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Calculation method of branch reactances in an electrical network using short-circuit current values

It is possible to determine branch reactances of a network when only short-circuit values are known. The paper presents the derivation of formulae for calculating branch reactances for symmetrical components. The results of short-circuit current flows of the first degree in all nodes (i.e. currents around each node) constitute a base for determining these reactances. The presented formulae do not take into consideration magnetic couplings occurring in parallel lines. Testing calculations prove that the branch reactances determined by means of such a method represent short-circuit currents with high accuracy. Such formulae have not been described so far.

1 Introduction

Distributions companies frequently possess only the results of short-circuit current calculations in nodes and short-circuit current flows in branches around subsequent nodes, i.e. flows of the first degree. They lack, however, a set of individual branch impedances for symmetrical components by means of which the short-circuit values have been calculated. In such a case the missing impedances may be determined by means of the formulae presented below using calculation results of short-circuit currents. Such formulae have not been described in literature so far [1–16].

2 Positive component reactance of the branch between the i-node and the j-node

In order to derive branch reactance formulae knowing short-circuit currents, the branch between nodes i and j will be analysed in two cases as shown in Fig. 1:

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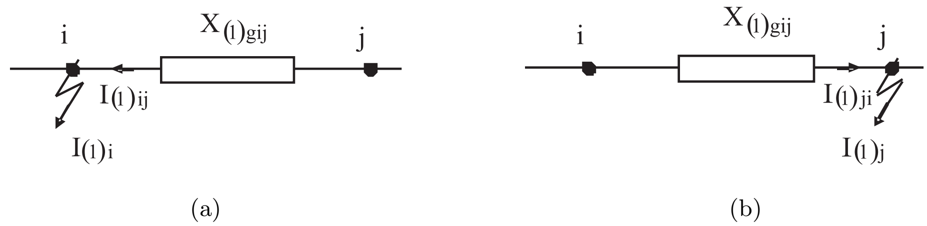

Figure 1. Branch equivalent circuit during a short circuit in the: (a) i-node, (b) j-node.

- during a short circuit in the i-node, Fig. 1(a);
- during a short circuit in the j-node, Fig. 1(b).

It is possible to derive the reactance $X_{(1)\,gij}$ for the positive component of the branch between the i-node and the j-node by analysing the voltage in the node without a short circuit. During a three-phase short circuit in the i-node, voltage in the j-node equals:

$$U_{(1)\,j} = E_{(1)} - X_{(1)\,ji} \cdot I_{(1)\,i} \tag{1}$$

or on the basis of Ohm's law:

$$U_{(1)\,j} = X_{(1)\,gij} \cdot I_{(1)\,ij} \ . \tag{2}$$

Using (1) and (2) the following is obtained:

$$E_{(1)} - X_{(1)\,ji} \cdot I_{(1)\,i} = X_{(1)\,gij} \cdot I_{(1)\,ij} \ , \tag{3}$$

where:

$E_{(1)}$	–	electromotive force,
$I_{(1)\,i}$	–	positive component of a three-phase short-circuit current in the i-node,
$I_{(1)\,ij}$	–	positive component of a three-phase short-circuit current flowing in the considered branch from the j-node to the i-node from the i-node side during a short circuit in the i-node,
$U_{(1)\,j}$	–	positive component of the j-node voltage during a three-phase short circuit in the i-node,
$X_{(1)\,ji}$	–	positive component of a mutual reactance between j- and i-nodes,
$X_{(1)\,gij}$	–	positive component of a branch reactance between j- and i-nodes.

During a three-phase short circuit in the j-node, voltage in the i-node equals:

$$U_{(1)\,i} = E_{(1)} - X_{(1)\,ij} \cdot I_{(1)\,j} = X_{(1)\,gij} \cdot I_{(1)\,ji} \,. \tag{4}$$

Assuming that $X_{(1)\,ji} = X_{(1)\,ij}$ and having eliminated the mutual reactance $X_{(1)\,ji}$ from (3) and (4), the reactance of the considered branch is obtained:

$$X_{(1)\,gij} = E_{(1)} \frac{I_{(1)\,i} - I_{(1)\,j}}{I_{(1)\,i} \cdot I_{(1)\,ji} - I_{(1)\,j} \cdot I_{(1)\,ij}} \,. \tag{5}$$

As can be seen from (5) it is possible to calculate the branch reactance between the nodes i and j, neither of which is the reference node, knowing the following values:

- three-phase short-circuit current in the i-node $I_{(1)\,i}$,
- three-phase short-circuit current in the j-node $I_{(1)\,j}$,
- three-phase short-circuit current flowing in the considered branch from the j-node to the i-node from the i-node side during a short circuit in the i-node $I_{(1)\,ij}$,
- three-phase short-circuit current flowing in the considered branch from the i-node to the j-node from the j-node side during a short circuit in the j-node $I_{(1)\,ji}$.

3 Positive component reactance of the branch between the *i*-node and the reference node

If one of the nodes in the analysed branch is a reference node (e.g. the j-node), the branch reactance can be calculated from (5), assuming that:

$$I_{(1)\,j} = \infty \,,$$

$$I_{(1)\,ji} \neq 0 \,.$$

Equation for the reactance of the branch between the i-node and the reference node will be the following:

$$X_{(1)\,gi0} = \frac{E_{(1)}}{I_{(1)\,i0}} \,, \tag{6}$$

where $I_{(1)\,i0}$ is positive component of a three-phase short-circuit current flowing in the considered branch from the reference node to the i-node during a short circuit in the i-node.

4 Zero component reactance of the branch between the *i*-node and the *j*-node

In order to derive the equation for the zero component reactance of the branch between the nodes i and j, a line-to-earth short circuit is analysed. During the line-to-earth short circuit in the i-node, voltage in the j-node equals:

$$U_{(0)\,j} = U_{(0)} + X_{(0)\,gji} \cdot I_{(0)\,ij} = -X_{(0)\,ii} \cdot I_{(0)\,i} + X_{(0)\,gji} \cdot I_{(0)\,ij} \tag{7}$$

or

$$U_{(0)\,j} = -X_{(0)\,ji} \cdot I_{(0)\,i}\ , \tag{8}$$

where:

- $X_{(0)\,ii}$ – zero component of the i-node self-reactance,
- $I_{(0)\,i}$ – zero component of the line-to-earth short-circuit current in the i-node,
- $I_{(0)\,ij}$ – zero component of a line-to-earth short-circuits current flowing in the considered branch from the j-node to the i-node from the i-node side during a short circuit in the i-node.

During a line-to-earth short circuit in the j-node, voltage in the i-node equals:

$$U_{(0)\,i} = -X_{(0)\,jj} \cdot I_{(0)\,j} + X_{(0)\,gji} \cdot I_{(0)\,ji} = -X_{(0)\,ij} \cdot I_{(0)\,j}\ . \tag{9}$$

Assuming that $X_{(0)\,ji} = X_{(0)\,ij}$ and having eliminated the mutual reactance, the reactance of the considered branch is obtained:

$$X_{(0)\,gij} = \frac{I_{(0)\,i} \cdot I_{(0)\,j} \cdot \left(X_{(0)\,jj} - X_{(0)\,ii}\right)}{I_{(0)\,i} \cdot I_{(0)\,ji} - I_{(0)\,j} \cdot I_{(0)\,ij}}\ . \tag{10}$$

Self-reactances of the i- or j-node for the zero component can be determined using line-to-earth (I_i^{1f}) and three-phase (I_i^{3f}) short-circuit currents in the i- or j-node:

$$X_{(0)\,ii} = E_{(1)} \frac{3 \cdot I_i^{3f} - 2 \cdot I_i^{1f}}{I_i^{3f} \cdot I_i^{1f}}\ . \tag{11}$$

The above equation has been derived assuming that all the impedances for positive and negative components are equal.

5 Zero component reactance of the branch between the *i*-node and the reference node

Zero component reactance of the branch between the i-node and the reference node has been derived by analysing the voltage in the i-node for the zero component:

$$X_{(0)\,gi0} = \frac{X_{(0)\,ii} \cdot I_{(0)\,i}}{I_{(0)\,i0}} \,. \tag{12}$$

The above formulae (5), (6), (10) and (12) do not take into consideration magnetic couplings occurring in parallel lines.

6 Method verification

The presented method will be verified by means of four test networks shown in Fig. 2. The verification algorithm is as follows:

1. Determining subsequent branch impedances of the equivalent circuit in the analysed network.
2. Calculation of short-circuit currents in all nodes and short-circuit current flows in branches around subsequent nodes.
3. Determining subsequent branch reactances using (5), (6), (10) and (12).
4. Calculation of short-circuit currents in all nodes and short-circuit current flows in branches around subsequent nodes using the previously determined reactances.
5. Cetermining errors in the short-circuit values obtained in steps 2 and 4.

As a result of the conducted analysis it may be stated that errors in determining short-circuit values equal zero in:

- networks without magnetically coupled branches, Fig. 2(a),
- networks with magnetically coupled branches operating with busbars connected at the sending and receiving end as shown in Fig. 2(b),
- networks with magnetically coupled branches operating with busbars not connected at the sending and/or receiving end presented in Fig. 2(c) and 2(d) if branch currents in the equivalent circuit shown in Fig. 3 are known.

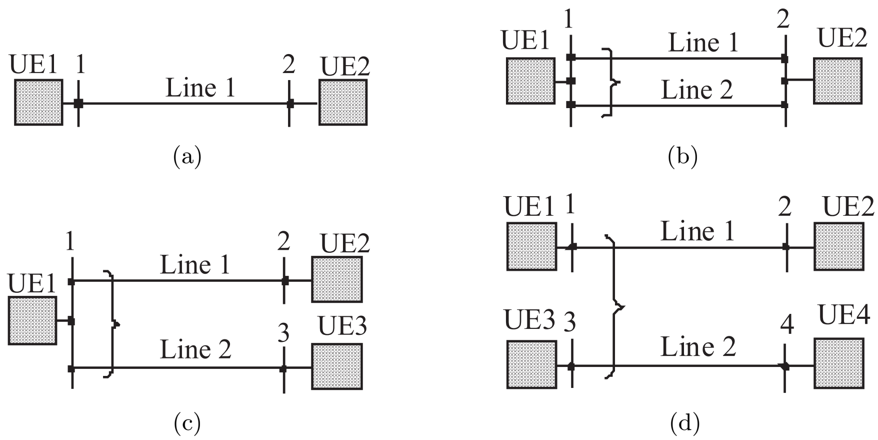

Figure 2. Test networks, symbol "}" signifies branch magnetic coupling.

For networks with magnetically coupled branches operating with busbars not connected at the sending and/or receiving end as shown in Fig. 2(c) or Fig. 2(d), where only currents in network branches are known, errors in determining short--circuit currents are less than 0.2%. As an example, results for the network from Fig. 2(d) are presented in Tab. 1.

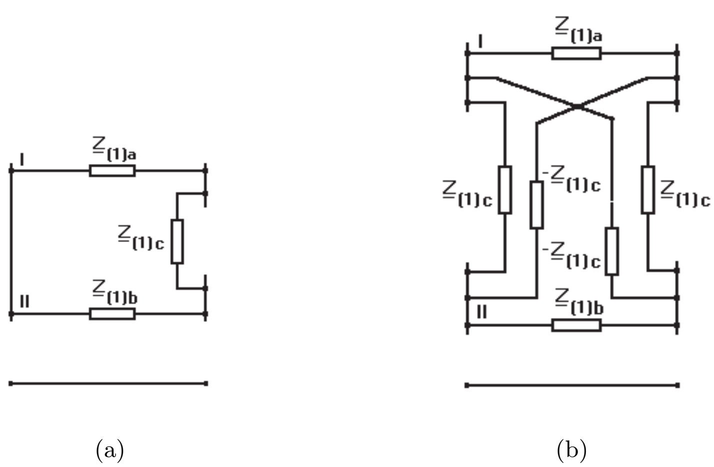

Figure 3. Equivalent circuit of a magnetically coupled line: (a) the line from Fig. 2(c); (b) the line from Fig. 2(d), where $\underline{Z}_{(1)\,c}$ signifies branch magnetic coupling.

In the above networks (modelled by means of branch reactances and calculated using the formulae presented in this paper) during a short circuit in one magnetically coupled line, voltage distortions do not occur in the other coupled line. This phenomenon is the only drawback of the method used here because in real networks voltage distortions occur in both magnetically coupled lines.

Table 1. Errors (%) of short-circuit current calculation for the network from Fig. 2(d) resulting from using the method presented in this paper.

Calculation point	Short-circuit in node 1		Calculation point	Short-circuit in node 2	
	Line-to-earth	Three-phase		Line-to-earth	Three-phase
Short-circuit place current	-0.019	-0.013	Short-circuit place current	-0.066	-0.040
Line 1 current	0.14	0.069	Line 1 current	0.11	0.040
UE1 current	0	0	UE2 current	-0.0030	0
Calculation point	Short-circuit in node 3		Calculation point	Short-circuit in node 4	
	Line-to-earth	Three-phase		Line-to-earth	Three-phase
Short-circuit place current	-0.033	-0.023	Short-circuit place current	-0.043	-0.030
Line 2 current	0.18	0.052	Line 2 current	0.16	0.045
UE3 current	-0.0011	0	UE4 current	-0.00088	0

7 Conclusions

The paper presents formulae based on the results of short-circuit current flows of the first degree in all nodes. The formulae are used to calculate branch reactances for symmetrical components and were derived assuming that:

- Resistances of transmission elements are negligible, which is true for a transmission system but does not have to be the case for a distribution system.
- Short-circuit calculations made for metallic faults serve as a basis for estimating branch reactances.
- All impedances for the positive and negative components are equal.
- Magnetic couplings occurring in parallel lines are not taken into account.

Using branch reactances calculated by means of the presented formulae, it could be stated that:

- Errors of short-circuit current calculation in any node or branch may be disregarded, even if the analysed network comprises magnetically coupled lines.
- Voltage is determined successfully except the following cases: when a short circuit occurs in one magnetically coupled line and the zero component voltage or the voltage containing this component in the other magnetically coupled line is needed. These cases, however, can be disregarded in practical calculations.

Both of the above conclusions prove the applicability of the presented formulae to branch reactance calculations. After calculating the network branch reactance by means of the presented method, arc and resistance faults can be obtained without having to introduce any simplifying assumptions.

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Metoda obliczania reaktancji gałęzi sieci elektroenergetycznej w oparciu o wartości prądów zwarciowych

S t r e s z c z e n i e

W sytuacji, gdy znamy jedynie wyniki obliczeń prądów zwarciowych można, wyznaczyć reaktancje gałęzi rozpatrywanej sieci. Referat przedstawia wyprowadzone wzory na obliczenia reaktancji gałęzi dla składowych symetrycznych. Wyniki rozpływów pierwszego stopnia prądu zwarciowego we wszystkich węzłach (tzn. prądy wokół każdego z węzłów) są podstawą do wyprowadzenia tych reaktancji. Prezentowane wzory nie uwzględniają sprzężeń magnetycznych, występujących w liniach równoległych. Obliczenia sprawdzające potwierdzają, że tak wyznaczone reaktancje gałęzi z dużą dokładnością odwzorowują prądy zwarciowe w rozpatrywanej sieci elektroenergetycznej. W znanej dotychczas literaturze brak jest tego typu wzorów.