the friction temperature is much higher. In addition the applied finite element mesh of the front brake disc and its hub is shown in Fig. 9c.

In both cases (i.e. braking on the road and on the roll-stand) the average temperature of the whole friction surfaces is 553°C (Figs. 9a and 9b), but the temperatures of the disc hub near wheel bearings are quite different (160°C for repeated brake applications on the road and 20°C for constant braking on the roll-stand).

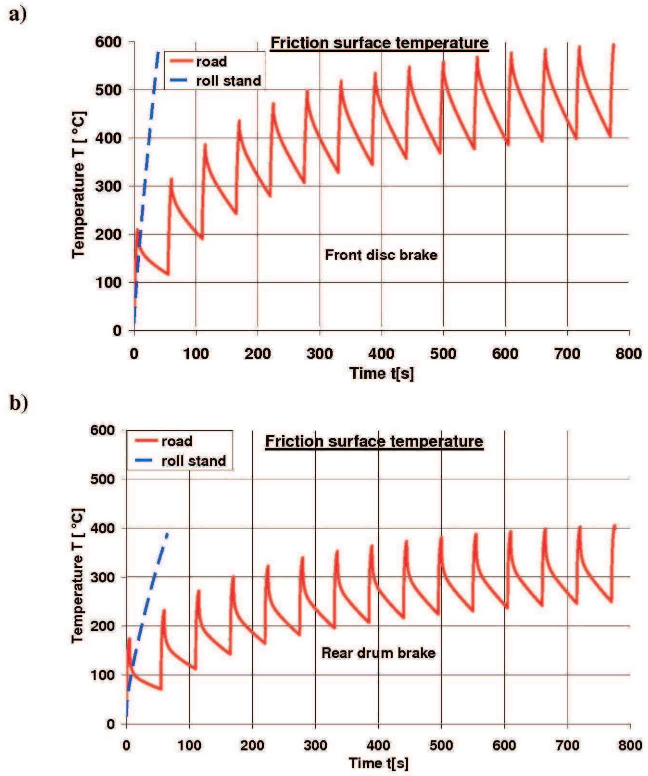


Fig. 8. Temperatures of the friction surfaces of car brakes during 15 repeated brake applications on the road (from $\mathbf{v}_1$ = 100 km/h to $\mathbf{v}_2$ = 50 km/h) and during constant braking on the *Clayton* roll-stand: test velocity $\mathbf{v}_T$ = 62 km/h, brake force $\boldsymbol{F}_b$=1000 N. Total braking time on the roll-stand: **a)** $\boldsymbol{t}_b$=40 s (for front disc brakes), **b)** $\boldsymbol{t}_b$=65 s (for rear drum brakes)

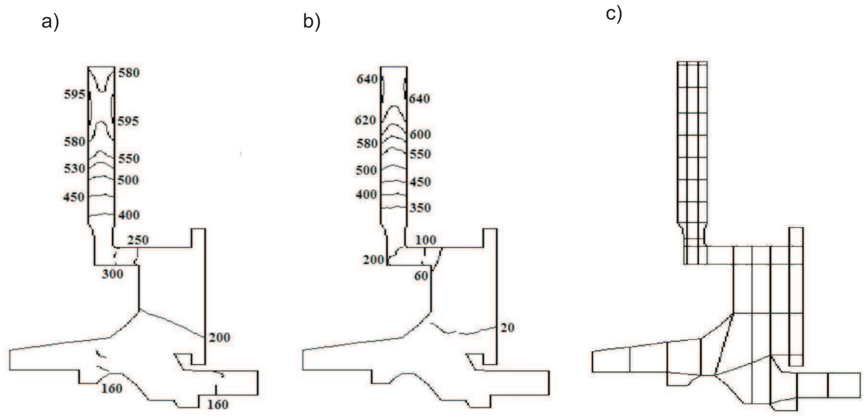


Fig. 9. Isothermal lines [°C] in the cross-section of front disc brake after: **a)** 15 repeated brake applications on the road (from $\mathbf{v}_1$ = 100 km/h to $\mathbf{v}_2$ = 50 km/h), **b)** braking on the *Clayton* roll-stand ($\boldsymbol{F}_b$ = 1000 N, $\mathbf{v}_T$ = 62 km/h, $\boldsymbol{t}_b$ = 40 s), **c)** applied finite element mesh of brake disc and hub

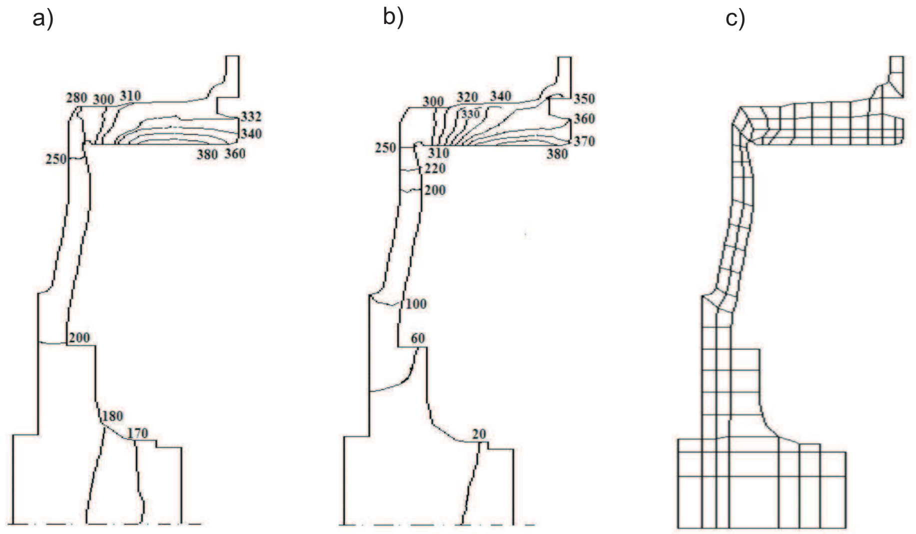


Fig. 10. Isothermal lines [°C] in the cross-sections of rear drum brake after: **a)** 15 repeated brake applications on the road (from $\mathbf{v}_1$ = 100 km/h to $\mathbf{v}_2$ = 50 km/h), **b)** braking on the *Clayton* roll-stand ($\boldsymbol{F}_b$ = 1000 N, $\mathbf{v}_T$ = 62 km/h, $\boldsymbol{t}_b$ = 65 s), **c)** applied finite element mesh of brake drum and a part of axle

Temperature isolines in the cross-section of the rear brake drum are presented in Figs. 10a and 10b. In both cases (i.e. braking on the road and on the roll-stand) the average temperature of friction surfaces is 379°C. For the drum brake the temperatures observed near wheel bearings are quite different (170°C in Fig. 10a and 20°C in Fig. 10b). In addition the applied finite element mesh of the rear brake drum and a part of axle is shown in Fig. 10c.

It should be noticed that road tests according [14] are much more dangers for the front and rear wheel bearings than shorter braking tests on the high-speed roll-stand.

The comparison of the simulation results for braking on the road and on the *Clayton* roll-stand (Figs. 9a, 9b and 10a, 10b) indicates the differences of qualitative character, although the *criterion II* is satisfied. Due to the long lasting schedule ($\boldsymbol{t}$ = 775 s) of repeated brake applications on the road and the resulted longer heat transfer time, the temperature distributions in materials of disc and drum brakes are more uniform than in the case of shorter braking on the roll-stand ($\boldsymbol{t}_b$ = 40 s for disc brakes and $\boldsymbol{t}_b$ = 65 s for drum brakes). In addition much higher temperatures near front and rear wheel bearings are observed in case of repeated brake applications on the road than after braking on the roll-stand.

For this reason a few successive cycles of repeated braking on the roll-stand could be planned using the proposed computational method.

7. Conclusions

The method presented above allows to achieve on a roll-stand the analogical thermal state of automotive brake, which is observed during braking in road conditions.

Basing on this method it is possible to determine for a high-speed roll-stand:
– braking time $\boldsymbol{t_b}$,
– brake force $\boldsymbol{F_b}$.

In addition the developed mathematical model and simulation program can be utilized for:
– analysis of heat transfer process in automotive brakes,
– evaluation or forecasting of the change in braking effectiveness due to the thermal state of brakes,
– designing of diagnostic tests for automotive brakes,
– optimization of brake design.

References

1. Brebbia C.A.: The Finite Element Method for Engineers. Pontech Press. Plymouth 1978.
2. Fensel P. A.: An axisymmetric finite element analysis of the mechanical and thermal stresses in brake drums, SAE Paper 740321, 1974.
3. Fukano A., Matsui H.: Development of Disc-Brake Method Using Computer Simulation of Heat Phenomena. SAE Paper 860634, 1986.
4. Garro A., Gavello E., Rossi F.: Thermal Transient Analysis of Disk Brake Problems Concerning a Mathematical Model. SAE Transactions 811328, 1981.
5. Hartter L., Schwartz H., Rhee S.: Evaluating copper alloy break discs by thermal modeling, SAE Transactions, Paper 740560, 1974.
6. Hinton E., Owen D.R.J.: Finite Element Programming. Academic Press, London, 1977.
7. Idogaki T., Kwai H., Harada H., Ioue H.: Measuring System of Transient Temperature Distribution on the Brake Disc Rotor. SAE Paper 870256, 1987.
8. Kays W., Bjorklund I.: Heat transfer from a rotating cylinder with and without crossflow, ASME Transactions, No 1, 1958.
9. Limpert R.: The thermal performance of automotive disc brakes, SAE Transactions, Paper 750873, 1975.
10. Lozia Z., Wolff A.: Thermal State of Automotive Brakes after Braking on the Road and on the Roll-Stand, SAE Special Publication No 1229 / SAE Technical Paper No 971040, p. 107-115, Detroit (USA), 1997.
11. Lozia Z., Wolff A.: Warunki odtwarzalności na stanowisku kontrolnym stanu termicznego hamulców występującego w czasie hamowania na drodze, Materiały międzynarodowej konferencji hamulcowej, Łódź, wrzesień 1991.
12. Newcomb T. P.: Temperatures reached in disc brakes, Jour. of Mech. Eng. Science, No 2, 1960.
13. Newcomb T. P.: Transient temperatures brake drums and linings, Proc. Auto. Div. Inst. Mech. Eng., No 7, 1958-59.
14. Regulation No 13 E/ECE/TRANS/ 505 of the United Nations, Economic Commission in Geneva.
15. Saumweber E.: Temperaturberechnung in Bremsscheiben fuer ein beliebiges Fahrprogramm, Leichtbau der Verkehrsfahrzeuge 13, Nr 3, 1969.
16. Staniszewski B.: Wymiana ciepła. Podstawy teoretyczne, PWN, Warszawa, 1980.

17. Timtner K.: Calculation of disc brake components using the finite element method with emphasis on weight reduction, SAE Transactions, Paper 790396, 1979.
18. Wiśniewski S.: Wymiana ciepła, WNT (wyd. 6), Warszawa, 2009.
19. Wolff A.: Teoretyczno-eksperymentalna analiza procesu wymiany ciepła w hamulcach samochodowych, (Theoretical and Experimental Analysis of the Heat Transfer Process in Automotive Brakes). Doctoral Thesis, Warsaw University of Technology, Faculty of Transport, Warsaw 1992 (in Polish).
20. Wolff A., Zagrodzki P.: Porównanie modeli symulacyjnych nieustalonej wymiany ciepła w hamulcach samochodowych, Materiały 5-tej Międzynarodowej Konferencji Naukowej: Simulation Research in Automobile Technology, Kazimierz Dolny, 22-24.05.1995.
21. Wolff A., Zagrodzki P.: Three-dimensional analysis of thermal processes in automotive brakes, Materiały Międzynarodowej Konferencji Naukowej pt. Inżynieria Systemów Transportowych, Wydział Transportu Politechniki Warszawskiej i Komitet Transportu Polskiej Akademii Nauk, Warszawa, 26-28.09.1995.
22. Zienkiewicz O.C., Morgan K.: Finite Elements and Approximation. John Wiley & Sons, Inc. New York, 1983.