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Optical fixing the positions of the off-shore objects applying the method of two reference points

Krzysztof Naus, Dariusz Szulc

Institute of Navigation and Maritime Hydrography
Polish Naval Academy
69 Smidowicza St., 81-103 Gdynia, Poland
e-mail: k.naus@amw.gdynia.pl; d.szulc@amw.gdynia.pl

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Abstract: The Paper presents the optical method of fixing the off-shore objects positions from the land. The method is based on application of two reference points, having the geographical coordinates defined. The first point was situated high on the sea shore, where also the camera was installed. The second point was intended for use to determine the topocentric horizon plane and it was situated at the water-level.

The first section of the Paper contains the definition of space and disposed therein reference systems: connected with the Earth, water-level and the camera system.

The second section of the Paper provides a description of the survey system model and the principles of the Charge Coupled Device – CCD array pixel's coordinates (plate coordinates) transformation into the geographic coordinates located on the water-level.

In the final section there are presented the general rules of using the worked out method in the optical system.

Key words: optical navigation, fixing position, land-based positioning

1. Introduction

Land-based monitoring positions of waterborne objects at sea (ships, yachts, buoys, beacons etc.) is one of the main tasks which are carried out by harbour boards, navies and border guards in a majority of states in the world. At present the task is usually performed using the radar systems. The radar systems offer many advantages, including obtaining immediate readings of the real values of bearings and distances to the objects under observation, with their motion elements determined as well. Unfortunately, that information is biased with errors, which consequently keep introducing the particular errors into the process of fixing the observed object positions. Moreover, in some specific situations, a use of radar electromagnetic radiation may be unreasonable. Taking into consideration intentions to improve accuracy of positioning the objects at sea and the need of secrecy of tracing thereof, introducing the optical methods of localization of the ships travelling at sea, within the traffic zones which remain under monitoring, seems advisable and valid.

Thus, in connection with the above conclusion, the Paper presents mathematical foundations of the optical method of land-based fixing the positions of waterborne objects at sea applying two reference points. The mathematical description of the method was based on the vector calculus. The Authors have characterized in this Paper neither the survey resolution analyses nor the coordinates determination error. They will be analyzed in details after the investigations in reality are carried out. The results are intended to be presented in the other paper.

2. Definition of the space and reference systems

Let E^3 be the three-dimensional Euclidean space over a domain of the real numbers R, E^3 associated with the Euclidean space, **O** point E^3, and $(\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3})$ the orthonormal basis of E^3. The following system is to be named the orthocartesian reference system (global reference mark) (Kryński and Rogowski, 2004) of E^3 space:

$$\Im = \{\mathbf{O}, (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)\}. \tag{1}$$

The point **O** is to define its origin (or the base point), and $(\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3})$ the base of $\Im$ system.

Let $\Im$ be the settled reference system of the space E^3. A set of numbers (x, y, z) defines the orthocartesian coordinates of the point **P** in relation to the reference system $\Im$, and:

$$\mathbf{P} = \overrightarrow{\mathbf{OP}} = x\,\mathbf{e}_1 + y\,\mathbf{e}_2 + z\,\mathbf{e}_3, \tag{2}$$

the vector of position **P** in relation to $\Im$.

Let $\Im^{\mathbf{G}} = \{\mathbf{O}^{\mathrm{G}}, (\mathbf{e}_1^{\mathrm{G}}, \mathbf{e}_2^{\mathrm{G}}, \mathbf{e}_3^{\mathrm{G}})\}$, $\Im^{\mathbf{H}} = \{\mathbf{O}^{\mathrm{H}}, (\mathbf{e}_1^{\mathrm{H}}, \mathbf{e}_2^{\mathrm{H}}, \mathbf{e}_3^{\mathrm{H}})\}$ and $\Im^{\mathrm{C}} = \{\mathbf{O}^{\mathrm{C}}, (\mathbf{e}_1^{\mathrm{C}}, \mathbf{e}_2^{\mathrm{C}}, \mathbf{e}_3^{\mathrm{C}})\}$ be three orthocartesian reference systems of the space E^3.

The reference system $\Im^{\mathbf{G}}$ (hereinafter referred to as the geocentric one) (Schwarz and Kryński, 1991) is connected with the Earth in a way to enable its versors $\mathbf{e}_1^{\mathrm{G}}, \mathbf{e}_2^{\mathrm{G}}, \mathbf{e}_3^{\mathrm{G}}$ to determine respectably the axes $\mathbf{O^GX^G}$, $\mathbf{O^GY^G}$, $\mathbf{O^GZ^G}$ of this system. The $\mathbf{O^GZ^G}$ axis would be in a line with the Earth rotation axis. The other axes ($\mathbf{O^GX^G}$ and $\mathbf{O^GY^G}$) would be laid out in the equator plane, whereat $\mathbf{O^GX^G}$ axis would be laid in 0º meridian plane and $\mathbf{O^GY^G}$ axis in 90ºE meridian plane (also known as earth-centered earth-fixedas coordinate system) (Felski, 1991).

The reference system $\Im^{\mathbf{H}}$ (hereinafter referred to as the horizontal topocentric reference system (Rogowski and Figurski, 2004; Czarnecki, 1996)) would be bounded with the point $\mathbf{P_H}$ (positioned at the water level), and its origin and basis would be defined in relation to the reference system $\Im^{\mathbf{G}}$.

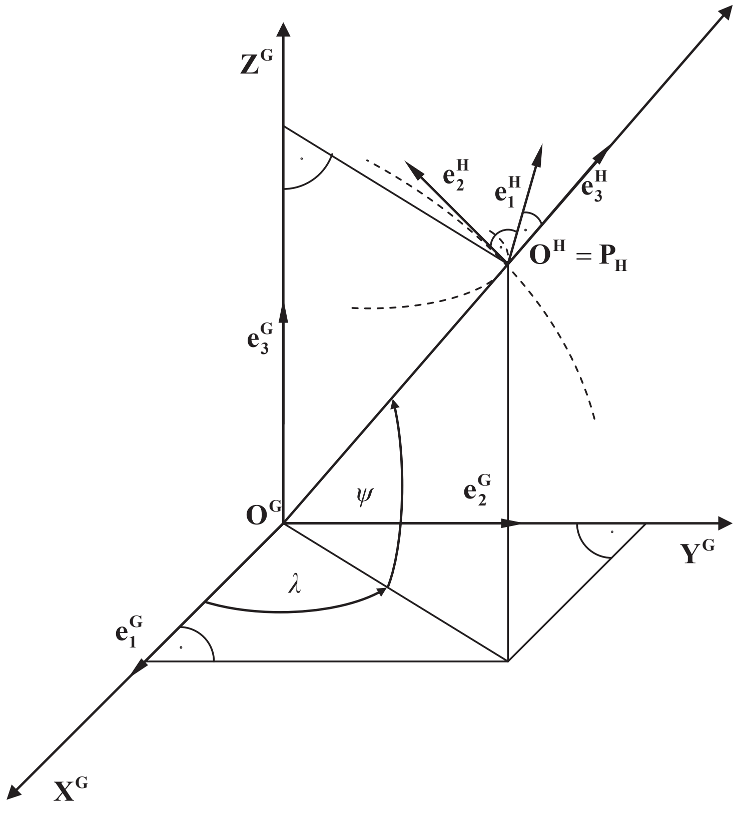

Fig. 1. Arrangement of the reference systems $\Im^{\mathbf{H}}$ and $\Im^{\mathbf{G}}$ of the space E^3 in relation to each other

The horizontal topocentric reference system $\Im^{\mathbf{H}}$ would be obtained in effect of transformation $\Im^{\mathbf{G}} = \left\{\mathbf{O}^{\mathbf{G}}, \left(\mathbf{e}_1^{\mathbf{G}}, \mathbf{e}_2^{\mathbf{G}}, \mathbf{e}_3^{\mathbf{G}}\right)\right\}$. This transformation would be a composition of the translation $\left\{\mathbf{O}^{\mathbf{H}}, \left(\mathbf{e}_1^{\mathbf{H}}, \mathbf{e}_2^{\mathbf{H}}, \mathbf{e}_3^{\mathbf{H}}\right)\right\}$ in relation to $\Im^{\mathbf{G}}$ and rotation of $\Im^{\mathbf{H}}$ with the centre $\mathbf{O}^{\mathbf{H}} = \mathbf{P}_{\mathbf{H}}$ in relation to the system $\left\{\mathbf{O}^{\mathbf{H}}, \left(\mathbf{e}_1^{\mathbf{G}}, \mathbf{e}_2^{\mathbf{G}}, \mathbf{e}_3^{\mathbf{G}}\right)\right\}$. It would be defined with the translation vector $\xi = \overrightarrow{\mathbf{O}^{\mathbf{G}}\mathbf{O}^{\mathbf{H}}}$ of the point $\mathbf{O}^{\mathbf{G}}$ to the point $\mathbf{O}^{\mathbf{H}}$ and rotation of the basis $\left(\mathbf{e}_1^{\mathbf{H}}, \mathbf{e}_2^{\mathbf{H}}, \mathbf{e}_3^{\mathbf{H}}\right)$ relatively to the basis $\left(\mathbf{e}_1^{\mathbf{G}}, \mathbf{e}_2^{\mathbf{G}}, \mathbf{e}_3^{\mathbf{G}}\right)$ performed separately for each versor in accordance to the following dependences (Naus, 2014):

$$\mathbf{e}_1^{\mathbf{H}} = -\sin\lambda\,\mathbf{e}_1^{\mathbf{G}} + \cos\lambda\,\mathbf{e}_2^{\mathbf{G}}, \tag{3}$$

$$\mathbf{e}_2^{\mathbf{H}} = -\sin\psi\cos\lambda\,\mathbf{e}_1^{G} - \sin\psi\sin\lambda\,\mathbf{e}_2^{G} + \cos\psi\,\mathbf{e}_3^{G}, \tag{4}$$

$$\mathbf{e}_3^{\mathbf{H}} = \cos\psi\cos\lambda\,\mathbf{e}_1^{G} + \cos\psi\sin\lambda\,\mathbf{e}_2^{G} + \sin\psi\,\mathbf{e}_3^{G}, \tag{5}$$

where:
ψ – geocentric latitude of the point $\mathbf{P_H}$,
λ – geographical longitude of the point $\mathbf{P_H}$.

The reference system $\Im^{\mathbf{C}}$ (hereinafter referred to as the camera reference system) would be joined with the camera rigidly and its origin and basis would be defined in relation to the reference system $\Im^{\mathbf{G}}$.

The direction of the versor $\mathbf{e}_3^{\mathbf{C}}$ would remain in one line with the optical axis of the camera. The other two versors ($\mathbf{e}_1^{\mathbf{C}}$ and $\mathbf{e}_2^{\mathbf{C}}$) would be laid in a plane parallel to the plane of the CCD array and intersecting the focusing point (projection centre) $\mathbf{P_f} = \mathbf{O^C}$. When facing $\mathbf{e}_3^{\mathbf{C}}$ direction, the versor $\mathbf{e}_2^{\mathbf{C}}$ would be directed left while the versor $\mathbf{e}_1^{\mathbf{C}}$ upwards. The versor $\mathbf{e}_2^{\mathbf{C}}$ would be laid out at a half of the array height, and the versor $\mathbf{e}_1^{\mathbf{C}}$ at a half of its width.

Fig. 2. The way of bonding the reference system $\Im^{\mathbf{C}}$ and the camera

3. Survey system model

Let the ellipsoidal coordinates (φ, λ, H) of the points $\mathbf{P_C}$ and $\mathbf{P_H}$ be known (measured with high accuracy using, for example, ASG-EUPOS system). The above coordinates are to be transformed to the orthocartesian ones,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \left(\dfrac{a}{\sqrt{1-e^2 \cdot sin^2\varphi}} + H \right) \cdot \cos\varphi \cdot \cos\lambda \\ \left(\dfrac{a}{\sqrt{1-e^2 \cdot sin^2\varphi}} + H \right) \cdot \cos\varphi \cdot \sin\lambda \\ \left[\dfrac{a}{\sqrt{1-e^2 \cdot sin^2\varphi}} \cdot \left(1-e^2\right) + H \right] \cdot \sin\varphi \end{bmatrix}, \tag{6}$$

describing layout of the points in relation to $\Im^{\mathbf{G}}$ (where a and b are the lengths of semimajor and semiminor axes of the reference ellipsoid, $e^2 = \dfrac{a^2 - b^2}{a^2}$ is the first eccentric's square).

In addition, the obtained orthocartesian coordinates of $\mathbf{P_H}$ would be transformed to the spherical ones,

$$\begin{bmatrix} \psi \\ \lambda \end{bmatrix} = \begin{bmatrix} arctg\left(\dfrac{z}{\sqrt{x^2 + y^2}} \right) \\ \lambda \end{bmatrix}, \tag{7}$$

further used for determination of the basis the reference system $\Im^{\mathbf{H}}$ (in conformity with the dependencies: 3, 4 and 5).

The point $\mathbf{P_C}$ would be situated on-shore (high on the shore), and point $\mathbf{P_H}$ at the water-level (Fig. 3).

Point $\mathbf{P_H}$ determines centre $\Im^{\mathbf{H}}$, whereas $\mathbf{P_C}$ a place of the camera installation (more precisely the camera projection centre). The versors $\mathbf{e_1^H}, \mathbf{e_2^H}$ of the system $\Im^{\mathbf{H}}$ determine a plane Π of the so-called topocentric horizon. The plane Π represents a part of the sea surface which can be observed from the point $\mathbf{P_C}$ using the camera.

Let us know a direction of the ray, projecting a singular point (pixel) onto the CCD camera array. The direction is defined by the normed (unit) vector **r**; its position is determined in relation to $\Im^{\mathbf{G}}$.

Applying the scalar product of the vectors $\overrightarrow{\mathbf{P_H P_C}}$ and **r** with the versor $\mathbf{e_3^H}$ of the basis $\Im^{\mathbf{H}}$ (which is at the same time the vector normal to Π) it is easy to calculate the following heights:

$$H = \overrightarrow{\mathbf{P_H P_C}} \circ \mathbf{e_3^H}, \tag{8}$$

$$h = \mathbf{r} \circ \mathbf{e_3^H}. \tag{9}$$

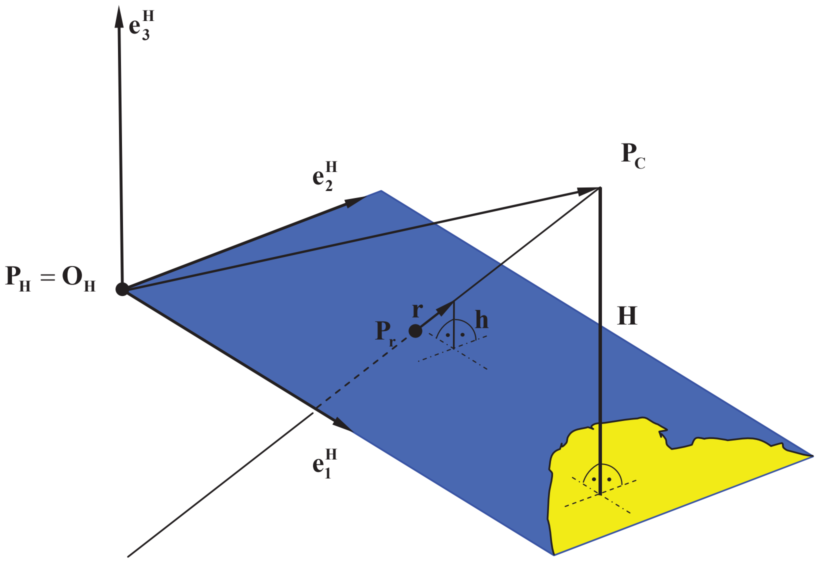

Fig. 3. Survey system model

Then, basing thereon (using the proportion condition), a position of the point of intersection $\mathbf{P_r}$ of the projecting ray and the plane Π in relation to $\Im^{\mathbf{G}}$, accordance to the dependence:

$$\mathbf{P_r} = \mathbf{P}_C - \frac{H}{h}\mathbf{r}\,. \tag{10}$$

This way obtained orthocartesian coordinates of the point $\mathbf{P_r}$ can be transformed to the ellipsoidal ones $(\varphi, \lambda,)$ applying the non-iterative Bowring's method (quick computing one) (Bowring, 1976):

$$\varphi = arctg\frac{z + \dfrac{a^2 - b^2}{b^2} b \sin^3\left(arctg\left(\dfrac{az}{b\sqrt{x^2+y^2}}\right)\right)}{\sqrt{x^2+y^2} - e^2 a \cos^3\left(arctg\left(\dfrac{az}{b\sqrt{x^2+y^2}}\right)\right)}, \tag{11}$$

$$\lambda = arctg\left(\frac{y}{x}\right), \tag{12}$$

allowing to determine finally the position of a waterborne object at sea, indicated as a pixel on the camera array.

4. Determination of the projecting ray direction

The projecting ray direction is originally determined applying the unit vector $\mathbf{r}^{\mathbf{C}}$, which location (position) is determined in relation to the camera reference system $\Im^{C} = \left\{\mathbf{O}^{C}, \left(\mathbf{e}_1^{C}, \mathbf{e}_2^{C}, \mathbf{e}_3^{C}\right)\right\}$. Then the vector $\mathbf{r}^{\mathbf{C}}$ is determined basing on the coordinates of two points, intersected by every projecting ray. The fixed focusing point $\mathbf{P_f}$ and the variable point $\mathbf{P'_f}$ are on the CCD array (Fig. 4.).

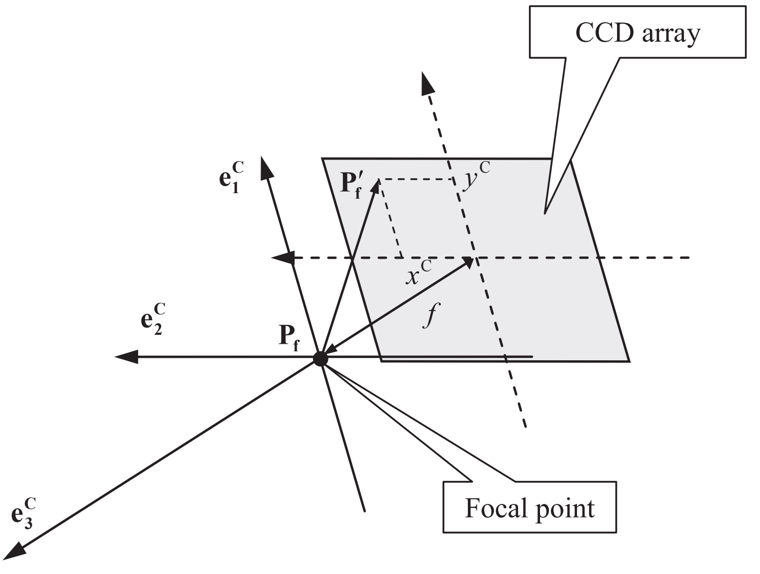

Fig. 4. Arrangement of the points $\mathbf{P_f}$ and $\mathbf{P'_f}$ which define the direction of the projecting ray in relation to $\Im^{\mathbf{C}}$ and the CCD array

To calculate the vector $\mathbf{r}^{\mathbf{C}}$ there is applied the following dependence:

$$\mathbf{r}^{C} = \frac{\overrightarrow{\mathbf{P_f P'_f}}}{\left|\overrightarrow{\mathbf{P_f P'_f}}\right|}. \tag{13}$$

This way obtained vector $\mathbf{r}^{\mathbf{C}}$ is secondarily transformed from the camera reference system $\Im^{\mathbf{C}}$ to the system $\Im^{\mathbf{G}}$. Anyhow, before it is done, it is necessary to determine the position of the camera reference system $\Im^{\mathbf{C}}$ in the locations space (places):

$$E^{3}_{\Im^{G}} = \left\{\mathbf{X} \in \mathrm{E},\ \mathbf{X} = \mathbf{O}^{G} + x\,\mathbf{e}_1^{G} + y\,\mathbf{e}_2^{G} + z\,\mathbf{e}_3^{G}, (x, y, z) \in \mathrm{R}^{3}\right\}. \tag{14}$$

To this end there is to be used a pair ($\mathbf{n} = \overrightarrow{\mathbf{P_C P_H}}$, $\mathbf{k} = \mathbf{e}_3^{\mathbf{H}}$) of non co-linear (especially the orthonormal) vectors and the point $\mathbf{P_C}$.

The versors of the basis of the system $\mathfrak{I}^C$ are to be obtained applying the dependence:

$$\mathbf{e}_3^C = \frac{\mathbf{n}}{|\mathbf{n}|}, \tag{15}$$

$$\mathbf{e}_1^C = \frac{\mathbf{k} \times \mathbf{n}}{|\mathbf{k} \times \mathbf{n}|}, \tag{16}$$

$$\mathbf{e}_2^C = \mathbf{e}_3^C \times \mathbf{e}_1^C. \tag{17}$$

An origin of the system $\mathbf{O}^C$ is corresponding to the point $\mathbf{P}_C$.

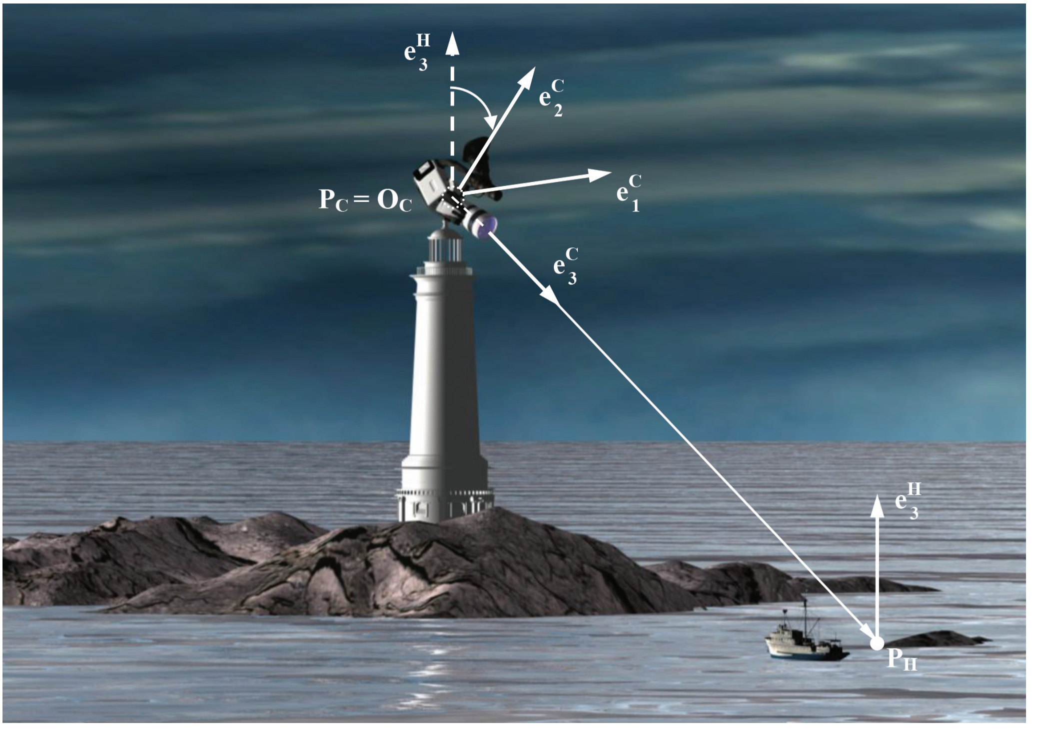

Fig. 5. Arrangement of $\mathfrak{I}^C$ in relation to the reference points $\mathbf{P}_C$ and $\mathbf{P}_H$ and the versor $\mathbf{e}_3^H$ of the basis $\mathfrak{I}^H$

Transformation from the vector $\mathbf{r}^C$ into the vector $\mathbf{r}$, comprising rotation and translation (displacement), is carried out applying the following dependence:

$$\mathbf{r} = \mathbf{r}^C \mathbf{R} - \mathbf{T}, \tag{18}$$

where:

$$\mathbf{R} = \begin{bmatrix} \mathbf{e_1^C} \\ \mathbf{e_2^C} \\ \mathbf{e_3^C} \end{bmatrix} = \begin{bmatrix} x_1^C & y_1^C & z_1^C \\ x_2^C & y_2^C & z_2^C \\ x_3^C & y_3^C & z_3^C \end{bmatrix}, \tag{19}$$

$$\mathbf{T} = \mathbf{P_C} = \begin{bmatrix} x_0^C \\ y_0^C \\ z_0^C \end{bmatrix}. \tag{20}$$

5. Conclusions

The described land-based method of fixing positions of waterborne objects at sea can be in future applied in any autonomous optical systems or the radar supporting systems as well as the Automatic Identification System (for example – in conditions where a possibility of using them is excluded).

It is simple to use upon construction of the optical system for any water area at sea. There is only required determination of coordinates of the two reference points and carrying out simple calibration of the camera optical system in relation to the above mentioned points (it comprises positioning the camera vertically and directing its optical axis toward the point $\mathbf{P_H}$).

However, a selection of the reference points is to depend on their positions in relation to each other. It may considerably affect precision of the survey (resolution of the survey). The point $\mathbf{P_C}$ should be situated possibly highest on the shore, whereas the point $\mathbf{P_H}$ at the possibly shortest distance from the point $\mathbf{P_C}$, measured within the topocentric horizon plane. With such configuration, the angles of incidence of the projection rays at the topocentric horizon plane are non acute. Thus, a singular pixel on the array imitates a smaller area (section) of the sea.

Moreover, on implementing the above method, it is necessary to take into consideration another factors connected with the optical system representation errors, which may negatively affect accuracy of fixing positions coordinates. A picture recorded with the camera may be geometrically distorted; its distortion depends on a range of:

– radial distortion coefficient and tangential distortion coefficient of the camera lens,
– afinism and non orthogonality of the coordinates system at the CCD array.

Therefore it is advisable to make corrections of the picture using the matrix of intrinsic parameters (also known as camera matrix) yet before applying the worked out method of fixing the waterborne objects at sea. Such operation should considerably improve accuracy of fixing the positions coordinates.

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Optyczne wyznaczanie pozycji obiektów przybrzeżnych z zastosowaniem metody dwóch punktów odniesienia

Krzysztof Naus, Dariusz Szulc

Instytut Nawigacji i Hydrografii Morskiej
Akademia Marynarki Wojennej
ul. Śmidowicza 69, 81-103 Gdynia
e-mail: k.naus@amw.gdynia.pl; d.szulc@amw.gdynia.pl

Streszczenie

W artykule przedstawiono optyczną metodę wyznaczania pozycji obiektów nawodnych z lądu. Oparto ją na dwóch punktach odniesienia o znanych współrzędnych geograficznych. Pierwszy umiejscowiono wysoko na brzegu morza i przeznaczono do zamontowania kamery. Drugi przeznaczono do określania płaszczyzny horyzontu topocentrycznego i umiejscowiono na poziomie lustra wody.

W pierwszej części artykułu zdefiniowano przestrzeń i rozmieszczone w niej układy odniesienia: związany z Ziemią, poziomem lustra wody i kamerą. Drugą część artykułu stanowi opis modelu układu pomiarowego oraz zasad transformacji współrzędnych piksela (tłowych) z matrycy CCD na współrzędne geograficzne punktu umiejscowionego na poziomie lustra wody. W części końcowej zaprezentowano ogólne zasady wykorzystywania opracowanej metody w systemie optycznym.